

LOGARITHMOLOGIA:  
OR THE WHOLE  
DOCTRINE  
OF  
LOGARITHMS,  
*Common and Logistical,*  
IN  
THEORY and PRACTICE.  


---

IN THREE PARTS.

PART. I. The THEORY of LOGARITHMS;  
Shewing their Nature, Origin, Construction, and Properties,  
demonstrated in various Methods, *viz.* 1. By Plain Arithme-  
tic. 2. By the Logarithmic Curve. 3. By Dr. HALLEY'S  
Infinite Series. 4. By Fluxions. 5. By the Properties of the  
Hyperbola. 6. By the Equiangular Spiral. 7. By a Loga-  
rithmic inspectional Scale of twenty-two Inches length. With  
the Construction of the artificial Lines of Numbers, Sines,  
and Tangents. Also the Nature and Construction of Logistical  
Logarithms. The whole illustrated and made easy by many  
and suitable Examples.

PART II. The PRAXIS of LOGARITHMS;  
Wherein all the Rules and Operations of Logarithmical Arith-  
metic, both Common and Logistical, by Numbers and Instru-  
ments, are copiously exemplified. Together with the Ap-  
plication thereof to the several Branches of Mathematical  
Learning.

PART III. A Three-fold CANON of LOGARITHMS;  
In a new and more compendious Method than any extant;  
*Viz.* { 1. A Canon of Logarithms of NATURAL NUMBERS.  
2. A Canon of Logarithms of SINES and TANGENTS.  
3. A Table of LOGISTICAL LOGARITHMS.

The whole being a *Compleat System* of this most useful Art;  
and enrich'd with all the Improvements therein from its Ori-  
ginal to the Present Time.

By BENJAMIN MARTIN,  
Author of the *Philological Library of Literary Arts and  
Sciences, &c.*

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T H E  
P R E F A C E.

**T**HO' Logarithms *may justly be esteem'd* the principal Invention of modern Ages, on account of their excellent and most extensive Use in Mathematical Literature, yet it may with equal Truth be said, that little more is known of them, generally speaking, than their practical Use in some Rules of common Arithmetic and trigonometrical Calculations; (and how few are perfect in this!) For (saith the great Improver of this Art, the learned Dr. Halley) " I find very few of  
 " those, who make constant use of Logarithms,  
 " have attain'd an adequate Notion of them;  
 " to know how to make or examine them, or  
 " to understand the Extent of the Use of  
 " them: contenting themselves with the Ta-  
 " bles of them, as they find them, without  
 " daring to question them, or caring to know  
 " how to rectify them, should they be found  
 " amiss; being, I suppose, under the Appre-  
 " hension of some great Difficulty therein, &c."

*For the sake of such Persons the following Treatise is principally intended; wherein they will find every thing necessary relating to the Theory and Practice of this admirable Art.*

*For, as to the Theory, (the principal Part, and so very rarely known) I have exhibited all the Methods, whereby it has been taught and explain'd by the Inventor, and several Improvers thereof, since his time; as by the Extraction of Roots; by the Logarithmic Curve from Dr. Keil; by an Infinite Series, from Dr. Halley; by the Method of Fluxions, from Mr. Ditton; by the Hyperbola, from Mr. Domky; by the Equiangular Spiral, from Halley, Wallis, &c. by the large Logarithmic Scale, of my own constructing: the like to which, for Largeness, was never before published; for a compleat Account of which see Chap. X. of the Theory.*

*I say, by all these various Methods I have endeavour'd to explain, illustrate, and facilitate the Knowledge of the Nature, Properties, and Construction of those excellent Numbers, called Logarithms. I have also exemplified the Manner of making Logarithms for the prime Numbers, by many and different Examples, and in several ways; and have taken all possible Care to render this most abstruse and difficult Part, as easy and intelligible, as the Nature of the Subject will admit.*

*Having*



## P R E F A C E.



*Having thus explain'd the Nature of Logarithms, I then shew how they are laid on Instruments, and thereby the Construction of the Artificial Lines of Numbers, Sines, and Tangents; first contrived by the famous Geometer, Mr. Gunter of Gresham-College; and for that reason they are still called Gunter's Line, and all together Gunter's Scale.*

*Lastly, I have largely explain'd the Nature, and shewn the Manner of making or constructing the Logistical Logarithms, according to Shakerley's and Street's Form thereof; and which I have not seen done by any other Hand.*

*These things together will, I hope, be allow'd to make a regular, universal, and compleat Theory of Logarithms, common and logistical, for Integers and Fractions, numerical and instrumental; and such, as for Brevity, yet Copiousness and Variety, has not been before extant.*

*As touching the Praxis, or Use of Logarithms, which makes the second Part of this Work, I have made it as compleat and perfect as possible; having illustrated all the Rules of Practice with all the Variety of Examples I could devise, that were necessary. And that none may be unappriz'd of the most extensive Service of Logarithms in the Mathematical Disciplines, I have applied them to the*  
Arithmetic

Arithmetic of all kinds of Numbers ; to Trigonometry, in the Solution of all Cases of Plain and Spherical Triangles ; to Mercator's Sailing particularly, showing how all its Cases may be resolv'd solely by the Canon of Logarithmic Tangents ; to the Mensuration of Superficies and Solids, &c. All which are fundamental Operations, and may each of them be extended or branched out into particular Sciences ; but that would have been too tedious a Task, and not absolutely necessary to my Design. I have therefore only applied the Doctrine of Mensuration to the Arts of Gauging, Timber-Measure, and Surveying ; because they are the most common and necessary Arts in Life ; and because the Use of Gunter's Scale, and Sliding-Rule (though before fully taught, and all along applied ; yet) in them is more extensive and various than in any other Arts : and therefore I have taken care not only to shew all the different ways of using those Instruments, but likewise the Rationale of every Operation ; a Matter of the greatest Importance, and too often neglected, in Books which treat thereof.

Lastly, I have in the last Chapter given a Variety of Examples of the Use of Logistical Logarithms in the practical Parts of Astronomy ; both with respect to Time and Motion, have made it appear that Street's Logistical Logarithms answer all the Ends of Shakerley's ; and how they are to be used along with the Common Logarithms of Numbers, Sines, and Tangents.

Tangents. *And throughout this second Part, as well as the first, you will find a great Variety of new and useful Particulars, not here to be express'd.*

*The Third Part of this Work consists of a three-fold Canon of Logarithms, viz. (1.) Of common Numbers from 1 to 10000; and is sufficient for any Number under 10000000 by proper Rules. (2.) Of Sines and Tangents to every Degree and Minute of the Quadrant. (3.) Of Logistical Logarithms of Mr. Street's Form. Concerning which Tables, I shall only observe two things in general, viz.*

*First, that they are here contrived in a new and most compendious Form, equally easy and useful as those of the common Form; tho' in this they take up but one half the room, as they do in that. An Abbreviation very commodious, and I hope will prove acceptable.*

*Secondly; the Correctness of these is a matter of the last Concern, and the greatest Argument to recommend them. In order to prove this, I need only say, that those large Tables of Mr. Sherwin's are granted to be the most correct of any extant, from the most careful and exquisite Method he took to make them so, which see in his Preface.*

*From these large Tables thus correct, I have made mine, every Figure of the two first Tables with my own Hand; in doing  
which*



*which, I discover'd several Errors, here and there, as I went along, in them, as exact as they were, which accordingly correcting in mine, I can, I presume, justly pronounce my Tables the most certain and exact, as well as the most compendious of any in being.*

*Having thus largely declared the several Parts of this Work, and shewn the Reader what an useful Variety he may expect to meet with both in the Theory, Praxis, and Tables, of this most useful and excellent Art; I must leave it to himself, to use or reject it, as he shall judge of the Merits thereof.*



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LOGA-



# LOGARITHMOLOGIA.

## PART I.

### *The Theory of LOGARITHMS.*

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#### CHAP. I.

##### *Of the Definition, Origin, and Nature of LOGARITHMS.*

1. **T**HE best *Definition* of those Numbers we call *Logarithms*, is contain'd in the very *Name* or *Word* (*Logarithm*) itself; for it is composed of the two *Greek Words*  $\lambda\acute{o}\gamma\omega\upsilon\ \acute{\alpha}\rho\iota\theta\mu\acute{o}\varsigma$ , which properly or literally signify, *a Number of Ratio's*: and a *Logarithm* is no other than a Number, which denotes or shews what Number of *Ratio's* is contain'd between *Unity*, and *that Number* of which it is said to be the *Logarithm*.

2. Whence 'tis evident, that in order to have a clear Notion of *Logarithms*, 'tis absolutely necessary to understand first, and that very well, what is meant by the Word *Ratio*, or *Ratio's*, as here used in the Definition of *Logarithms*, and making an essential part thereof.

3. *Ratio*, then, is a certain mutual *Habitude* of *Magnitudes* of the same kind, according to *Quantity*. This is *Euclid's* Definition: in which four things must be observ'd; as (1.) he saith *Ratio* is a certain *mutual Habitude*; by which he means no more than

B

what



what we commonly call the *Proportion of any two things of a like sort to each other*, when by us they are compar'd together. (2.) He useth the general Word *Magnitude* to denote, that all *Subjects of Quantity*; as *Numbers, Lines, Superficies, and Solids*, are capable of such *Ratio, Habitude, or Proportion*, as aforesaid. (3.) He adds this Restriction, *of the same kind*; thereby insinuating there can be no *Ratio or Proportion of Quantity*, between Magnitudes of a *different kind*; thus we cannot compare a *Line* to a *Superficies*; because the Quantity of a *Line* is estimated in *Length* only, but the *Quantity* of a *Superficies* ariseth from the joint Consideration of *Length and Breadth*, or the *Product* of each, and so importeth *Space*; which is entirely different from a *Line*, and therefore these two things cannot be the *Terms of Ratio or Comparison*. (4.) Lastly, he says, this *Ratio* is according to *Quantity*; that is, we compare Magnitudes in this Case, only to observe and maintain the *Proportion of Greatness or Bulk* which is between them; or to find *how often, or how many times*, one *lesser Magnitude* is contain'd in another *greater Magnitude*; neglecting all other Considerations and Affections of the said Magnitudes.

4. Having thus consider'd the general Nature of *Ratio's or Proportions*; I shall apply it to *Numbers*, as they are immediately the Subject of *Logarithms*. The *Ratio* therefore, or *Proportion* of a *Number to a Number* is two-fold; for first the *Ratio* of a *greater Number* to a *lesser* may consist in the *Addition* of some certain Number to that *lesser Number*; thus the *Ratio* of 6 to 2 is made by adding 4 to 2. And if from Unity you begin the constant Addition of the same Number, suppose 2, you will then have a *Series of Numbers*, whose Differences will be the same, as 1, 2, 4, 6, 8, 10, 12, &c. and such Numbers are said to be in *Arithmetical Proportion* or *Progression*;

*gression*; and this *common Difference* of the Terms, as here 2 is called the *Ratio* of the *Progression*.

5. Secondly, the *Ratio* of a *greater Number* to a *lesser* may consist in a *Multiplication* of the *Lesser Number* by some other Number; thus the *Ratio* of 12 to 4, is made by multiplying 4 by 3; and if from *Unity* you begin a *constant Multiplication* by the *same Number*, suppose 2, you will then have a *Series* of Numbers, as 1, 2, 4, 8, 16, 32, 64, &c. which are said to be in *Geometrical Proportion*, or *Progression*; and the *common Multiplier*, as here 2, is called the *Geometrical Ratio* of this *Progression*.

6. Wherefore in the two *Series* or *Progressions* of Numbers, *viz.*

Arith. 1, 2, 4, 6, 8, 10, 12, 14, &c.

Geom. 1, 2, 4, 8, 16, 32, 64, 128, &c.

'tis easy to observe, that as the second Term exceeds the first by one *Ratio*, so the third Term exceeds the first by two *Ratio's*, the fourth by three *Ratio's*, the fifth by four *Ratio's*, &c. Thus in the *Arithmetical Series*, the *Ratio* of 4 to 1, is *double* of the *Ratio* of 2 to 1; the *Ratio* of 6 to 1, is *triple* the *Ratio* of 2 to 1; the *Ratio* of 8 to 1, is *quadruple* the *Ratio* of 2 to 1, &c. And in the *Geometrical Series*, the *Ratio* of 4 to 1 is the *duplicate* of the *Ratio* of 2 to 1; the *Ratio* of 8 to 1 is *triplicate* of the *Ratio* of 2 to 1, and the *Ratio* of 16 to 1 is *quadruplicate* of the *Ratio* of 2 to 1, and so on. Where 'tis to be observed, that the Words, *double*, *triple*, *quadruple*, &c. are proper to the *Ratio's* of the *Arithmetical Series*, and imply the *Addition* of them only; but the Terms *duplicate*, *triplicate*, *quadruplicate*, &c. are proper to the *Geometrical Series*, and imply the *Multiplication* of those *Ratio's*.

7. What has been thus far related of the *Doctrine* of *Ratio's*, is sufficient for our present purpose, *viz.* the understanding the *Nature* of *Logarithms*. For

suppose a Series of Numbers in *Arithmetical Progression*, beginning from 0, and whose *Ratio*, or common Difference, is Unity or 1; and to them be adapted a Series in *Geometrical Progression*, beginning from *Unity*; and whose common Ratio be any assign'd Number, suppose 2, as before; then will those two Series stand as below:

viz.  $\left\{ \begin{array}{l} \text{Arith. } 0. 1. 2. 3. 4. 5. 6. 7. 8. 9 \text{ \&c.} \\ \text{Geom. } 1. 2. 4. 8. 16. 32. 64. 128. 256. 512. \text{ \&c.} \end{array} \right.$

8. 'Tis evident the Numbers in the first Series shew the Number of Ratio's between their correspondent Numbers and Unity in the second Series. For instance, the Figure 2 in the first Series, shews the Ratio's between its corresponding Number 4 and 1, in the lower Series, are 2; the Numbers 5, 7, 9, in the upper Series, shew the Number of Ratio's between their corresponding Numbers 32, 128, 512, and 1 or Unity, in the lower Series, to be respectively 5, 7, and 9; or that the Ratio is so often repeated from Unity to those Numbers, and consequently so often compounded in them; or farther, that the Ratio of 32 to 1 is compounded of five times the Ratio of 2 to 1; and the Ratio of 128 to 1, of seven times the Ratio of 2 to 1; and the Ratio of 512 to 1, of nines times the Ratio of 2 to 1.

9. Wherefore since the Numbers in the upper Series shew the Number of Ratio's contain'd between their corresponding Numbers and Unity in the lower Series; therefore those Numbers in the upper Series in *Arithmetical Progression* shall be the *Logarithms* of the Numbers in the lower Series of *Geometrical Progression*, and that according to the Definition of *Logarithms* foregoing.

10. From the said Series, 'tis farther manifest, that the Product of any two Terms in the lower Series corresponds to the Sum of their respective Terms in the upper Series. See the following Examples.

Arith.



$$\text{Arith. or } \left\{ \begin{array}{l} 2 + 3 = 5; 2 + 4 = 6; 3 + 6 = 9. \\ \text{Logar.} \end{array} \right.$$

$$\text{Geom. or } \left\{ \begin{array}{l} 4 \times 8 = 32; 4 \times 16 = 64; 8 \times 64 = 512. \\ \text{N.Num.} \end{array} \right.$$

Also if any two Numbers in the lower Series be divided the one by the other, the Quotient thence arising shall correspond to the Difference of the respective Numbers in the upper Series. Examples in

$$\text{Arith. } 5 - 3 = 2; 6 - 2 = 4; 9 - 3 = 6.$$

$$\text{Geom. } 32 \div 8 = 4; 64 \div 4 = 16; 512 \div 8 = 64.$$

11. And universally, if any four contiguous Numbers be taken in the second Series, as the Product of the Extremes is equal to the Product of the Means; so in the first Series of the four corresponding Numbers, the Sum of the Extremes will be equal to the Sum of the Means; as in the Examples below.

$$\text{Arith. } 1 + 4 = 2 + 3 = 5; 3 + 6 = 4 + 5 = 9.$$

$$\text{Geom. } 2 \times 16 = 4 \times 8 = 32; 8 \times 64 = 16 \times 32 = 512.$$

and *vice versa*.

12. Again the *Square*, *Cube*, &c. of any Number in the lower Series of Geometricals will be answer'd by *double*, *triple*, &c. the corresponding Number in the upper Series in Arithmetical Ratio; for Example;

$$\text{Square } \left\{ \begin{array}{l} \text{Arith. } 2 \times 2 = 4; 4 \times 2 = 8; 0 \times 2 = 0. \\ \text{Geom. } 4 \times 4 = 16; 16 \times 16 = 256; 1 \times 1 = 1. \end{array} \right.$$

$$\text{The Cube } \left\{ \begin{array}{l} \text{Arithmet. } 1 \times 3 = 3; 3 \times 3 = 9. \\ \text{Geomet. } 2 \times 2 \times 2 = 8. 8 \times 8 \times 8 = 512. \end{array} \right.$$

And the Converse of this Article is also true; as is evident enough without example.

13. It appears then by these six last Articles, that *Logarithms are a Series of Numbers in Arithmetical Progression, so fitted and adapted to another Series of Numbers in Geometrical Progression, as that each*

## 6      *Of the Definition, Origin, and*

*Term of the first shall expound (or be the Exponent of) the Ratio of its correspondent Term to Unity in the second Series. And that on this very Principle: For every Addition, Subtraction, Multiplication, or Division of the Logarithmic Numbers there corresponds a mutual Multiplication, Division, Involution, and Extraction of the respective Terms in the Geometrical Series.*

14. Now 'tis a Matter entirely arbitrary or indifferent, what Number be made the *first Term* in either Series; for since the *first* are made by *equal Additions*, the *latter* by *equal Multiplication*, be the *Ratio* what it will, the *former* will still be the *Logarithms* of the *latter*; as is evident in the Table adjoin'd.

		Series of Logarithms.					
6	1	1	3	6	7	0	0
18	2	2	5	7	12	1	10
54	4	3	7	8	17	2	20
162	8	4	9	9	22	3	30
486	16	5	11	10	27	4	40
1458	32	6	13	11	32	5	50
4374	64	7	15	12	37	6	60
13122	128	8	17	13	42	7	70
39366	256	9	19	14	47	8	80
118098	512	10	21	15	52	9	90
354294	1024	11	23	16	57	10	100

15. Wherefore to the *same Series of Proportionals* there may be an *infinite Number* of *Series* or *Scales of Logarithms* contrived; and *vice versa*. Yet of *all* those *infinite* kinds of *Logarithms*, only those whose *first Term* is 0, and the *common Difference* 1, 10, 100, &c. are adapted for use. Because if the *first Term* be a significant Figure, we must necessarily have respect to it in use; and so, in this case,

four

four Terms of the *Logarithmical Series* becomes unavoidable; whereas if 0 be the first Term, three other Terms only suffice in the *Multiplication* of any two *Proportionals* whatever: for the *Sum* of their Logarithms will point out the *Product*, and shew its *Place*, that is, its *Distance* from, or *Ratio* to, the first Term of *that Scale* of *Proportionals*. But if the *first Term* in the Scale of Logarithms were significant, it must be subducted from the *Sum* of the Mean, in order to find the *Product* of the two *Proportionals*, &c. as before: and so we should perpetually have *double Labour* in every Operation. All this is evident from the different Series of Logarithms in the foregoing Table.

16. This being the Nature, and such the wonderful Properties of those Numbers called Logarithms, 'tis natural to suppose that he who first discover'd them, would make such a *noble Discovery* as subservient as possible to the Uses of Life, for the general Benefit of Mankind, but more immediately of *Artists* or *Mathematicians*.

17. Now to do this, 'twas necessary to calculate and fit a *Scale* or *Canon* of *Logarithms* to all Numbers which Men commonly make use of in Business, that so the *Fatigue* and *Labour* of *Multiplying*, *Dividing*, &c. large *Sums* or *Numbers* might be

Proport.	Logarith.
1	0.0000000
10	1.0000000
100	2.0000000
1000	3.0000000
10000	4.0000000
100000	5.0000000
1000000	6.0000000

avoided by only the Addition or Substraction, &c. of Logarithms. And of consequence nothing less than a *general Table* of *Logarithms* for all Numbers between Unity, or 1 and 1000000 or 10000000, could suffice; because no Number must here be wanting, as in the other intermitting Series 1, 2, 4, 8, 16, &c. since general Use requires them all, and therefore their Logarithms.



8      *Of the Definition, Origin, &c.*

18. Towards this 'twas easy to find Logarithms for a Series of Numbers proceeding in a *Decuple Proportion* or *Ratio* from Unity, as 1, 10, 100, 1000, &c. in the Table above. For putting  $0 =$  Logarithm of 1; and  $1 =$  Logarithm of 10; the Logarithm of 100 will be 2; and of 1000, 3; and so on from what I have already said.

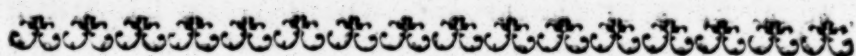
19. But the *Difficulty* all consisted in finding the Logarithms for the intermediate Numbers between 1 and 10, 10 and 100, 100 and 1000, &c. or those of them which are called *Prime Numbers*, because they being once found, the Logarithms of all others are easily obtain'd, as we shall see by and by.

20. Now tho' the Numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, are not in *Geometrical Proportion* (but indeed in an *Arithmetical one*) and so their Logarithms not to be obtain'd like the Logarithms of those which are so, yet as the *Ratio* of 10 to 1, shews only what *Distance*, or *how far* 10 is from Unity in the Scale of Proportion, and that the Numbers 2, 3, 4, 5, &c. possess severally a certain part of that Distance, if the said Ratio or Distance of 10 to 1, be supposed to be divided into a vast number of *equal Parts*, suppose 10000000, &c. 'tis evident a certain number of those equal Parts are to be allotted to the Numbers 2, 3, 4, 5, &c. which shall as truly express the *Distances* from, or the *Ratio's* of those several Numbers to Unity, as that of 10 to Unity is express'd by the whole Number 10000000.

21. This indefinite Number (10000000, &c.) of *equal Parts*, into which the whole Ratio of 10 to 1, is divided, may be conceived as a Number of so many *small Ratio's* or *Ratiunculae*, since they are all equal to each other. And so since the whole Number of *Ratiunculae* between 10 and 1, is the Logarithm of 10 to 1; therefore that Number of those *Ratiunculae* which lie between, or express the Distance of 2, 3, &c. to 1, shall also be as much the  
*Logarithms*

## Chap. II. Of making a Table of LOGARITHMS. 9

*Logarithms* of those Numbers 2, 3, &c. or, rather, of their Ratio's to Unity. And thus it appears that even the *Logarithms* of 2, 3, 4, 5, &c. (and for the same Reason) 11, 12, 13, &c. 21, 22, 23, &c. 101, 102, 103, &c. may be found to as great Exactness as is necessary. The Method of doing this by Numbers, shall be the Subject of the next Chapter.



### C H A P. II.

*Of the Method of making a Table of Logarithms by plain Arithmetic.*

1. **H**AVING explain'd the Nature and Properties of *Logarithms*, and shewn not only how they are adapted to Series of Geometrical Proportionals, but also by what means they are to be calculated for all Numbers from Unity to any large Number either above or below it:

2. The *Method* or *Process* it self, then, of doing this, is the next thing to be propos'd; an *arduous Task* this, to him that first aggress'd it! At such an Enterprize, he indeed might truly have said, *Hic Labor, hoc Opus est*. And this any but an *inhuman Reader* will be convinced of, by viewing the following *tedious* and *laborious Process* for gaining the *Logarithm* of one single Prime Number, and that only to seven Places of Figures.

3. For, as I said before, 'tis sufficient to produce the *Logarithms* of *Prime Numbers* only, because, by the *Addition*, *Subtraction*, &c. of these, we obtain the *Logarithms* of all *Composit Numbers*, with Ease; as will appear further on. Now, tho' there are but three Prime Numbers, *viz.* 2, 3, 5, 7, between

1 and 10 ; yet, because the Logarithm of no Number between 1 and 10 can be found by the Logarithms of any of the Proportionals 1, 10, 100, 1000, &c. it follows, that the Logarithm of any one of the nine Digits is found with equal Difficulty for the first. Therefore I shall give an Example in finding the Logarithm for the Number 9.

4. The Number 10, then, being already supposed at such a Distance from Unity as contains 10000000 equal Parts or *Ratiunculæ*, which is the Logarithm of 10 ; therefore to find the Logarithm of 9, is to find *how many*, or *what Number* of those *Ratiunculæ* are contain'd between 1 and 9, which is done in the following Manner.

5. First, make  $A=1$ , whose Log. is 0.0000000 ; and  $B=10$ , which Logarithm 1.0000000 ; as in the Table below.

Secondly, between A and B, or 1 and 10, find a *Mean Geometrical Proportional*  $C=3.1622777$  ; the Logarithm whereof will be half the Logarithm of 10, viz. 5000000.

Thirdly, because C is much less than 9, find another *Mean Proportional*  $D=5.623413$  between B and C ; whose Logarithm will be an *Arithmetical Mean* between the Logarithms of B and C, viz. 0.7500000.

Fourthly, because D is still much less than 9, find another *Geometrical Mean* between B and D, viz.  $E=7.4989421$ , whose Logarithm is an *Arithmetical Mean* between those of B and D, viz. 0.87500000.

Fifthly, since E is yet much less than 9, find yet another *Geometrical Mean*  $F=8.6596432$ , between B and E ; the Logarithm of which will be an *Arithmetical Mean* 0.93750000, between those of B and E.

6. And thus continue finding *Mean Geometrical Proportionals* between the Numbers *next Greater* and *next Lesser* than 9, till you arrive to the Number 9 itself,



Chap. II. *a Table of* LOGARITHMS. 11

itself, which shall be clear of all other Figures, but *Cyphers*, to the Number of Places proposed ; which will happen at the twenty-sixth Trial, as you see in the following Table.

Proportionals.		Logarithms.
A	1.0000000	0.0000000
C	3.1622777	0.5000000
B	10.0000000	1.0000000
B	10.0000000	1.0000000
D	5.6234132	0.7500000
C	3.1622777	0.5000000
B	10.0000000	1.0000000
E	7.4989421	0.8750000
D	5.6234132	0.7500000
B	10.0000000	1.0000000
F	8.6596432	0.9375000
E	7.4989421	0.8750000
B	10.0000000	1.0000000
G	9.3057204	0.9687500
F	8.6596432	0.9375000
G	9.3057204	0.9687500
H	8.9768713	0.9531250
F	8.6596432	0.9375000
G	9.3057204	0.9687500
I	9.1398170	0.9609375
H	8.9768713	0.9531250
I	9.1398170	0.9609375
K	9.0579777	0.95703125
H	8.9768713	0.9531250
K	9.0579777	0.95703125
L	9.0173333	0.95507812
H	8.9768713	0.9531250
L	9.0173333	0.95507812
M	8.9970796	0.95410156
H	8.9768713	0.9531250

C 2

L

*Of the Method of making*

Proportionals.		Logarithms.
L	9.0173333	0.95507812
N	9.0072008	0.95458984
M	8.9970796	0.95410156
N	9.0072008	0.95458984
O	9.0021388	0.95434570
M	8.9970796	0.95410156
O	9.0021388	0.95434570
P	8.9996088	0.95422363
M	8.9970796	0.95410156
O	9.0021388	0.95434570
Q	9.0008737	0.95428467
P	8.9996088	0.95422363
Q	9.0008737	0.95428467
R	9.0002412	0.95425415
P	8.9996088	0.95422363
R	9.0002412	0.95425415
S	8.9999250	0.95423889
P	8.9996088	0.95422363
R	9.0002412	0.95425415
T	9.0000831	0.95424652
S	8.9999250	0.95423889
T	9.0000831	0.95424652
V	9.0000041	0.95424271
S	8.9999250	0.95423889
V	9.0000041	0.95424271
X	8.9999650	0.95424080
S	8.9999250	0.95423889
V	9.0000041	0.95424271
Y	8.9999845	0.95424217
X	8.9999650	0.95424080
V	9.0000041	0.95424271
Z	8.9999943	0.95424223
Y	8.9999845	0.95424217





foregoing operose Method: No; for tho' 'tis not impossible, 'tis impracticable by that Method; but it is the Product of modern Invention; an Instance of which will be given in due Place.

8. Having thus found the Log. of  $9=0.9542425$  if it be divided by 2, the Quotient or Half,  $0.4771212$  will be the Logarithm of 3, by the Converse of Article 12. Chap. I. The Double of the Logarithm of 9 is the Logarithm of  $81=1.9084850$  by the same Article. And thus by a continual Multiplication of this one Logarithm by 2, 3, 4, 5, &c. you gain the Logarithms of all the Powers of Nine.

9. 'Tis farther evident, that if the Ratio of 10 to 1, 100 to 10, 1000 to 100, &c. be supposed only Unity or 1, as expressed in the Table of Art. 17. Chap. I. the Logarithm of Nine, viz.  $0.95424225$  will be no other than a Decimal Fraction, whose Denominator is the said Ratio, or Unity, with Cyphers annex'd, thus  $\frac{9542425}{10000000}$ .

Therefore also the Log. of any Number, viz.	{	3=0.4771212	} may be expres'd Frac- tion wise, as follows, viz.	{	3=0. $\frac{4771212}{10000000}$
		9=0.9542425			9=0. $\frac{9542425}{10000000}$
		10=1.0000000			10=1.0000000
		81=1.9084850			81=1. $\frac{9084850}{10000000}$
		100=2.0000000			100=2.0000000
		729=2.8627275			729=2. $\frac{8627275}{10000000}$
		1000=3.0000000			1000=3.0000000

10. Hence 'tis obvious that the Logarithms are only the Decimal Parts of several Ratio's, 1, 2, 3, &c. of 10, 100, 1000, &c. to Unity; and that those Ratio's themselves make the *integral Part* of the *mixt Decimal Logarithm*, (as I may call it.) These *integral Parts* of those *mixt Decimal Logarithms*, then are what we call the *Indexes*, or rather *Indices*, of those Logarithms, or *fractional Parts*; wherefore, the Ratio's of 10, 100, 1000, 10000, 100000, &c. to Unity, are the *Indices of or belonging to the Logarithms of all intermediate Numbers immediately*

mediately above those Proportionals. So the Ratio of 10 to 1, viz. 1, is the *Index* of the Logarithms of all Numbers between 10 and 100. And 2 (the Ratio of 100 to 1) is the *Index* of all Logarithms of Numbers, from 100 to 1000, and so on.

11. From whence 'tis easy to observe, That the *Index* of any Logarithm contains a Number of Units less by one than is the Number of Figures in that Number which belongs to the said Logarithm.

Therefore the Number  $\left\{ \begin{array}{c} 9 \\ 81 \\ 729 \\ 1000 \end{array} \right\}$  the Number of whose Figures are  $\left\{ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\}$  hath the *Index* of its Log.  $\left\{ \begin{array}{c} 0. \\ 1. \\ 2. \\ 3. \end{array} \right\}$

12. Now as this *Index* is thus called, from its indicating or shewing how many Places of Figures there are in the Number of the Logarithm; so, on the contrary, the Number of the Logarithm, as plainly shews what the *Index* of the Logarithm must be. And therefore, since in the Tables of Logarithms, all the Numbers of those Logarithms are appositely placed in proper Columns by them, 'tis entirely needless to print the *Indices*, as being well enough known by the Numbers; and thus they are (for this Reason) omitted in several of the said Tables; and consequently in those we have here, in so concise a Form, now first of all made publick.

13. What has been said hitherto, relates altogether to the Logarithms of *Whole Numbers*; but the same *Doctrine* is equally applicable both in *Theory* and *Practice*, to *fractional Numbers*; for in Effect, the Properties of *Decimal* and *Whole Numbers* are the same. Thus the Series of *decuple Proportionals*, 1, 10, 100, 1000, 10000, &c. may be continued as well below Unity, as above it; for the following Rank of Numbers, viz.  $\frac{1}{10000}$ ,  $\frac{1}{1000}$ ,  $\frac{1}{100}$ ,  $\frac{1}{10}$ , 1, 10, 100, 1000, 10000, &c. which may be thus express'd, 0,0001, 0,001, 0,01, 0,1, 1, 10, 100, 1000, 10000,

10000, &c. are all in a *Geometrical Decuple Ratio or Proportion*; and consequently equally distant from Unity on each Side, and so their Logarithms equidifferent, and the same. The Ratio's then, in the Scale above Unity, may be called *Positive Ratio's*, and those below Unity, *Negative Ratio's*; and thus the *Index* of a Logarithm of a *Whole Number*, or *Integer*, shall be *positive*, but the *Index* of the Logarithm of *Fraction* shall be *negative*.

14. And since *Vulgar Fractions*, *Duodecimal*, *Sexagesimal*, &c. Fractions, are all reducible to *Decimals*; 'tis evident that all *Numerical Arithmetic* whatsoever is subject to, and manageable by this Art of *Logarithmical Arithmetic*. The only Difficulty being in adapting or readily finding the proper *Index* to the Logarithm of a *Decimal Number*. For, since the *Index* in the Logarithm of *Integers*, shews only the *Number of Places or Figures* in the said *Integers*, 'tis plain those *Indices decrease*, as the *Places of Figures* in the *Integer* decrease, and intirely vanish when those *Places of Figures* become *Unity* or *one*, that is, the *Index* is then 0; and consequently cannot serve for *Decimal Numbers*.

15. Therefore some new kind of *Indices* or *Characteristics* must be invented, which shall be proper only to *Decimal Numbers*, as the other are to *Integers*; and such as shall as readily discover the *Number of Cyphers* to be *prefix'd* to the *significant Figures* in the *Decimal*, as the other determine the *Number of Figures* in the *Integer*, are in the same degree useful for *Decimals*, as they for *Integers*. For as to the *significant Figures* of the *Decimal*, the *Logarithm* it self discovers them; all therefore that is farther necessary, is to procure *such Indices* as shall at all times denote *how far*, or *how many Places* from *Unity*, the *first significant Figure* of the *Decimal* must stand; or, which amounts to the same, *how many Cyphers* must be *prefix'd* to compleat the true *Value* of the *Decimal*.



16. The *Indices* for this Purpose are therefore judg'd best, *which being subtracted from 9, shall have a Remainder expressing the Number of Cyphers to be prefix'd*; thus the *Index* of a *Decimal* whose first Figure (to the Left,) is significant,

Integers	{	47500=	4.6766936
		4750=	3.6766936
		475=	2.6766936
Decimals	{	47,5=	1.6766936
		4,75=	0.6766936
		,475=	9.6766936
		,0475=	8.6766936
		,00475=	7.6766936
		,000475=	6.6766936
		,0000475=	5.6766936

must be 9, because  $9-9=0$ , that is *no Cypher* is to be prefix'd. And thus, a *Decimal* that has 1, 2, 3, 4, &c. *Cyphers*, must have the *Indices*, accordingly 8, 7, 6, 5, &c. because  $9-8=1$ ,  $9-7=2$ ,  $9-6=3$ ,  $9-5=4$ , &c. denoting the *Cyphers* to be prefix'd to the *Decimal Numbers*. This is evident by the Examples in the Table above; but it is proper to dott these new *Indices* on each Side, to distinguish 'em from those which belong to the Logarithms of *Integers*.

17. From all that has been said hitherto, 'tis evident, that while the *significant Figures* in any Number whatever, *Integral* or *Decimal*, are the same, the *Logarithm* of those Figures will be the same also; the Variation occasion'd by the differing Nature of the Number, being only in the *Indices*, as denoted in the said Table. But something farther will be said of the *Nature* and *Variety of Indices*, when we come to treat of the *Practical Rules of Addition and Subtraction* of Logarithms; I shall now proceed to the next Chapter, wherein the *mysterious Nature*, or *Doctrine of Logarithms* will be farther illustrated, and render'd more obvious to the Senses by *Geometrical Schemes* and *Demonstrations*.



## C H A P. III.

*The Doctrine of the Nature and Properties of Logarithms farther explained and illustrated, by means of the LOGARITHMIC CURVE.*

1. **I**N explaining the Nature of *Logarithms* from the *Logarithmic Curve*, 'twill be expedient to represent *Proportional Quantities* by *Letters or Species*; as being suited to a more *universal Theory* than *Numbers*, and better applicable to *Right Lines*, by which alone both *Numbers* and their *Logarithms* are represented in the *Geometrical Method* now before us.

2. In a Series of *Proportionals*, then, increasing from Unity, let the first be  $=a$ ; then will the Series be  $1, a^1, a^2, a^3, a^4, a^5, \&c.$  where 'tis evident the *Indices* or *Exponents* of the several *Powers* of  $a$ , are a Series of *Numbers* in *Arithmetical Progression*, each whereof shews the *Place* or *Distance* of its *Term* from Unity; thus the *Term*  $a^4$  is shewn by its *Index* 4 to be in the *fourth Place* from Unity; and  $a^5$  is in the *fifth Place*; or  $a^4$  and  $a^5$  are *four* and *five times* more distant from Unity than the *first Term*  $a$ ; which is here the *common Ratio* also.

3. From hence you observe, that the *Exponents* of the *Powers* of the *Terms* in the Series  $1, a^1, a^2, a^3, a^4, a^5, a^6, a^7, \&c.$  are the *Logarithms* of those *Terms* respectively. Let the *Exponent* be  $=e$ ; then  
 { the *Terms*,  $1, a, aa, aaa, aaaa, aaaaa, aaaaaa, \&c.$   
 { the *Expon.*  $0, e, 2e, 3e, 4e, 5e, 6e, \&c.$   
 Therefore

{ As  $axaxaa=a^7$ ;  $axa^5=a^6$ ; and  $axaxaa\sqrt{a}=a^5\sqrt{a}$ ,  
 { So  $2e+3e=5e$ ;  $e+5e=6e$ ; and  $2e+3\frac{1}{2}e=5\frac{1}{2}e$ .

and so every where, agreeable to the Nature of *Logarithms*

### Chap. III. of LOGARITHMS explained. 19

garithms before describ'd, in the two preceding Chapters.

4. If between 1 and  $a$  there be put one *mean Proportional*, viz.  $\sqrt{a}$ , its *Index* or *Exponent* must be wrote  $\frac{1}{2}$ ; thus  $\sqrt{a} = a^{\frac{1}{2}}$ ; for 0,  $\frac{1}{2}$ , 1, have an *Arithmetical Ratio*; and so the Terms 1,  $a^{\frac{1}{2}}$ ,  $a$ , are *Geometrical Proportionals*. Thus, a *mean Proportional* between  $a$  and  $aa$ , is  $a\sqrt{a} = a^{1\frac{1}{2}} = a^{\frac{3}{2}}$ . Also if you conceive two *mean Proportionals* between 1 and  $a$ , they shall be  $a^{\frac{1}{3}}$ ,  $a^{\frac{2}{3}}$ ; for  $1 : a^{\frac{1}{3}} :: a^{\frac{2}{3}} : a$ ; and 0,  $\frac{1}{3}$ ,  $\frac{2}{3}$ , 1, have an *equal Difference*.

5. Moreover the same Series of *Geometrical Proportionals* may be continued both ways, and be made *decreasing* as well as *increasing*; that is, it may as well *descend below Unity* to the *left Hand*, as *ascend* above it to the *Right*. Thus the Terms  $a^{\frac{1}{5}}$ ,  $a^{\frac{1}{4}}$ ,  $a^{\frac{1}{3}}$ ,  $a^{\frac{1}{2}}$ ,  $\frac{1}{a}$ , 1,  $a$ ,  $a^2$ ,  $a^3$ ,  $a^4$ ,  $a^5$ , &c. are all in the same *Geometrical Progression*. And since the Distance of  $a^2$  from *Unity* towards the *right Hand* is *positive*, and denoted by the *positive Exponent*  $+2$ ; so  $a^{\frac{1}{2}}$  being equally distant on the *contrary Side*, or *below Unity*, and this Distance being *negative*, therefore the Exponent thereof may be thus *negatively* express'd,  $a^{-2} = a^{\frac{1}{2}}$  1, and so  $a^{\frac{1}{3}}$  is the same as  $a^{-3}$ ; thus the Series  $a^{-5}$ ,  $a^{-4}$ ,  $a^{-3}$ ,  $a^{-2}$ ,  $a^{-1}$ , 1,  $a$ ,  $a^2$ ,  $a^3$ ,  $a^4$ ,  $a^5$ , &c. is the same as the last foregoing.

6. Therefore, in those Series, if  $a$  represent any *Right Line* (see Fig. 1.)  $aa$ , or the *Square* thereof, is not to be taken as a *Quantity of two Dimensions*, or *Surface*, viz. a *Geometrical Square*; but only as a *Line* that is a *third Proportional* to some *Line* taken as *Unity*, and the *Line*  $a$ . So likewise  $aaxa$ , or  $aaa$ , is not a *Quantity of three Dimensions*, or a *Geometrical Cube*; but a *Line* that the *fourth Term* or *Proportional* in a *Geometrical Series*, whose *first Term* is 1,



and *second a.* And thus you are to conceive of all the rest; viz. simply as *Proportionals of Length* only.

7. This being premised then, if on any Line, as AN (Fig. 2.) both ways indefinitely extended from A, be taken the equal Distances  $AC=CE=EG=GI=IL=LN$ , to the *right Hand*; and  $Ac=ce=eg$ , &c. on the left; and if on the Points  $g, e, c, A, C, E, G, I, L, N$ , be erected to the Right Line  $gN$ , the Perpendiculars  $gh, ef, cd, AB, CD, EF, GH, IK, LM, NO$ , which let be in a continual *Geometrical Proportion* to each other, and represent *Numbers*, whereof  $AB=1$ , or *Unity*. The Tops of these Lines being duely join'd, from what is call'd the *Logarithmic Curve*, viz.  $hBHO$ ; by which we are farther to explain and illustrate the *Nature and Properties of Logarithms*.

8. But a more compleat Idea of the *Logarithmical Curve*, may be conceiv'd by a twofold Motion of the Line  $AB$ ; the one *equable*, the other *accelerated* or *retarded* in any given *Geometrical Ratio*: For Example, if the Right Line  $AB$  moves uniformly along the  $AN$ , so that the End  $A$  describes *equal Spaces*  $AC=CE=EG$ , &c. in *equal Times*; while in the same Time the said Line  $AB$  so *increases*, that the *Increments* thereof generated in *equal Times*, be *proportional* to the *whole* increasing Line; that is, if  $AB$  in moving forward to  $ab$ , be *increased* by the *Increment*  $ob$ , and in an *equal Time*, when it comes to  $CD$ , the *Increment* thereof is  $Dp$ ; and  $Dp : ob :: bo : AB$ ; then the End  $B$  of the said Line  $AB$  thus *continually increasing* or *decreasing* in the *same Ratio*, will describe the *Logarithmic Curve*. For since  $AB : bo :: ab : Dp :: DC : dq$ , &c. it shall be, by *Composition of Ratio*, as  $AB : ab :: ab : DC :: DC : dc$ ; and so on.

9. Since (from what has been said) the Line  $AB=1$ , and the other Lines  $CD, EF, GH$ , &c. proceed from thence to *increase* in a *Geometrical Ratio*,  
and

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and their Distances AC, CE, EG, &c. are all *equal* to each other (by *Art. 7.*) Therefore it follows that the Distances or Lines AC, AE, AG, &c. are the *Logarithms* of those Numbers represented by the Right Lines CD, EF, GH, &c. according to the *Definition of Logarithms*, Chap. I. Art. 1, and 13. For if  $AC=1$ , then  $AE=2$ ,  $AG=3$ , &c. and so in the Series of *proportional Numbers* AB, CD, EF, GH, IK, LM, NO, &c. we have the *Logarithms*

0, AC, AE, AG, AI, AL, AN, } &c.  
0, 1AC, 2AC, 3AC, 4AC, 5AC, 6AC, }

10. As *Logarithms*, then, are the *Exponents* of the *Ratio's* of *proportional Numbers* to *Unity* in any Series; or shew the *Place*, *Power*, or *Order* of the *Numbers* with respect thereto; 'tis plain that the *Logarithm* of AB or *Unity* must be  $=0$ , because *Unity* is not distant from it self. Also if the *Ratio* of CD to AB be  $=x$ , then shall the *Ratio* of the Number EF to  $AB=x \times x=xx$ ; or *Duplicate* of the former; and the *Ratio* of GH to  $AB=x \times x \times x=x^3$ , or *TriPLICATE* of the first *Ratio*  $x$ . Thus the *Ratio* of NO to  $AB=x^6$ . And consequently the *Numbers*, their *Ratio's*, and *Logarithms*, will stand in the following Order, *viz.*

Prop. N°. AB, CD, EF, GH, IK, LM, NO, &c.  
Ratio's . . . 1,  $x$ ,  $x^2$ ,  $x^3$ ,  $x^4$ ,  $x^5$ ,  $x^6$ , &c.  
Logarithms 0, AC, 2AC, 3AC, 4AC, 5AC, 6AC, &c.

11. If four Numbers be such, that the Distance between the *first* and *second*, be equal to the Distance between the *third* and *fourth*, (be the Distance between the *second* and *third* what it will) then these Numbers will be *proportional*; for let the four Numbers be AB, CD, LM, and NO; then because  $AC=LN$ , it will be  $AB:DP::LM:OT$ , by *Art. 8.* Therefore (by *Composition of Ratio*)  $AB:DC::LM:NO$ . And thus of any other four Numbers in the *Geometric Series*. The *Converse* also of this Proposition is as manifestly true.

12. Since  $AB=1$ , and its Logarithm  $=0$ ; and since *Unity* is to the *Multiplier* as the *Multiplicand* is to the *Product*, in every Multiplication; therefore to every *Addition* of Logarithms, there corresponds a *Multiplication* of their Numbers. Thus,

Numbers,  $CD \times EF = GH$ ; and  $EF \times IK = NO$ .

Logarithms,  $AC + AE = AG$ ; and  $AE + AI = AN$ .

And in *Division*, since *Unity* is to the *Divisor* as the *Quotient* to the *Dividend*; then (by *Art. 11.*) for every *Subtraction* or *Difference* of Logarithms, there corresponds a *Division* of their Numbers. Thus,

Numbers,  $GH \div EF = CD$ ; and  $NO \div EF = IK$ .

Logarithms,  $AG - AE = AC$ ; so  $AN - AE = AI$ .

agreeably to what was shewn in *Chap. I. Art. 10.*

13. Again, since *Unity*, any assum'd Number, the *Square* thereof, the *Cube*, the *Biquadrate*, &c. are all continual *Proportionals*, their *Distances* from each other will be equal to one another; and are therefore as the Numbers  $AB, CD, EF, GH, \&c.$  in the Series. Consequently if the Distance or Logarithm of the Number  $CD$  be multiplied by 2, 3, 4, &c. there will answer an *Involution* of the said Number to the *Square, Cube, Biquadrate, &c. Power.*

Thus for the Square

Numb.  $CD \times CD = EF$ ; and  $EF \times EF = IK$ .

Log.  $AC \times 2 = AE$ ; so  $AE \times 2 = AI$ .

For the Cube

Numb.  $CD \times CD \times CD = GH$ ; and  $EF \times EF \times EF = NO$ .

Log.  $A \times 3 = AG$ ; and so  $AE \times 3 = AN$ .

For the Biquadrate

Numb.  $CD \times CD \times CD \times CD (= EF \times EF) = IK$ .

Log.  $AC \times 4 = (AE \times 2) AI$ .

Which is the same as was observed in *Chap. I. Art. 12.*

14. If the equal Distances  $AC, CE, EG, GI, IL$ , be bisected, and in the Points of Bisection  $a, c, e, g, i$ , there



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there be erected the Perpendiculars *ab*, *cd*, *ef*, *gb*, *ik*; these, by means of the Curve, will be all *Proportionals*; and the Number LM will be in the *tenth Place* from *Unity* or AB: if then we put  $LM=10$ , and suppose its *Ratio* to *Unity* be 10000000, such *Ratiuncule*, as that of *no* to AB, or *Unity* is 1. Then will the *Numbers* and their *Logarithms* be as express'd in the following Table.

Numbers.	Logarithms.
AB = 1	0.
<i>ab</i> = 1.259, &c.	Aa = 1000000
CD = 1.585, &c.	AC = 2000000
<i>cd</i> = 1.996, &c.	Ac = 3000000
EF = 2.512, &c.	AE = 4000000
<i>ef</i> = 3.163, &c.	Ae = 5000000
GH = 3.982, &c.	AG = 6000000
<i>gb</i> = 5.072, &c.	Ag = 7000000
IK = 6.310, &c.	AI = 8000000
<i>ik</i> = 7.944, &c.	Ai = 9000000
LM = 10.	AL = 10000000.

Numbers.	Logarithms.
AB = 1	0
CD = 2	AC = 3010300
EF = 3	AE = 4771212
GH = 4	AG = 6020600
IK = 5	AI = 6989700
LM = 6	AL = 7781512
NO = 7	AN = 8450980
PQ = 8	AP = 9030900
RS = 9	AR = 9542425
TV = 10	AT = 10000000

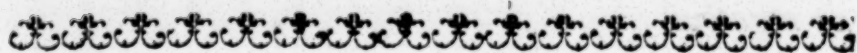
15. If BKV (Fig. III.) be the *Logarithmic Curve*, and  $AB=1$ ,  $TV=10$ , or  $10AB$ ; and all the intermediate Lines CD, EF, GH, &c. be as *Digits* 2, 3, 4, &c. to 10; these Lines will be situated

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ated at *unequal Distances* from each other ; and so their *Logarithms* AC, AE, AG, &c. not proceeding to increase by equal Differences, shew the Numbers AB, CD, EF, &c. are not *ordinately* (or all of them) in a *Geometrical Ratio* or *Proportion*. If the Ratio of 10 to 1, or TV to AB be supposed to consist of 10000000 *Ratiunculae*, and consequently the whole Logarithm AT of 10000000 *equal Parts*, then the Distances of each of the *nine Digits* from Unity, viz. AC, AE, AG, &c. shall consist of such Numbers of those small *equal Parts*, as are expressed in the Table, opposite to the said Digits.

16. Yet since some of the Digits are in a *Geometric Ratio*, as 1, 2, 4, 8, their Logarithms will be *equidifferent* ; so  $AC=CG=GP$  ; and because  $1:3::3:9$ , there is  $AE=ER$  ; and thus it appears that tho' the nine Digits are not all in continual *Geometrical Proportion*, yet some of them are so ; and the rest are some of *those Proportionals*, of which there be 10000000 between  $AB=1$ , and  $TV=10$ . If the *first Term* from Unity be called  $x$ , the second will be  $x^2$ , the third  $x^3$ , &c. and since the Number 10 is the 10000000th Term of the Series, it will be  $TV=x^{10000000}=10$ . Also  $CD=x^{3010300}=2$  ; and  $EF=x^{4771212}=3$  ; and so on. Whence every Digit is *some Power* of that Number which is the *first Proportional* from Unity : The *Exponents* or *Indices* of the Powers being the *Logarithms* of the Numbers, agreeable to Art. 10. of this.





# CHAP. IV.

*The Nature of Logarithms and their Indices, when the Numbers are Fractions, farther explain'd by the LOGARITHMIC CURVE.*

1. **W**E have hitherto principally considered the Nature and Properties of Logarithms of *whole Numbers* or *Integers*, and have observ'd that in the *Decuple Series* 1, 10, 100, 1000, &c. the Terms have their Ratio's to Unity *affirmative* or *positive*, viz. 1, 2, 3, &c. or thus, +1, +2, +3, &c. the contrary of which happens when the Number of the Logarithm is not *integral*, but *fractional*, or expresses only some *fractionate* Part of Unity. For there the Series being continued on the *other Side*, or *below Unity*, hath the *Indices* of the Powers of the Terms of a *Quality* directly *opposite* to the former; and therefore as *those Indices* were *positive*, so *these* will be of a *negative Nature*, and import the Terms to be *below* the *State of Unity*, or rather of *Integrity*; and will be affected with the Sign —, as —1, —2, —3, &c. as is evident Chap. III. Art. 5.

2. Wherefore since AB represents *Unity*, (Fig. II.) all the *Numbers* in the *Series* towards the *right Hand*, or *above Unity*, CD, EF, GH, &c. being *integral*, and having the *Ratio* greater than *Unity*, will have the Logarithms thereof *positive*, viz. +AC, +AE, +AG, &c. But those *Numbers* or *Terms* on the *Left*, or *below Unity*, cd, ef, gh, &c. being *fractionate*, and having the *Ratio* less than *Unity* or *decreasing*, will have the Logarithms thereof *Negative*, viz. —Ac, —Ae, —Ag, &c. And so the *Indices*

E

of



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of those *Logarithms* will be respectively *affirmative* or *negative*.

3. Since then (as hath been all along shewn) the *Indices* of the *Logarithms* of *Integers*, as being *affirmative*, must be *added*, that so their *Sum* may indicate the *Product* of the *Numbers* multiplied; as  $AC + AE = AG$ , which shews that its *Number*  $GH = CD \times EF$ , the *Numbers* multiplied; so if the *Index* of one *Logarithm* be *negative*, and the *Index* of another be *affirmative*, the *Difference* of those two *Indices* must be taken for the *Product* of the *Numbers*. Thus to multiply the *Integer*  $GH$  by the *Decimal*  $cd$ , their *Indices* being  $+AG$ , and  $-Ac$ , their *Difference*  $AG - Ac = AE$ , and shew the *Number*  $EF$  is the *Product* of the other two. And here, because the greater *Index*  $GH$  is *affirmative*, the *Difference* also  $AE$  is *affirmative*, and the *Product*  $EF$  to be an *Integer* or on the right *Hand* of *Unity*.

4. But if the *Decimal*  $gh$  be to be multiplied by the *Integer*  $CD$ , whose *affirmative Logarithm* or *Index*  $AC$  is the *negative Index* of the *Decimal*, viz.  $Ag$ , their *Difference*  $Ag - AC = Ae$ , is *negative* also, and so shews the *Product*  $ef$  will be a *Decimal*, or below *Unity*. Again, if both the *Indices* of *Logarithms*, whose *Numbers* are to be multiplied, are *negative*, their *Sum* shall be a *negative Index* whose *Logarithm* points out the *Product* (in this Case) always a *Decimal*, or in the Series below *Unity*  $AB$ . For Example, to multiply the two *Decimals*  $cd$ ,  $ef$ , the *Sum* of their *negative Indices*  $Ac + Ae = Ag$  is *negative*, and shews the *Decimal Product*  $gh$ , ever below *Unity*.

5. The Reason of all which is very plain; for since *Unity* is to the *Multiplier* as the *Multiplicand* to the *Product*, and the *Logarithm* of *Unity* is  $=0$ ; *Unity*, any two *Numbers* and their *Product*, together with their *Indices*, will stand as follows:

$$1. \begin{cases} \text{Ind. } o, AC, AE, AG, \\ \text{Num. } AB : CD :: EF : GH, \end{cases}$$

therefore  $\begin{cases} AC + AE = AG + o = AG. \\ CD \times EF = GH \times 1 = GH. \end{cases}$

$$2. \begin{cases} o, -Ac, +AG, +AE, \\ AB : cd :: GH : EF. \end{cases}$$

therefore  $\begin{cases} AG - Ac = AE + o = AE, \\ GH \times cd = EF \times 1 = EF. \end{cases}$

$$3. \begin{cases} o, +AC, -Ag, -Ae, \\ AB : CD :: gh : ef, \end{cases}$$

therefore  $\begin{cases} AC - Ag = o - Ae = -Ae, \\ CD \times gh = 1 \times ef = ef. \end{cases}$

$$4. \begin{cases} o, -Ac, -Ae, -Ag, \\ AB : cd :: ef : gh, \end{cases}$$

therefore  $\begin{cases} Ae + Ac = o - Ag = -Ag, \\ ef \times cd = 1 \times gh = gh. \end{cases}$

6. From whence 'tis evident, if the Numbers are both *Integers*, the *Product* will fall in the *Scale* above (or be greater than) either; if they be of *different Sorts*, the *Product* will fall between them, above *Unity* if the greater Number be an *Integer*, or below *Unity*, if it be a *Decimal*. Lastly, if both the Numbers be *Decimals*, the *Product* will fall below *Unity* and them both, or will be less than either of them. These things well observ'd and understood, make all the *Mystery* of the *Arithmetic* of *Fractions* by *Logarithms* vanish, where it is taught with the Use of *negative Indices* to *Logarithms* of *fractional Numbers*, as it is to be found in some Books. Note, I have all along in this Chapter, said *affirmative* or *negative Indices*, and not *affirmative* and *negative Logarithms*; because the *Indices* only shew the *Quality* of the *Numbers*, viz. whether above or below *Unity*, greater or lesser than it, i. e. whether they be *Integers*, or wholly *Fractions*; and therefore these *Indices* must be carefully added or subtracted, as occasion requires,

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and as before directed; but the *Logarithms* shewing the *Distances* of *Numbers* from *Unity* indifferently, or without respect to the *Order* of *above* or *below*, are *always* to be *added*.

7. But since this Method of *negative* or *different Indices* to *Logarithms* is attended with *Addition*, *Subtraction*, and other *intricate Cautions* peculiar to themselves, it can't be recommended so much as another Method *more approv'd*, and therefore *more generally used*; wherein the *Indices* of *all Logarithms*, (both of *fractional* as well as *integral Numbers*) undergo the same common *Management* with their *Logarithms*; and so can't be so *difficult* to *Learners* as the *other*; tho' it is also attended with *particular Rules*, as you'll find farther on, and is as follows:

	Numbers.	Indices.	Log <sup>s</sup> .
Fractions.	ut =,00000000001	o —10 0	90.00000000
	rs = ,0000000001	tr = — 9 1.	91.00000000
	pq = ,000000001	tp = — 8. 2.	92.00000000
	no = ,00000001	tn = — 7. 3.	93.00000000
	lm = ,0000001	tl = — 6. 4.	94.00000000
	ik = ,000001	ti = — 5. 5.	95.00000000
	gh = ,0001	tg = — 4. 6.	96.00000000
	ef = ,001	te = — 3. 7.	97.00000000
	cd = ,01	tc = — 2. 8.	98.00000000
	ab = ,1	ta = — 1. 9.	99.00000000
Integers.	AB = 1	tA = 0 10.	100.00000000
	CD = 10	tC = + 1. 11.	101.00000000
	EF = 100	tD = + 2. 12.	102.00000000

8. Let AB be =1, or *Unity* (Fig. IV.) and let tu be a *fractional* Number 10 times as far *below* *Unity* AB, as CD is *above* it; then will At=10AC, and supposing the Number CD=10, 'tis evident tu= $\frac{1}{10000000000}$ , or ,00000000001; if then, instead of





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oooooooooooo ooooooooooooo ooooooooooooo 1 and 1, will of course be *positive*; and so in this Class, the *Indices* of *Logarithms* of all *decimal Numbers* (greater by far than *Use*, or even *Curiosity*, can desire) will be *positive* also, as well as those of the *Logarithms* of *integral Numbers*; and therefore in short, we have obtain'd *positive Indices* for the *Logarithms* of *all Numbers* in *general*, and so the *Trouble* of *Addition* and *Subtraction* of *different Parts* of *Logarithms* at the same time, is avoided, as was proposed.

11. But (as I have before observed, Chap. II. Art. 16.) the *Indices* of the *Logarithms* of *Decimals* should be distinguished from the *Indices* of *Logarithms* of *Integers* in *Operations*; and this I think is best done as there directed, *viz.* by fixing a *Dott* on *each Side* the said *Indices*. And thus manifestly appears the *Reason* of all that is deliver'd in Chap. II. from Art. 13. to the End.

12. Now in *multiplying*, *dividing*, &c. of *Fractions*, the *Factors* may be either *both Decimal*; or *one Decimal*, and the *other* an *Integer*; if *both* the *Factors* are *Decimal*, the *Product* will be *Decimal*, in *Multiplication*. If the *Factors* be of *different Sorts*, and the *Ratio* of *Unity* to the *Decimal* be either *less*, *equal to*, or *greater* than the *Ratio* of *Unity* to the *Integer*; then shall the *Product* be *greater*, *equal to*, or *less* than *Unity*; i. e. it will be *Integer*, *Unity*, or *Decimal*, in *multiplying*; all which will be evident by the following *Examples*, serving as so many *Rules* for the *right ordering* of *Indices* in *Operations*.

$$\begin{array}{l}
 13. \left\{ \begin{array}{l} AB : cd :: ab : ef \\ tA. .tc. .ta. .te. \\ 10. .8. .9. .7. \end{array} \right\} \text{theref.} \left\{ \begin{array}{l} cd \times ab = 1 \times ef = ef. \\ .tc. + .ta. = tA + te. \\ .8. + .9. = 10. + .7. \\ \text{That is } \dots \dots \dots .8. + .9. - 10. = .7. = \\ \text{Index of } ef. \end{array} \right.
 \end{array}$$

$$2. \left\{ \begin{array}{l} AB : ab :: EF : CD \\ tA. .ta. tE. tC. \\ 10. .9. 12. 11. \end{array} \right\} \text{theref.} \left\{ \begin{array}{l} ab \times EF = 1 \times CD = CD. \\ .ta. + tE = tA. + tC. \\ .9. + 12. = 10. + 11. \\ \text{And consequently} \dots .9. + 12. - 10 = 11. = \\ \text{Index of CD.} \end{array} \right.$$

$$3. \left\{ \begin{array}{l} AB : ab :: CD : AB \\ tA. .ta. tC. tA. \\ 10. .9. 11. 10. \end{array} \right\} \text{theref.} \left\{ \begin{array}{l} ab \times CD = 1 \times 1 = 1 \\ .ta. + tC = tA + tA. \\ .9. + 11 = 10 + 10. \\ \text{Therefore} \dots .9. + 11. - 10 = 10 = \\ \text{Index of 1.} \end{array} \right.$$

$$4. \left\{ \begin{array}{l} Ab. cd :: CD : ab. \\ tA. .tc. tC. .ta. \\ 10. .8. 11. .9. \end{array} \right\} \text{theref.} \left\{ \begin{array}{l} CD \times cd = 1 \times ab = ab. \\ tC. + .tc. = tA. + .ta. \\ 11. + .8. = 10. + .9. \\ \text{and therefore again} \dots 11. + 8. - 10 = .9. = \\ \text{Index of ab.} \end{array} \right.$$

14. From these Observations well consider'd, 'tis easy to apprehend the Truth of what is deliver'd in Art. 12. above. And since AB, in these *Examples* of the *Products*, hath for the *Index* of its Logarithm 10, 'tis equally obvious what the *Indices* of the *Logarithms* of the *Products* would be, were the said *Index* of the Logarithm of AB made 100. In this *present Case* were  $tA = 10$ , if we reject 10, the *Indices* of the Logarithms of *integral Products* will be the same as if the *Logarithms* began at AB or Unity; as is plain in the 2d and 3d *Examples*; and accordingly if  $tA = 100$ , we reject 100, and the Case is the same.

15. But since 'tis most convenient to have the *Indices* of all Logarithms of *integral Numbers* to begin from *Unity* in the simple Order, 0, 1, 2, 3, &c. as if the *Logarithms* did really begin from thence; so 'tis but rejecting 10 from the *said Indices* in their present State, and what we desire is obtain'd.

Thus



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$$\begin{array}{l} \text{Thus the foregoing Cafes} \left\{ \begin{array}{l} 1. \quad .8. + .9. - 10 = .7. \\ 2. \quad .9. + 12. - 10 = 11. \\ 3. \quad .9. + 11. - 10 = 10. \\ 4. \quad 11. + .8. - 10 = .9. \end{array} \right. \\ \text{by rejecting 10 from the Indices of the Logarithms of Integers, becomes} \left\{ \begin{array}{l} .8. + .9. - 0 = .17. \\ .9. + 2 - 0 = 11. \\ .9. + 1 - 0 = 10. \\ 1 + 8 - 0 = .9. \end{array} \right. \begin{array}{l} .7. \\ 1. \\ 0. \\ .9. \end{array} \end{array}$$

Then from the *Sums* .17. 11. 10. .9. again rejecting 10 (where 'tis found) we have the *Remainders* .7. 1. 0. .9. the *true Indices* of the *Logarithms* of the *Products*, as required.

16. If either *Factor*, or the *Product* of them, exceeds the Limit of  $\frac{1}{10000000000}$ , or is less than ,0000000001, we shall find it most convenient to use the *Indices* of those *Logarithms* of which the *Index* of the *Logarithm* of *Unity* or AB is = 100, viz. those *Indices* in the 3d Class in Table of Art. 8. hereof. And if, in all the foregoing Cafes, instead of rejecting 10, we now reject 100, we shall have the *Indices* of the *Logarithms* of the *Products* the same kind as before.

$$\begin{array}{l} \text{Thus} \left\{ \begin{array}{l} 1. \quad .98. + .99. - 100 = .97. \\ 2. \quad .99. + 102 - 100 = 101. \\ 3. \quad .99. + 101 - 100 = 100 \\ 4. \quad 101. + .98. - 100 = .99. \end{array} \right. \\ \text{by rejecting 100, becomes} \left\{ \begin{array}{l} .98. + .99. - 0 = .197. \\ .99. + 2 - 0 = 101. \\ .99. + 1 - 0 = 100. \\ 101. + .98. - 0 = 199. \end{array} \right. \begin{array}{l} .97. \\ 1. \\ 0. \\ .99. \end{array} \end{array}$$

Thus here you see the Effect the same as above, Art. 15. if .97. .99. be deducted from 100, as .7. .9. were from 10, the *two middle Indices* being the same in both Cafes.

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17. In *Division*, the *Divisor* is to *Unity* as the *Dividend* is to the *Quotient*; and so the *Distance* between *Unity* and the *Divisor* is equal to that between the *Dividend* and *Quotient*. If then the *Fraction*  $ef$  be divided by  $ab$ , because  $aA = ec$ , therefore  $cd$  is the *Quotient*, the *Index* of whose *Logarithm* is  $tc$ ; but  $tc = tA + te - ta$ . Also if the *Integer*  $CD$  be divided by the *Fraction*  $ab$ , because  $aA = CE$ , therefore  $EF$  is the integral *Quotient* whose *Logarithm* is  $tE$ ; but  $tE = tA + tC - ta$ . Again, let the *Fraction*  $ab$  be divided by the *Integer*  $CD$ , because  $CA = ac$ , therefore the *Fraction*  $cd$  is the *Quotient*; whose *Logarithm*  $tc = tA + ta - tC$ ; that is, in every Case, the *Logarithm* of the *Divisor* being taken from the *Logarithm* of the *Dividend*, if to the *Remainder*, you add the *Logarithm* of *Unity*, the *Sum* will be the *Logarithm* of the *Quotient*: which is but the *Converse* of the *Rules* for the *Logarithms* in *Multiplication*, as is evident by *Inspection* of *Art.* 13. foregoing. And the *Methods* there mention'd for duely adjusting the *Indices*, are to be equally observed here.

18. In *Involution*, or raising the *Powers* of *Fractions*, 'tis evident that the *Distance* between *Unity* and the *Root*, is equal to the *Distance* between the *Root* and the *first Power*, the *first* and *second Power*, the *second* and *third Power*, and so on. Therefore since  $Aa = ab = ce = eg$ , &c.  $cd = \text{Square of } ab$ ,  $ef = \text{Cube}$ ,  $gh = \text{Biquadrate}$ , &c. *Power* of the *Root*  $ab$ . But since  $AB : ab :: ab : cd :: cd : ef :: ef : gh$ , &c. therefore the *Logarithms*  $tA + tc = 2ta$ , and so  $tc = 2ta - tA$ . That is, the *Logarithm* of the *Square* of the *Root* is equal to double the *Logarithm* of the *Root* less the *Logarithm* of *Unity*. Again,  $\frac{ta + te}{2} = tc = 2ta - tA$ , and so  $ta + te = 4ta - 2tA$ ; that is,  $te = 3ta - 2tA$ ; in Words, the *Logarithm* of the *Cube* is equal to triple the *Logarithm* of the *Root*, less double the *Logarithm*

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garithm of *Unity*. Moreover,  $\frac{tc+tg}{2} te = 3ta - 2tA$ , therefore  $tc + tg = 6ta - 4tA$ ; but  $tc = 2ta - tA$ , subduct this from the *last Equation*, there will remain  $tg = 4ta - 3ta$ ; or the Logarithm of the *Bi-quadrato*.

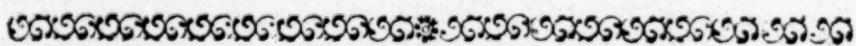
19. And *universally*, if the *Power* of a *Fraction* be  $x$ , and the *Logarithm* =  $L$ , the *Logarithm* of the *Power*  $x$  shall be  $= xL - x - 1 \times tA$ , or  $= xL - x tA + tA$ . Thus, suppose you would know the Logarithm of the *Square* of the *Fraction*  $dc$ ; here  $x = 2$ , and  $L = tc$ , therefore  $2tC - tA = tg$ , the Logarithm of the *Square*  $gh$ , as required. If the Logarithm of the *Cube* be desired of the *Fraction*  $ef$ , we have  $x = 3$ ,  $L = te$ , and so  $3te - 2tA = tr$ , the Logarithm of the *Cube*  $rs$ , as desired. And thus you proceed for the *Logarithms* of *other Powers*.

N. B. The *Indices* of Logarithms of *all Powers* of *Fractional Numbers* (I mean such as are *purely* so) must be *doubly dotted*, since those *Powers* always fall below the *Root*, which is supposed a *pure Fraction*.

20. *Evolution*, or the *Extraction* of the *Roots* of *Powers*, is just the *reverse* of the foregoing *Process*. For suppose the *Fraction*  $cd$  were given, whose *square Root* was required. Because  $AB : ab :: ab : cd$ , therefore  $AB \times cd = ab \times ab$ ; and so  $\sqrt{AB \times cd} = ab$ ; therefore the Logarithm  $\frac{tA+tc}{2} = ta$ , is the Logarithm of  $ab$  the *square Root* sought. Also if the *Cube Root* of the *Fraction*  $ef$  be required; because  $AB : ab :: cd : ef$ , we have  $AB \times ef = ab \times cd$ ; and so the Logarithms  $tA + te = ta + tc$ . But  $tA + te - ta = tc = 2ta - tA$ , that is,  $3ta = 2tA - te$ , and consequently  $\frac{2tA+te}{3} = ta$  the Logarithm of  $ab$  the *Cube Root* required.



21. And universally, if the Logarithm  $L$  of the Root of any Power  $x$ , of any Fraction  $rs$ , be required, we have this Theorem  $\frac{tr + xtA - tA}{x} = L$ , or  $\frac{tr + \frac{tA}{x} - tA}{x} = L = te$ , that is, in Words, the Number  $x - 1$   $xtA$  added to the Index of the Logarithm of the Fraction, the Logarithm thus augmented being divided by  $x$ , the Quotient shall be the Logarithm of the Root sought. Or, since  $tA = 10, 100$ , the Number  $x - 1$  prefix'd to the Index of the Logarithm of the Power, and the Logarithm thus divided by  $x$ , the Quotient shall be the Logarithm of the Root sought.



## CHAP. V.

*The original Construction of Logarithms by the Lord Neper, and the Alteration thereof to the present Form by himself and Mr. Briggs, explain'd and illustrated by the LOGARITHMIC CURVE.*

1. **T**HE noble Inventer of Logarithms, the Lord Neper, having duely contemplated their wonderful Nature, first constructed and publish'd a Canon thereof; but those far different from what we now commonly use. And this was no Wonder, since scarcely any thing receives its *Invention*, and *utmost Perfection* at the same time.

2. In the first kind of Logarithms that Neper published, the *first Term* of the *continual Proportionals*, was placed only *so far* distant from *Unity*, as *that Term* exceeded *Unity*. Thus, for Example, if  $on$  be the *first Term* of the Series from *Unity*  $AB$  (see Fig. I.) the Logarithm thereof, or the Distance  $An$ , or  $By$ , was by him put equal to  $vy$  or the *Increment*

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of the Number  $n$  y above Unity. If then we suppose  $vn=1.0000001$ , the Excess of this Number above  $AB=1$ , is  $0.0000001$ , which therefore, by him, was made its *Logarithm*; that is,  $An=0.0000001$ .

3. From hence, by Computation, the Number 10 will be the 23025850th *Term* of the *Series*; which Number therefore is the *Logarithm* of 10 in this *Form* of *Logarithms*: and expresses its Distance from Unity in such Part whereof  $vy$ , or  $An$  is one. Also the *Logarithm* of 2 (in this *Form*) is 6931471; of 3, is 10986122; of 4, is 13862943, &c.

4. But this *Position* of the *Ratio* of the *Terms* is entirely at *Pleasure*; for the *Distance* of the first *Term* may have any given *Ratio* to its *Excess* above *Unity*; that is,  $An$  may be indifferently less, equal to, or greater than  $vy$ ; and according to that various *Ratio* (which may be supposed at pleasure) between  $An$ , or  $By$  and  $vy$ , i. e. the *Increment* of the first *Term* above *Unity*, and the *Distance* of the same from *Unity*, there will be produced different *Forms* of *Logarithms*.

5. The *Logarithms* of this first *Form*, were found by the sagacious *Inventer* not to answer the *Design* in the best manner as could be wish'd; and therefore he changed 'em into another more convenient *Form*, wherein he put the Number 10, not as the 23025850th *Term* of the *Series*, but the 10000000th *Term*: And after *Neper's* Death, the learned and indefatigable Mr. Briggs, with great pains, made and publish'd a *Canon* of *Logarithms* according to this new *Form*. Now since in this *Canon* the *Logarithm* of 10 is 1.0000000, and since 1, 10, 100, 1000, &c. are *Proportionals*, they shall be equidistant from each other; wherefore the *Logarithm* of 100 shall be 2.0000000; of 1000, 3.0000000, and the *Logarithm* of 10000 will be 4.0000000, and so on. And this *Form* of *Logarithms* hath been ever since in use, and are those in present Use; the Nature and Properties

Properties of which we have been hitherto explaining.

6. The *Rationale* of the Method by which Mr. Briggs computed his Logarithms, is best explain'd from the *Logarithmic Curve*, according to Dr. Keil, as follows. In the *Logarithmic Curve* HBD (Fig. V.) let there be three *Proportionals* AB, ab, qs, very nearly equal to one another; that is, let their Differences have a *very small Ratio* to the said *Ordinates*, (for such are those *Proportionals*,) and then the *Differences* of the *Logarithms* will be *proportional* to the *Differences* of the *Ordinates*; that is, it will be  $sr : bc :: Br : Bc :: Aq : Aa$ . For since the *Ordinates* AB, ab, qs, are *nearly equal* to one another, they will be *very nigh* to one another; and so the *Parts* of the *Curve* Bs, Bb, intercepted between them, will *nearly coincide* with a *Right Line*; for it is possible that the *Ordinates* may be so near to each other, that the *Difference* between the *Part of the Curve* and the *Right Line* subtending it, may have to that *Subtense*, a *Ratio less* than any given *Ratio*. Consequently the *Triangles* Bcb, Brs, may be taken for *Right-lined* ones, and will be *equiangular*: and therefore, since ab is parallel to qs, they will be *similar*, and their *homologous Sides proportional*; viz.  $rs : bc :: Br : Bb$ , or  $Aq : Aa$ .

7. From hence, by the way, appears also the *Reason* of the *Correction* of *Numbers* and *Logarithms* by *Differences* and *proportional Parts*. For putting  $AB=1$ , or *Unity*, 'tis evident, that the *Logarithms* Bc, Br, are *proportional* to the *Differences* cb, rs, of the *Numbers* AB, ab, rs; as we shall hereafter prove by *Facts*, in the *practical Part*.

8. If a *mean Proportional* be found between 1 and 10, and then again another *Mean* between that and *Unity*; and if proceeding thus, you continually find a *mean Proportional* between the *Mean last found* and *Unity*, bisecting the *Logarithms* still as you proceed (in



### 38 Construction of LOGARITHMS,

(in the manner of the *Example* in Chap. II. Art. 6.) you will at last get a Number whose *Distance* from Unity shall be less than the  $\frac{1}{10000000000000000}$  Part of the Logarithm of 10.

9. After Mr. Briggs, in this manner, had made 54 *Extractions* of the *Square Root*, he arrived to the Number 1.00000 00000 00000 12781 91493 20032-3442, and its Logarithm was 0.00000 00000 00000-05551 11512 31257 82702. Suppose this Logarithm be equal to  $Aq$  or  $Br$ ; and let the Number found by this Extraction, be  $=qs$ ; and then its *Excess* above Unity will be  $=rs$ .

$$\text{That is, } \begin{cases} Aq = 0.00000\ 00000\ 00000\ 05551\ 11512- \\ \quad 31257\ 82702. \\ qs = 1.00000\ 00000\ 00000\ 12781\ 91493- \\ \quad 20032\ 3442. \\ rs = 0.00000\ 00000\ 00000\ 12781\ 91493- \\ \quad 20032\ 3442. \end{cases}$$

10. Now by means of *these* Numbers the Logarithms of all *other* Numbers may be found in the following Manner. Between the *given* Number (whose Logarithm is to be found) and *Unity*, find (by the Extraction of Roots, as above) so many *mean Proportionals* till at last a Number be obtain'd so little exceeding Unity, that there be 15 *Cyphers* next after it, and as many *significant Figures* after those. Suppose the small Number thus found be  $ab$ , and let the *significant Figures* with the 15 *Cyphers* prefix'd before them, denote the Difference  $bc$ ; then say, as the Difference  $rs$  is to the Difference  $bc$ , so is the *given* Logarithm  $Br$ , to  $Bc$  the Logarithm sought for the Number  $ab$ . If now this Logarithm  $Bc$  or  $Aa$ , be continually doubled the same number of times as there were *Extractions* of the *Square Root*, you'll have at last the Logarithm of the Number propos'd as required.

11. If the *Tangent*  $TB$  be drawn to touch the *Curve* in  $B$ , then may the *Subtangent*  $AT$  be found by

by the Numbers above in *Art. 9.* For since AB, Br, are parallel to rs, AT; therefore the *Right-lin'd Triangles* Brs and BAT, are similar, and so as  $sr : rB :: AB : AT$ , the Subtangent; but since  $AB=1$ , therefore  $\frac{rB}{sr} = AT$ ; or thus;

As  $rs = 0.00000\ 00000\ 00000\ 127819149320032-$   
3442,

Is to  $Br = 0.00000\ 00000\ 00000\ 055511151231257-$   
8270,

So is  $AB = 1.00000\ 00000\ 00000\ 00000\ 00000\ 00000$   
0000,

to  $AT = 0.434294481903251827651128918916-$   
6051.

12. If the *proportional Right Lines* GH, EF, AB, CD, (Fig. V.) are Ordinates to the Axis CV of the *Logarithmic Curve*, and if their *Ends* FH, DB, be join'd by *Right Lines*, which produced meet the *Axis* in the Points P and K, then the *Right Lines* GP, KA, will be *always equal*. For since  $GH : EF :: AB : CD$ ; it will be, as  $GH : Ft :: AB : DR$  (by Chap. II. Art. 8.) But because of the *similar Triangles* PGH HtF, as also KAB BRD, we have  $PG : Ht (:: GH : Ft :: AB : DR) :: KA : BR$ . But  $Ht = BR$ , and therefore  $PG = AK$ ; which was to be demonstrated.

13. If the *Right Lines* CD, EF *equally* accede to AB, GH, so that the Point D at last may coincide with B, and the Point F with H, then the *Right-Lines* DBK, FHP, which did *before* cut the Curve, will *now* only *touch* it in the Points B and H; that is, they will be chang'd into the *Tangents* BT, and HV. And so the *Right-Lines* AT, GV, will *always be equal* to each other; and so the *Subtangent* AT, or GV, in whatever Part of the Axis it be, is *always one constant given Length*; and this is one of the most remarkable and useful Properties of the *Logarithmic*

*garithmic Curve* : For the *different Species* or *Forms* of those Curves are determined by the *Subtangents*.

14. If the Excess *cb* of any Number *ab* extremely near Unity, or but a *small matter* exceeding it, be given, the *Logarithm* of its Distance from Unity *Aa* or *Bc*, will be known by means of the constant *Subtangent* *AT* ; for by Art. 11. we have  $bc : Bc :: AB : AT$  ; therefore  $AT \times bc = Bc \times AB = Bc$ , the *Logarithm* required. Thus also  $AT \times rs = Br$ , the *Logarithm* of the Number *qr* ; and so the *Logarithm* of any *prime Number* 2, 3, 7, 11, 13, &c. may be found independently of the *Logarithm* of any other Number.



## CHAP. VI.

*A Method of constructing the Logarithms derived and demonstrated from the Nature of Numbers only, by Dr. Edm. Halley.*

1. **T**HE *admirable Method* now before us, is one of the many great and wonderful Inventions and Discoveries of the celebrated Dr. Halley, the present *Astronomer Royal*, and *Fellow of the Royal Society* ; and whose Name amongst the *Literati* will be had in *everlasting Remembrance*. This Method not only comprehends all the Improvements made by others by means of the *Hyperbola* and other *Geometrical Figures*, but shews with great Accuracy from the *pure Properties* of Numbers (as most natural and agreeable in a *Business* purely *Arithmetical*, as the *Logarithmotechny* is) how the *Logarithms* may be produced to any desired Number of Places, with far greater *Ease* and *Expedition* than by any Method before known. According to him, therefore,



2. Suppose an *infinite Number* of equal *Ratio's* or *Ratiunculæ* between any two Terms in a continued *Scale* or *Series* of *Proportionals*; and those *Ratiunculæ* express the *Ratio* of those two Terms, as of 1 to  $1+x$ . If then between *Unity* (1) and any *Number* proposed ( $1+x$ ) there be taken any *Infinity* ( $n$ ) of mean *Proportionals*, the infinitely little *Augment* or *Decrement* of the first of these *Means* from *Unity* will be a *Ratiuncula* or *Fluxion* ( $\dot{x}$ ) of the *Ratio* of *Unity* to the said *Number*; and the Terms of the *Series* will stand thus; viz. 1.  $1+\dot{x}$ .  $\overline{1+\dot{x}}^2$ .  $\overline{1+\dot{x}}^3$ .  $\overline{1+\dot{x}}^4$ . &c. to  $\overline{1+\dot{x}}^n$ ; and the

Exponents  $\left\{ \begin{array}{l} 0. \quad 1. \quad 2. \quad 3. \quad 4. \quad \&c. \text{ to } n; \text{ or thus,} \\ 0. \quad \dot{x}. \quad 2\dot{x}. \quad 3\dot{x}. \quad 4\dot{x}. \quad \&c. \text{ to } n\dot{x}. \end{array} \right.$

From whence 'tis evident that not only the *Number* ( $n$ ) of the *Proportionals* or *Ratiunculæ*, but also their *Sum* ( $n\dot{x}$ ) may be put for the *Logarithm* of  $1+x$ . And thus also  $N\dot{y}$  may be put = *Logarithm* of  $1+y$ , and consequently it will be, as  $L, 1+x : L, 1+y :: n\dot{x} : N\dot{y}$ .

3. If  $\dot{x}=\dot{y}$ , that is, if the *Ratiunculæ* composing divers *Ratio's* have the same *Magnitude*, then are those *Ratio's* proportional to the *Numbers* of *Ratiunculæ* contain'd between their Terms, viz.  $L, 1+x : L, 1+y :: n : N$ . For if  $x=1$ , and  $y=2$ ; then  $1+x=2$ , and  $1+y=3$ ; and if  $n=30103$ , &c.  $N$  will be found = 47712, &c. that is, if there be the *infinite Number* 30103, &c. of *Ratiunculæ* in the *Ratio* of 1 to 2; there shall be the *infinite Number* 47712, &c. of the same *Ratiunculæ* in the *Ratio* of 1 to 3.

4. On the contrary, if  $n=N$ , then  $L, 1+x : L, 1+y :: \dot{x} : \dot{y}$ , that is, suppose the *infinite Number* of *Ratiunculæ* in one *Ratio* equal to the *infinite Number* of *Ratiunculæ* in any other *Ratio*, then are the *Logarithms* of those *Ratio's* directly as their *Fluxions*,

G

or

or as the *Magnitudes* of the *Ratiunculae* respectively. For instance, let the Ratio of 10 to 1, 100 to 1, 1000 to 1, &c. all and every of them be supposed to consist of 5 *Ratiunculae*; as follows: Thus,

Ratio's	0.	1.	2.	3.	4.	5.	&c.
Terms	1.	10.	100.	1000.	10000.	100000.	&c.
Ratiunc.	5v.	5w.	5x.	5y.	5z.		&c.

'Tis plain the *Ratiuncula* *v* of the whole Ratio of 10 to 1 is  $\frac{1}{5}$ ; of 100 to 1, is  $\frac{2}{5}$ , &c. that is, the *Ratiunculae* are  $v=\frac{1}{5}$ ,  $w=\frac{2}{5}$ ,  $x=\frac{3}{5}$ ,  $y=\frac{4}{5}$ ,  $z=\frac{5}{5}$ ; but those *Fractions* are as the *natural Numbers* 1, 2, 3, 4, 5. Wherefore the Logarithms of the Ratio's of 10 to 1, 100 to 1, 1000 to 1, &c. are directly as the *natural Numbers* 1, 2, 3, 4, 5, &c. and so may be expressed by them.

5. Since then the *Logarithms* of Ratio's are as their *Fluxions*, therefore the Logarithm of any Number is found by taking the *Difference* of Unity and the *infinite Root* of that Number; that is, because  $1+x$  is the first Term from Unity, or *Ratiuncula*,  $\overline{1+x}^n = 1+x$ , is the *infinite Power* to be resolv'd; and  $\sqrt[n]{1+x}$ , or  $\overline{1+x}^{\frac{1}{n}} = 1+x$ , and so  $\overline{1+x}^{\frac{1}{n}} - 1 = x =$  Logarithm of  $1+x$ .

6. In order to extract the Root of the infinite Power  $1+x$ , (which, to some, may seem *strange* and next to *impossible*) we must make use of Sir *Isaac Newton's* celebrated Theorem for that purpose. Supposing then the Power be  $1+x$ , according to his Theorem,  $\overline{1+x}^{\frac{1}{n}} = 1 + \frac{1}{n}x + \frac{1-n}{2nn}xx + \frac{1-3n+2nn}{6n^3}x^3 + \frac{1-6n+11nn-6n^3}{24n^4}x^4$ , &c. the Root of the Power  $1+x$ , when the *Index* (*n*) is *finite*; but (*n*) being in the present Case *infinite*, all the Terms of the *Coefficients*, wherein (*nn*) is found a Divisor (as being *infinitely infinite*) will vanish, as being *infinitely less* than

than nothing. But the Co-efficient  $\frac{1-n}{2nn} = \frac{1}{2nn} - \frac{1}{2n} =$   
 $-\frac{1}{2n}$ ; and  $\frac{1-3n+2nn}{6n^3} = \frac{-1}{6nn} + \frac{1}{3n} = \frac{1}{3n}$ ; Also the  
 Coefficient  $\frac{1-6n+11nn-6n^3}{24n^4} = \frac{1}{12nn} - \frac{1}{4n} = \frac{1}{4n}$ ; &c.

Wherefore the foregoing Root will become  $1 + \frac{1}{n}x -$   
 $\frac{1}{2}nx^2 + \frac{1}{3}nx^3 - \frac{1}{4}nx^4 + \frac{1}{5}nx^5$ , &c. That is,  
 $\frac{1}{n}xx - \frac{1}{2}xx + \frac{1}{3}xxx - \frac{1}{4}x^4 - \frac{1}{5}x^5$ , &c.  $= 1 + x^n - 1 = x =$   
 Logarithm of  $1+x$ .

7. And whereas the *infinite Index* ( $n$ ) may be taken  
 at pleasure, an *Infinity* of different Scales of Loga-  
 rithms may be produced; and those different Loga-  
 rithms will be to one another as  $\frac{1}{n}$ , or *reciprocally* as  
 the *Indices* ( $n$ ). And as it hath been shewn (in  
 Chap. V.) that in making the *first kind of Logarithms*  
 by *Neper*, the *infinite Index* of the Logarithm of 10  
 would be 23025850, &c. But in making the se-  
 cond sort after by *Briggs*, the said *infinite Index* was  
 put = 10000000, &c. Consequently in the fore-  
 going Series for Logarithms, if  $n=10000000$ , &c.  
 the Lord *Neper's* Logarithms will be produced; and  
 the Series will be *simply*  $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$ , &c.  
 On the contrary, if  $n=23025850$ , &c. then *Briggs's*  
*Logarithms* will arise from the Series; and because  
 $\frac{1}{n} = \frac{1}{23025850}$ , &c.  $= 0.434229448$ , &c. therefore  
 $\frac{1}{n}$  (=AT) is the *Subtangent* of the *Logarithmic Curve*  
 for the *Briggian Logarithms*, as is plain from Chap. I.  
 V. Art. 11. Whence if a Logarithm of *Neper's*  
*Form* be multiplied by 0.434229448, &c. or divided  
 by 2.3025850, &c. it is converted into a Logarithm  
 of *Briggs's Sort*, or those in present Use.

8. If the Logarithm of a decreasing Ratio be  
 sought, as of 1 to  $1-x$ , the Power being  $1-x$ , its  
*infinite Root* will be  $\frac{1}{1-x^n} = 1 + \frac{1}{n}x + \frac{1}{2}nx^2 + \frac{1}{3}nx^3 +$   
 $\frac{1}{4}nx^4 +$



$\frac{1}{2}nx^4 - \frac{1}{3}nx^5$ , &c. that is,  $\frac{1}{n} \times x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5$ ,

&c.  $= 1 - 1 - x^n = x$  = the Logarithm of  $1 - x$ ; the first Term next below Unity, or Root of this infinite decreasing Series being  $1 - x$ . And so in this Case, according as the infinite Index ( $n$ ) is made  $= 10000000$ , &c. or  $2.3025850$ , &c. so Neper's or Briggs's Logarithms of those negative Numbers will be produced.

9. Instead of the Terms  $1 : 1 + x$ , let  $a : b$  express the Terms of any Ratio universally; and make  $a + b = s$ , and  $a - b = d$ ; and since it is  $1 : 1 + x :: a : b$ , therefore  $a + ax = b$ , and so  $x = \frac{b-a}{a} = \frac{d}{a}$ . Again because (in the decreasing Ratio) it is  $1 : 1 - x :: b : a$ ; therefore  $b - bx = a$ , and so we have again  $x = \frac{b-a}{b} = \frac{d}{b}$ . Whence the Logarithm of the same Ratio  $a : b$ , may be doubly express'd; viz. for the encreasing Ratio, the Series will be

$$\frac{1}{n} \times \frac{d}{a} - \frac{d^2}{2a^2} + \frac{d^3}{3a^3} - \frac{d^4}{4a^4} + \frac{d^5}{5a^5}, \text{ \&c. or,}$$

$$\frac{1}{n} \times \frac{d}{b} + \frac{d^2}{2bb} + \frac{d^3}{3b^3} + \frac{d^4}{4b^4} + \frac{d^5}{5b^5}, \text{ \&c. for}$$

the decreasing Ratio; all which is evident from the three last Articles.

10. But if we suppose the Ratio of  $a$  to  $b$ , viz.  $\frac{a}{b}$  composed of two Parts; viz. of the Ratio of  $a$  to the Arithmetical Mean between the Terms  $a$  and  $b$ , and of the Ratio of the said Arithmetical Mean to the other Term  $b$ ; that is, suppose  $\frac{a}{b} = \frac{a}{\frac{1}{2}s} \times \frac{\frac{1}{2}s}{b}$ , (for  $\frac{1}{2}s = \frac{a+b}{2}$  is the Arithmetical Mean between  $a$  and  $b$ ;) then the Sum of the Logarithms of those two Ratio's,  $\frac{a}{\frac{1}{2}s}$ ,  $\frac{\frac{1}{2}s}{b}$ , will be equal to the Logarithm of the Ratio of  $a$  to  $b$ . Or,  $L, \frac{a}{\frac{1}{2}s} + L, \frac{\frac{1}{2}s}{b} = L, \frac{a}{b}$ . And

And thus also we have  $L, \frac{1}{2}s + L \frac{b}{\frac{1}{2}s} = L \frac{b}{a}$ .  
 Now because the Ratio of  $\frac{1}{2}s$  to  $b$  is encreasing, therefore  $1 : 1 + x :: \frac{1}{2}s : b$ ; and so  $1 + x = \frac{b}{\frac{1}{2}s}$ ; consequently  $x = \frac{b}{\frac{1}{2}s} - 1 = \frac{b - \frac{1}{2}s}{\frac{1}{2}s} = \frac{d}{s}$  a-new; again, because the Ratio of  $\frac{1}{2}s$  to  $a$  is decreasing; therefore  $\frac{1}{2}s : a :: 1 : 1 - x$ , and so  $1 - x = \frac{a}{\frac{1}{2}s}$ ; and  $x = 1 - \frac{a}{\frac{1}{2}s} = \frac{\frac{1}{2}s - a}{\frac{1}{2}s} = \frac{d}{s}$ .

11. Therefore since  $x = \frac{d}{s}$  for both Ratio's, viz. of  $a$  to  $\frac{1}{2}s$ , and  $\frac{1}{2}s$  to  $b$ , we shall have (by the foregoing Rule, Art. 9.)  $\frac{1}{n} \times \frac{d}{s} + \frac{d^2}{2s^2} + \frac{d^3}{3s^3} + \frac{d^4}{4s^4} + \frac{d^5}{5s^5}$ , &c. = (A)  $L, a$  to  $\frac{1}{2}s$ , and  $\frac{1}{n} \times \frac{d}{s} - \frac{d^2}{2s^2} + \frac{d^3}{3s^3} - \frac{d^4}{4s^4} + \frac{d^5}{5s^5}$ , &c. = (B)  $L, \frac{1}{2}s$  to  $b$ . Then  $\frac{1}{n} \times \frac{2d}{s} * + \frac{2d^3}{3s^3} * + \frac{2d^5}{5s^5}$ , &c. = (A + B)  $L, a$  to  $b$ .

Thus you have a Series expressing the Logarithm of the Ratio of  $a$  to  $b$ , whose Sum is  $s = a + b$ , and Difference  $d = a - b$ : and this Series converges twice as swift as the former in Art. 8. and therefore is more proper for making or examining of Logarithms, which it performs with great Expedition, when  $d$  the Difference is but the 100th Part of  $s$  the Sum; the first Step  $\frac{2d}{s}$  sufficing for 7 Places of the Logarithm, and the second  $\frac{2d^3}{3s^3}$  for 12 Places.

12. Because the Difference of the Logarithms of the Ratio's of  $a$  to  $\frac{1}{2}s$ , and  $\frac{1}{2}s$  to  $b$  is the Logarithm of the Ratio of  $a$  to  $b$  to  $\frac{1}{4}ss$ ; or thus, because

$$\frac{\frac{1}{2}s}{b}$$

$\frac{\frac{1}{2}s}{b}) \frac{a}{\frac{1}{2}s} (= \frac{ab}{\frac{1}{4}ss})$ , therefore the  $L, \frac{a}{\frac{1}{2}s} - L, \frac{\frac{1}{2}s}{b} = L, \frac{ab}{\frac{1}{4}ss}$   
 $(A-B) = \frac{1}{n} \times \frac{2d^2}{2s^2} * + \frac{2d^4}{4s^4} * + \frac{2d^6}{6s^6}$ , &c. but  
 half the Ratio  $\frac{ab}{\frac{1}{4}ss}$  is the Ratio  $\frac{\sqrt{ab}}{\frac{1}{2}s}$  (for  $\frac{\sqrt{ab}}{\frac{1}{4}s} \times \frac{\sqrt{ab}}{\frac{1}{2}s} =$   
 $\frac{ab}{\frac{1}{4}ss}$ ), that is, the Ratio of the *Geometrical Mean*  
 $\sqrt{ab}$  to the *Arithmetical Mean*  $\frac{1}{2}s$ ; consequently  
 the Logarithm of  $\frac{\sqrt{ab}}{\frac{1}{2}s} = \frac{1}{n} \times \frac{d^2}{2s^2} * + \frac{d^4}{4s^4} * + \frac{d^6}{6s^6}$ ,  
 &c. which is a Theorem of good Dispatch for find-  
 ing the Logarithm of  $\frac{1}{2}s$ .

13. But the same Logarithm may yet be much more  
 advantageously obtained by a Method like the for-  
 mer. For if we make the Terms of the Ratio  
 $\frac{ab}{\frac{1}{4}ss} = \frac{A}{B}$ , and put  $S = ab + \frac{1}{4}ss$ , the Sum of the  
 Terms, and  $D = ab - \frac{1}{4}ss$ ; 'tis evident the  
 Logarithm of  $\frac{A}{B} (= \frac{ab}{\frac{1}{4}ss}) = \frac{1}{n} \times \frac{2D^2}{S} + \frac{2D^3}{3S^3} +$   
 $\frac{2D^5}{5S^5} + \frac{2D^7}{7S^7}$ , &c. by Art. 11. But because  $\frac{1}{4}ss =$   
 $\frac{1}{4}a^2 + \frac{1}{2}ab + \frac{1}{4}b^2$ , therefore  $D = ab - \frac{1}{4}ss = ab -$   
 $\frac{1}{4}a^2 + \frac{1}{2}ab + \frac{1}{4}bb = \frac{1}{4}a^2 - \frac{1}{4}ab + \frac{1}{4}bb = \sqrt{\frac{1}{2}a - \frac{1}{2}b}$   
 $= \frac{1}{4}d^2 = 1$ , in the present Case of finding the Loga-  
 rithms of Prime Numbers; for suppose the Loga-  
 rithm of 23 be sought, then  $a = 22$ ,  $b = 24$ ,  $\frac{1}{2}s = 23$ ,  
 and  $d = 2$ ; also  $A = ab = 528$ , and  $B = \frac{1}{4}ss = 529$ ,  
 and therefore  $ab - \frac{1}{4}ss = A - B = D = \frac{1}{4}d^2 = 1$ . Where-  
 fore, since  $D = 1$ , the Series above becomes  $\frac{1}{n} \times \frac{2}{S} +$   
 $\frac{2}{3S^3} + \frac{2}{5S^5} + \frac{2}{7S^7}$ , &c. = the Logarithm of the  
 Ratio of  $\frac{ab}{\frac{1}{4}ss}$ ; and so the half of it, viz. the Series  
 $\frac{1}{n} \times \frac{1}{S} + \frac{1}{3S^3} + \frac{1}{5S^5} + \frac{1}{7S^7}$ , &c. = the Loga-  
 rithm



rithm of the Ratio of  $\sqrt{ab}$  to  $\frac{1}{2}s$ . And this converges much sooner than any Theorem hitherto invented, and beyond which nothing better can be hoped for, in the *great Author's* Opinion.

14. The *Logarithm* given to find what *Ratio* it expresses, is a Problem solv'd with *like Ease*, and demonstrated by a *like Process* to that foregoing for finding the Logarithm of a *given Ratio*. For as the Logarithm of the Ratio of 1 to  $1+x$  was proved to be  $\frac{1}{1+x^n}-1$ , (by Art. 5.) and that of the Ratio of 1 to  $1-x$  to be  $1-\frac{1}{1-x^n}$  (by Art. 8.) so the Logarithm, which we will call  $L$ , being given, since  $L=\frac{1}{1+x^n}-1$ , therefore  $L+1=\frac{1}{1+x^n}$  in the first Case; and  $1-L=\frac{1}{1-x^n}$ , in the latter: Consequently  $\frac{1}{1+L^n}=1+x$ , and  $\frac{1}{1-L^n}=1-x$ . That is, according to Sir *Isaac's Theorem*,  $1+nL+\frac{1}{2}n^2L^2+\frac{1}{6}n^3L^3+\frac{1}{24}n^4L^4+\frac{1}{120}n^5L^5$ , &c.  $=1+x$ ; and also  $1-nL+\frac{1}{2}n^2L^2-\frac{1}{6}n^3L^3+\frac{1}{24}n^4L^4-\frac{1}{120}n^5L^5$ , &c.  $=1-x$ ; consequently the Number  $1+x$  or  $1-x$  is readily known by those Series, be the *Species* of the Logarithm what it will. That is, whether it be *Neper's* Logarithm, where  $n=100000$ , &c. and so  $1+x=1\pm L+\frac{1}{2}L^2\pm\frac{1}{6}L^3+\frac{1}{24}L^4\pm\frac{1}{120}L^5$ , &c. or whether  $n=23025850$ , &c. for *Briggs's* Logarithm.

15. If one Term of the *Ratio*, whereof  $L$  is the Logarithm, be given, the other Term will easily be had by the same Rule. Let  $a$  = the *least Term* of the *Ratio*, and  $b$  = the greatest; then because it is  $1:1+x::a:b$ ; and so  $1+x=\frac{b}{a}=1+L+\frac{1}{2}L^2+\frac{1}{6}L^3+\frac{1}{24}L^4$ , &c. if  $n=1000000$ , &c. and therefore  $b=a+aL+\frac{1}{2}aL^2+\frac{1}{6}aL^3$ , &c. if  $a$  were given; but if  $b$  were given, because  $1:1-x::b:a$ ,  
therefore

therefore  $1 - x = \frac{a}{b}$ ; and so  $a = b - bL + \frac{1}{2}bL^2 - \frac{1}{6}bL^3 + \frac{1}{24}bL^4, \&c.$  Wherefore by the Help of the *Tables*, the Number belonging to any Logarithm will be *exactly had* to the utmost Extent of the *Tables*.

16. Suppose  $\frac{b}{a} = N$ , the Number belonging to the given Logarithm  $L$  of the Ratio  $\frac{b}{a}$ , then  $\frac{b}{N} = a$ , let the Logarithm of the Ratio  $\frac{b}{N}$  be  $\mathfrak{E}$ , and let the Term  $b$  be known; then (*per Rule*, Art. 15.) we have  $1 : 1 - x :: b : N$ , and so  $1 - x = \frac{N}{b}$ , and  $N = b - b\mathfrak{E} + \frac{1}{2}b\mathfrak{E}^2 - \frac{1}{6}b\mathfrak{E}^3, \&c.$  if  $\mathfrak{E}$  be *Neper's* Logarithm, but  $N = b - bn\mathfrak{E} + \frac{1}{2}bn^2\mathfrak{E}^2 - \frac{1}{6}n^3b\mathfrak{E}^3, \&c.$  if  $n = 2.3025, \&c.$  as in *Briggs's* Logarithm. But if the Ratio be  $\frac{a}{b} = N$ , then  $\frac{a}{N} = b$ , and so  $1 + x = \frac{N}{a}$ , therefore  $N = a + a\mathfrak{E} + \frac{1}{2}a\mathfrak{E}^2 + \frac{1}{6}a\mathfrak{E}^3, \&c.$  Or  $N = a + an\mathfrak{E} + \frac{1}{2}an^2\mathfrak{E}^2 + \frac{1}{6}n^3a\mathfrak{E}^3, \&c.$  Note, here  $a$  and  $b$  denote the Number belonging to the nearest next lesser or next greater Logarithm than the given Logarithm  $L$ , and the Logarithm  $\mathfrak{E}$  is the *Difference* of those Logarithms; wherefore as  $\mathfrak{E}$  is less, the Series converges the swifter; and finds the Number  $N$  of the Logarithm  $L$ , much sooner and easier than the Rule in Art. 15.

17. In the foregoing Series  $a + a\mathfrak{E} + \frac{1}{2}a\mathfrak{E}^2 + \frac{1}{6}a\mathfrak{E}^3, \&c. = N$ ; the three first Steps may be abridg'd thus,  $a + \frac{a\mathfrak{E}}{1 - \frac{1}{2}\mathfrak{E}} = a + a\mathfrak{E} + \frac{1}{2}a\mathfrak{E}^2$ , very nearly, and may serve with *Exactness* enough for Numbers not exceeding 14 Places, which is more than sufficient for *common Use*. Therefore we may take

$a +$

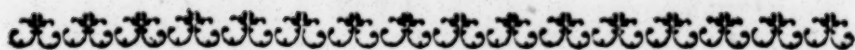
$a + \frac{a^{\frac{1}{2}}}{1 - \frac{1}{2}} = N$ , or  $b - \frac{b^{\frac{1}{2}}}{1 + \frac{1}{2}} = N$ ; and if the Index  $n$  taken for Briggs's Logarithms, we shall have  $a + \frac{a^{\frac{1}{2}}}{\frac{1}{n} - \frac{1}{2}} = N$ , or  $b - \frac{b^{\frac{1}{2}}}{\frac{1}{n} + \frac{1}{2}} = N$ ; that is (putting  $z = \frac{1}{n} = .43429$ , &c.)  $\frac{za + \frac{1}{2}a^{\frac{1}{2}}}{z - \frac{1}{2}} = N$ , or  $\frac{zb - \frac{1}{2}b^{\frac{1}{2}}}{z + \frac{1}{2}} = N$ ; which Equation may be resolv'd into the following Analogies;

$$\text{viz. } \begin{cases} z - \frac{1}{2} : z + \frac{1}{2} :: a : N; \text{ or,} \\ z + \frac{1}{2} : z - \frac{1}{2} :: b : N. \end{cases}$$

18. If more Steps of *this Series* be desir'd, it will be found as follows, viz.  $a + \frac{a^{\frac{1}{2}}}{1 - \frac{1}{2}} - \frac{\frac{1}{2}a^{\frac{3}{2}}}{1 - \frac{1}{2}} + \frac{\frac{1}{2}a^{\frac{5}{2}}}{1 - \frac{1}{2}}$ , &c.  $= N$ ; also the Rule  $1 + nL + \frac{1}{2}nnLL + \frac{1}{6}n^3L^3$ , &c. may be thus contracted, viz.  $1 + 2 + nL + \frac{1}{2}nnLL \times \frac{1}{2}nL = N$ . What is said concerning this Method of making either *Logarithms* or *Numbers*, I presume is sufficient to render it very intelligible to any common Capacity, and to shew the admirable *Usefulness* and *Excellency* thereof beyond any other, hitherto invented.







## C H A P. VII.

*The \*Logarithmic Series aforegoing, demonstrated also by \*FLUXIONS, and from the Nature of the \*Hyperbola.*

1. **T**HE preceding Series for the Logarithms, which has been demonstrated *purely* from *Arithmetical Principles*, or the *Properties of Numbers*, may also be prov'd from the *Doctrine of the Fluxions of Logarithms*. For the Writers on *Fluxions* variously demonstrate the *Fluxion of the Logarithm of any Number* is equal to the *Fluxion of that Number (whose Logarithm it is) divided by the said Number it self*.

2. Let the Number proposed be  $1+x$ , the Fluxion of which is  $\dot{x}$ , therefore the *Fluxion of its Logarithm* will be  $= \frac{\dot{x}}{1+x}$ ; from whence the foregoing infinite Series for the Logarithm of the Number  $1+x$  may be derived, as follows. The Logarithm of  $1+x$  is equal to the *flowing Quantity* or *Fluent* of the said Fluxion  $\frac{\dot{x}}{1+x}$ . But  $\frac{\dot{x}}{1+x} = \dot{x} \times \frac{1}{1+x}$ ; and  $\frac{1}{1+x} = 1+x) 1 (=1-x+xx-x^3, \&c.$

$$\begin{array}{r}
 1+x \\
 \cdot -x \\
 \hline
 -x-xx \\
 \cdot +xx \\
 \hline
 +xx+x^3 \\
 \cdot -x^3 \\
 \hline
 -x^3-x^4 \\
 \cdot +x^4, \&c.
 \end{array}$$

3. The Quotient then  $1 - x + x^2 - x^3 + x^4, \&c.$   
 $= \frac{1}{1+x}$ ; but  $\dot{x} \times 1 - x + x^2 - x^3 + x^4, \&c. = \dot{x} -$   
 $x\dot{x} + x^2\dot{x} - x^3\dot{x} + x^4\dot{x}, \&c. = \frac{\dot{x}}{1+x}$ . The *Fluent* there-

fore of that *infinite fluxionary Series* (by the *inverse Method of Fluxions*) is found to be  $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5, \&c.$  which therefore is the Logarithm of the Number  $1+x$ ; and is the very same with that Series in the foregoing Chap. VI. Article 7. for *Neper's Logarithms*.

4. Again, if the Ratio be *decreasing*, or the Number be  $1-x$ , the Fluxion of this also is  $\dot{x}$ , and therefore the Fluxion of its Logarithm  $\frac{\dot{x}}{1-x}$ . But

$\frac{\dot{x}}{1-x} = \dot{x} \times \frac{1}{1-x} = 1+x+x^2+x^3, \&c. \times \dot{x} = \dot{x} +$   
 $x\dot{x} + x^2\dot{x} + x^3\dot{x}, \&c.$  The *Fluent* of which is  $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4, \&c.$  the same Series as that in Article 8. of the *preceding Chapter*, for *Neper's Logarithm* of  $1-x$ .

5. The *same Series* is likewise deduced from the Nature or Equation of the *Hyperbola*. For let FCH be an *Hyperbola* (Fig. VI.) AE, AI, the *Asymptotes*; draw BC, DC parallel to AI and AE; also draw the Ordinate EF parallel to the Ordinate BC, or *Asymptote* AI. Let  $AB=a$ ,  $EF=y$ , and  $BE=x$ . The Equation  $aa=ay+xy$  expresses the Nature of the *Hyperbola* between the *Asymptotes*. Now the Fluxion of the Space between the *Abscissa*, *Ordinate*, and *Curve*, is always equal to the *Product* of the Ordinate into the *Fluxion* of the *Abscissa*; that is, in this Case,  $=y\dot{x}$ . Therefore to determine the Fluxion of the *Asymptotic Space* contain'd between the *Abscissa* BE, the *Ordinates* EF, BC, and the *Curve* of the *Hyperbola* FC, we have  $\frac{aa}{a+x} = y$ , therefore  $\frac{aa\dot{x}}{a+x} =$   
 $y\dot{x} =$  Fluxion of the said Space FCBE.

6. But  $\frac{aax}{a+x} = \frac{aa}{a+x} \times x$ , and  $\frac{aa}{a+x} = a+x) aa$   
 $(=a-x+\frac{xx}{a}-\frac{x^3}{aa}+\frac{x^4}{a^3}, \&c.$  Wherefore  $\frac{aax}{a+x} =$   
 $ax - xx + \frac{x^3x}{a} - \frac{x^3x}{aa} + \frac{x^4x}{a^3}, \&c.$  But the *flow-*  
*ing Quantity* of this *fluxionary Series* is  $ax - \frac{1}{2}x^2 +$   
 $\frac{x^3}{3a} - \frac{x^4}{4aa} + \frac{x^5}{5a^3}, \&c. =$  the Space FCBE. Sup-  
 pose  $a=1$ , then  $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5, \&c. =$  the  
 said *Asymptotic Space* FCBE, as before. But 'tis evi-  
 dent this is again the very same Series as was invented  
 by Dr. Halley for the *Logarithm of the Number*  $1+x$ .

7. The *Asymptotic Spaces*, then, are with respect  
 to the *Abscissæ*, as *Logarithms* in respect to *Numbers*.  
 That is, since  $AB=1$ , and the *Logarithm of 1* is  $=0$ ,  
 the Spaces  $BabC$ ,  $BcdC$ ,  $BegC$ ,  $BbkC$ , &c.  
 are the *Logarithms* of the *Numbers*  $Aa$ ,  $Ac$ ,  $Ae$ ,  
 $Ab$ , &c. Again, because the *Abscissæ* are in a *re-*  
*ciprocal* Proportion of the *Ordinates*, that is,  
 $AB : AE :: EF : BC$ ; therefore the *Asymptotic Spaces*  
 are in respect of the *Ordinates* as *Logarithms* in re-  
 spect of *Numbers*: Yet so, that while the *Ordinates*  
 $BC$ ,  $ab$ ,  $cd$ ,  $eg$ ,  $bk$ ,  $EF$ , decrease in a *Geometrical*  
*Ratio*, the Spaces  $BabC$ ,  $BcdC$ , &c. may in-  
 crease in an *Arithmetical Ratio*. And since in *Ne-*  
*per's Logarithms*  $\frac{1}{n} \times x - \frac{1}{2}xx + \frac{1}{3}xxx - \frac{1}{4}x^4, \&c.$   
 $n=100000$ , &c. 'tis plain his *Logarithms* become  
 the same with the *Hyperbolic Logarithms* just now  
 consider'd.





C H A P. VIII.

*The Method of constructing Logarithms by the Infinite Series, exemplified and illustrated.*

1. **T**HE Manner of raising *Theorems* for the Construction of Logarithms hath been sufficiently explain'd; it therefore remains that we illustrate the same by proper Examples. The *Theorems* for doing this directly are,

Theorem I.  $\frac{1}{n} \times x = \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5, \&c.$   
 $= L, \overline{1 \pm x}.$

Theorem II.  $\frac{1}{n} \times \frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5} + \frac{2d^7}{7s^7}, \&c.$   
 $= L, \frac{a}{b}.$

Theorem III.  $\frac{1}{n} \times \frac{d^2}{2s^2} + \frac{d^4}{4s^4} + \frac{d^6}{6s^6} + \frac{d^8}{8s^8},$   
 $\&c. = L.$

Theorem IV.  $\frac{1}{n} \times \frac{1}{s} + \frac{1}{3s^3} + \frac{1}{5s^5} + \frac{1}{7s^7},$   
 $\&c. = L$

Note, in these *Theorems*,  $\frac{1}{n}$  is all along applied to adapt them to *all sorts of Logarithms*.

2. Since  $n =$  the Logarithm of 10, we must therefore first suppose  $n = 10000000, \&c.$  and thence *Neper's Logarithms* will be produced; and so these are the first sort of Logarithms which Nature affords: The others, as *Briggs's Logarithms*, &c. are, made from them. In order then to find a *Briggian Logarithm*, 'tis necessary first to find *Neper's Logarithm* of 10. This may be done several Ways, either by the *Number 10 it self*, or by its *component Parts*. If we attempt it by the *Number 10 it self*, then because

$$1 + x =$$

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$1+x=10$ , we shall have  $x=9$ , which, because it is greater than 1, will occasion that the first Theorem will not converge; and so the second Theorem must be used. In this  $\frac{a}{b}=\frac{1}{10}$ , and therefore  $s=1+10=11$ , and  $d=10-1=9$ . And thus the second Series for *Neper's* Logarithm of 10 will be  $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$ , &c.  $= \frac{18}{11} + \frac{1458}{1331} + \frac{117858}{1677161}$ , &c.  $= 2.30258$ , &c. the Logarithm sought.

3. But this Series converging so extremely slow, renders the Business very tedious, and therefore the said Logarithm must be attempted from the component Parts of 10. And since  $8 \times 1\frac{1}{4}=10$ , and  $2 \times 2 \times 2=8$ , therefore  $3L, 2+L, 1\frac{1}{4}=L, 10$ . Consequently by finding the Logarithm of 2 and  $1\frac{1}{4}$ , we find the Logarithm of 10. Now *Neper's* Logarithm of 2, is found either by Theorem I. which converges very slowly; or by Theorem II. which converges much faster; and therefore to be chosen. Here  $\frac{a}{b}=\frac{1}{2}$ , and  $a+b=s=3, a-b=d=1$ , and so

the Theorem  $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$ , &c.  $= \frac{2d}{s} + \frac{\frac{1}{3} \times \frac{2d^3}{s^3}}{\frac{1}{3} \times \frac{2d^5}{s^5}}$ , &c.  $= \frac{2}{3} + \frac{1}{3} \times \frac{2}{3^3} + \frac{1}{3} \times \frac{2}{3^5}$ , &c.  $= L, 2$ ; this multiplied by 3, is  $= \frac{6}{3} + \frac{1}{3} \times \frac{6}{3^3} + \frac{1}{3} \times \frac{6}{3^5}$ , &c.  $= 3L, 2$ . Now put  $\frac{6}{3^3}=A$ , and because  $\frac{1}{3^2} \times \frac{6}{3^3} = \frac{6}{3^5}$ , and  $\frac{1}{3^2} = \frac{1}{9}$ , therefore also put  $\frac{1}{9}A=B$ , and so  $B=\frac{6}{3^5}$ ; and thus  $\frac{1}{9}B=C, \frac{1}{9}C=D$ , and so on. Whence the Theorem will become  $2 + \frac{1}{3}A + \frac{1}{9}B + \frac{1}{27}C + \frac{1}{81}D + \frac{1}{243}E + \frac{1}{729}F$ , &c.  $= 3L, 2$ .

4. By the same Theorem II. we obtain *Neper's* Logarithm of  $1\frac{1}{4}$ ; for because  $\frac{5}{4} = 1\frac{1}{4}$ , therefore  $\frac{1}{1\frac{1}{4}} = \frac{4}{5}$   $\left( = \frac{4}{5} = \frac{a}{b} \right)$ . Whence  $a + b = s = 9$ , and  $d = a - b = 1$ ; and so the Theorem  $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$ , &c.  $= \frac{2}{9} + \frac{1}{3} \times \frac{2}{9^3} + \frac{1}{5} \times \frac{2}{9^5}$ , &c. But because  $\frac{2}{9} = \frac{6}{3^3} = A$ ;  $\frac{2}{9^3} = \frac{6}{3^7} = \frac{6}{3^5} \times \frac{1}{9} = \frac{1}{9}B = C$ ;  $\frac{2}{9^5} = \frac{6}{3^{11}} = \frac{6}{3^9} \times \frac{1}{9} = \frac{1}{9}D = E$ , &c. therefore the said Theorem will become  $A + \frac{1}{3}C + \frac{1}{5}E + \frac{1}{7}G + \frac{1}{9}I + \frac{1}{11}L$ , &c.  $= L$ ,  $1\frac{1}{4}$  as required. If now this Series be added to the foregoing (in Art. 3.) we shall have the Theorem  $2 + \frac{1}{3}A + \frac{1}{5}B + \frac{1}{7}C + \frac{1}{9}D + \frac{1}{11}E + \frac{1}{13}F + \frac{1}{15}G + \frac{1}{17}H$ , &c.  $= 3L$ ,  $2 + L$ ,  $1\frac{1}{4} = L$ , 10. See the Operation in the Table below.

	2. = 2.00000000000000
A = 0.22222222222222	$\frac{1}{3}A = .2962962962962$
B = . . . 246913580246	$\frac{1}{5}B = . . . 49382716049$
C = . . . 27434842249	$\frac{1}{7}C = . . . 13064210595$
D = . . . . 3048315805	$\frac{1}{9}D = . . . . 338701756$
E = . . . . . 338701756	$\frac{1}{11}E = . . . . . 98531420$
F = . . . . . 37633529	$\frac{1}{13}F = . . . . . 2894887$
G = . . . . . 4181503	$\frac{1}{15}G = . . . . . 876124$
H = . . . . . 464612	$\frac{1}{17}H = . . . . . 27330$
I = . . . . . 51624	$\frac{1}{19}I = . . . . . 8454$
K = . . . . . 5763	$\frac{1}{21}K = . . . . . 273$
L = . . . . . 637	$\frac{1}{23}L = . . . . . 85$
M = . . . . . 71	$\frac{1}{25}M = . . . . . 3$

Thus *Neper's* Log. of 10 = 2.3025850929940, &c.

5. The Logarithm thus found (if continued on) will be 2.302585092994045684017991454684364207601101488628772976033328, &c.  $= n$ ; and therefore



fore in making the *Briggian Logarithms*, we shall have  $\frac{1}{n} = 0.434294481903251827651128918916605082294397005803666566114454$ , &c. the Reciprocal of the former; which henceforth let be call'd  $z$ ; that is, let  $\frac{1}{n} = z$ . And now we are prepar'd to find the Logarithm (of any Form) of any other Number.

6. For Example, let it be required to find *Briggs's* Logarithm of 2, to 10 Places of Figures. In order to this, the *Index*  $\frac{1}{n}$  must be assum'd of a *Figure or two* more than the intended Number of Places in the Logarithm. The *second* Theorem is most proper for this Purpose; for here again  $\frac{a}{b} = \frac{1}{2}$ ,  $d = 1$ , and  $s = 3$ , and also  $z = 0.434294481903$ ; and the Theorem  $\frac{1}{n} \times \frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$ , &c.  $= z \times \frac{2}{3} + \frac{\frac{1}{3} \times \frac{2}{3^3} + \frac{1}{5} \times \frac{2}{3^5}}{\frac{1}{3} \times \frac{2}{3^3} + \frac{1}{5} \times \frac{2}{3^5}}$ , &c.  $= L, 2$ , and therefore  $\frac{z}{3} + \frac{\frac{1}{3} \times \frac{z}{3^3} + \frac{1}{5} \times \frac{z}{3^5}}{\frac{1}{3} \times \frac{z}{3^3} + \frac{1}{5} \times \frac{z}{3^5}}$ , &c.  $= \frac{1}{2} L, 2$ . Here put  $\frac{z}{3} = A$ , and because  $\frac{z}{3^3} = \frac{z}{3} \times \frac{1}{3^2} = \frac{1}{9} \times A = B$ , and thus  $\frac{z}{3^5} = \frac{z}{3^3} \times \frac{1}{3^2} = \frac{1}{9} B = C$ , and so on. Therefore the said Theorem will here again become  $A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D + \frac{1}{9}E + \frac{1}{11}F$ , &c.  $= \frac{1}{2} L, 2$ , as is evident from the following Operation.

$$\frac{1}{3}z =$$

$\frac{1}{3}z = A = 0.144764827301$	$A = 0.144764827301$
$B = 16084980811$	$\frac{1}{3}B = 5361660270$
$C = 1787220090$	$\frac{1}{5}C = 357444018$
$D = 198580010$	$\frac{1}{7}D = 28368572$
$E = 22064445$	$\frac{1}{9}E = 2451605$
$F = 2451605$	$\frac{1}{11}F = 222873$
$G = 272400$	$\frac{1}{13}G = 20953$
$H = 30266$	$\frac{1}{15}H = 2017$
$I = 3362$	$\frac{1}{17}I = 197$
$K = 373$	$\frac{1}{19}K = 19$
$L = 41$	$\frac{1}{21}L = 1$
The Sum is . . . . . $\frac{1}{2}L, 2 = 0.150514997826$	
Multiplied by . . . . . 2	
The <i>Briggian</i> Logarithm of 2 = 0.301029995652	

7. This Logarithm may yet be much easier and sooner obtain'd by this Consideration, *viz.* That  $\frac{1}{2}^{10} = \frac{1}{1024}$ , and  $\frac{1000}{1024} \times \frac{1}{1000} = \frac{1}{1024}$ ; therefore  $L \frac{1000}{1024} + L \frac{1}{1000} = L \frac{1}{1024} = L \frac{1}{10} = L \frac{1}{2} = L 2$ . But

$L \frac{1000}{1024} = L \frac{125}{128}$ . Wherefore put  $\frac{125}{128} = \frac{a}{b}$  a-new, then  $a + b = s = 253$ , and  $a - b = d = 3$ ; thus Theorem II. will converge much faster, and will become, in

Numbers,  $z \times \frac{2}{1} \times \frac{3}{253} + \frac{2}{3} \times \frac{3^3}{253^3} + \frac{2}{5} \times \frac{3^5}{253^5}, \&c.$

Or if  $\frac{d}{s} = y$ , the said Theorem, in *Species*, is  $2zy + \frac{2}{3}zy^3 + \frac{2}{5}zy^5, \&c.$  Suppose  $2zy = A$ , then  $\frac{2}{3}zy^3 = \frac{1}{3} \times 2zyxy^2 = \frac{1}{3}Axy^2 = B$ , also  $\frac{2}{5}zy^5 = \frac{1}{5} \times 2zyxy^2xy^2 = \frac{1}{5}Bxy^2 = C$ , and so on; and thus the Theorem is  $A + \frac{1}{3}Ay^2 (=B) + \frac{1}{5}By^2 (=C) + \frac{1}{7}Cy^2 (=D) + \frac{1}{9}Dy^2, \&c.$

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$$8. \text{ Wherefore } \begin{cases} 2zy=A= & 0.01029947387912 \\ \frac{1}{3}Axyy=B= & \dots & 48271995 \\ \frac{1}{3}Bxyy=C= & \dots & 4072 \end{cases}$$

The Sum is the Logarithm }  $\dots 0.01029995663980$   
of  $\frac{1000}{1024} (=L\frac{125}{128}) =$

Add the Logarithm of  $\frac{1}{1000} \dots 3.00000000000000$

And  $\frac{1}{10}$  of that Sum will }  $\dots 0.30102999566398.0$   
be  $L_2 =$

Thus you see 3 Steps of the Series thus ordered, are sufficient for 14 Places of Figures, whereas before (Art. 6.) 11 Steps produced the Logarithm true only to 10 Places.

9. Let the next Example be to find the *Briggian Logarithm* of 3. This may also be done by Theorem II. where  $\frac{a}{b} = \frac{1}{3}$ , and  $d=2$ , and  $s=4$ ; also

$\frac{d}{s} = \frac{1}{2}$ ,  $z=0,43429$ , &c. as before. Then  $z \times \frac{2^d}{s} +$

$\frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$ , &c.  $= \frac{2}{3} \times \frac{z}{2} + \frac{2}{5} \times \frac{z}{8} + \frac{2}{7} \times \frac{z}{32}$ , &c.

$= L_3$ . The half thereof  $\frac{z}{2} + \frac{1}{3} \times \frac{z}{8} + \frac{1}{5} \times \frac{z}{32}$ ,

&c.  $= \frac{1}{2} L_3$ . Put  $\frac{z}{2} = A$ , then  $\frac{z}{8} = \frac{z}{2} \times \frac{1}{4} =$

$\frac{1}{4} A = B$ , and  $\frac{z}{32} = \frac{z}{8} \times \frac{1}{4} = \frac{1}{4} B = C$ , and so on.

Whence  $A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D + \frac{1}{9}E$ , &c.  $= \frac{1}{2} L_3$ .

But this Series converges so very slow, that as many Steps will be necessary as you intend Places of Figures in the Logarithm, and more; therefore Recourse must be had to Theorem IV. which here comes into play, because the Logarithms on each Side of it are known, *viz.* the Logarithm of 2 and 4.

10. Therefore (according to Theorem IV.)  $a=2$ ,  $b=4$ ,  $\frac{a+b}{2} = \frac{1}{2}s = 3$ ,  $ab=8$ , consequently  $\frac{1}{4}ss=9$ ,

and so  $\frac{1}{4}ss + ab = S = 17$ . Wherefore  $\frac{z}{S} + \frac{1}{3} \times \frac{z}{S^3}$

+



$\frac{1}{3} \times \frac{z}{s^3} + \frac{1}{7} \times \frac{z}{s^7}, \&c. = L \sqrt{\frac{8}{3}}$ , put  $\frac{z}{s} = A$ ,  
 then  $\frac{1}{3} \times \frac{z}{s^3} = \frac{1}{3} \times A \times \frac{1}{s^2} = B$ ,  $\frac{1}{5} \times \frac{z}{s^5} = \frac{1}{5} \times B \times \frac{1}{s^2}$   
 $= C$ , &c. Also  $\sqrt{\frac{8}{3}} \times \frac{1}{\sqrt{8}} = \frac{1}{3}$ , wherefore  $L \sqrt{\frac{8}{3}} +$   
 $L \frac{1}{\sqrt{8}} = L 3$ . Therefore  $L \frac{1}{\sqrt{8}} + A + B + C + D$ ,  
 &c.  $= L 3$ . See the Operation following.

$$\begin{array}{l} \text{Thus } \left\{ \begin{array}{l} L \frac{1}{\sqrt{8}} = \frac{L_2 + L_4}{2} = \dots\dots 0.4515449934959 \\ \frac{1}{3} \times \frac{z}{s} = A = \dots\dots\dots 255467342296 \\ \frac{1}{3} \times \frac{1}{s^2} \times A = B = \dots\dots\dots 294656680 \\ \frac{1}{5} \times \frac{1}{s^2} \times B = C = \dots\dots\dots 611744 \\ \frac{1}{7} \times \frac{1}{s^2} \times C = D = \dots\dots\dots 1511 \end{array} \right. \end{array}$$

The Sum is the Log. of  $\dots\dots 3 = 0.477121254719.0$

11. But this Logarithm may yet much sooner  
 and with less Trouble be found, by the Artifice used  
 in Art. 7. For the Ratio  $\frac{2^{15}}{5 \times 3^8} = \frac{32768}{32805} = \frac{a}{b}$ , and  
 so  $a - b = 37$ , and  $a + b = 65573$ , and since  
 $\frac{2^{15}}{5 \times 3^8} \times \frac{5}{2^{15}} = \frac{1}{3^8}$ , therefore  $L \frac{2^{15}}{5 \times 3^8} + L \frac{5}{2^{15}} =$   
 $L 3^8$ ; but  $L \frac{5}{2^{15}} = L 2^{15} - L 5$ . And  $L \frac{2^{15}}{5 \times 3^8}$   
 $= \frac{2zd}{s} + \frac{2zd^2}{3s^3}, \&c.$

$$\text{Therefore } \left\{ \begin{array}{l} L 2^{15} - L 5 = \dots\dots\dots 3.81647993062 \\ \frac{2zd}{s} = \dots\dots\dots 0.00049010708 \end{array} \right.$$

The Sum is the Logarithm of  $3^8 = 3.81697003770$

And  $\frac{1}{8}$  thereof is the Log. of  $3 = 0.47712125471$ ;  
 thus the *first Step* of Theorem II. gives the Loga-  
 rithm true to 11 Places.

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12. The next Example shall be that which Dr. Halley has given for finding the Logarithm of 23, which is done by Theorem IV. In this Case,  $a=22$ ,  $b=24$ ,  $\frac{1}{2}s=23$ ,  $\frac{1}{2}ss=529$ ,  $ab=528$ , and  $\frac{1}{2}ss+ab=1057=S$ . And  $\frac{2}{S} + \frac{2}{3S^3} + \frac{2}{5S^5}$ , &c.  $= L \sqrt{\frac{ab}{\frac{1}{2}s}}$ . But  $\sqrt{\frac{ab}{\frac{1}{2}s}} \times \frac{1}{\sqrt{ab}} = \frac{1}{\frac{1}{2}s}$ ; therefore  $L \frac{\sqrt{24+22}}{\frac{1}{2}s} + L \frac{1}{\sqrt{24+22}} = L \frac{1}{\frac{1}{2}s}$ ; and because  $2 \times 2 \times 2 \times 3 = 24$ , and  $2 \times 11 = 22$ , therefore also  $3L, 2 + L3 = L24$ ; and  $L2 + L11 = L22$ . And  $\frac{L24+L22}{2} = L \sqrt{ab}$ ; therefore, (proceeding in the Operation according to Art. 10.) we have

$$\left\{ \begin{array}{l} \frac{L24+L22}{2} = \dots 1.36131696126690612945009172669805 \\ \frac{2}{S} = A = \dots 41087462810146814347315886368 \\ \frac{1}{3} \times \frac{1}{SS} \times A = B = \dots 12258521544181829460074 \\ \frac{3}{5} \times \frac{1}{SS} \times B = C = \dots 6583235184376175 \\ \frac{5}{7} \times \frac{1}{SS} \times C = D = \dots 4208829765 \\ \frac{7}{9} \times \frac{1}{SS} \times D = E = \dots 2930 \end{array} \right.$$

The Sum  $= L23 = \dots 1.36172783601759287886777711225117$  which Logarithm is true to 32 Places of Figures, and thus you may proceed for any other.

13. In making Logarithms for *Prime Numbers*, the *Artifice*, or *greatest Advantage* consists in finding such a *Ratio* or *Fraction*, whose *Terms* are the *greatest* possible and their *Difference* the least; and the *Number* whose Logarithm is sought, or some *Power* thereof, is an *aliquot Part* or *Submultiple* of one of the *Terms* of the said *Ratio* or *Fraction*. For this once obtain'd, the Logarithm is soon acquired by Theorem

rem II. with ease. Thus the Fraction  $\frac{211}{80 \times 29} = \frac{b}{a}$ ; where  $d=1$ , and  $s=4641$  the Series

will converge very swift for the Logarithm of  $\frac{211 \times 11}{80 \times 29}$

but  $\frac{211 \times 11}{80 \times 29} \times \frac{80 \times 29}{11} = \frac{211}{1} = 211$ , therefore  $L80 +$

$L29 - L11$  added to the Series, gives the Logarithm of 211. But the *Ratio* or *Fraction*  $\frac{5387041}{5387040} =$

$\frac{121 \times 211^2}{80 \times 54 \times 29 \times 43}$  will make the Series converge very much

sooner than before; for here  $d=1$ , and  $s=10774081$ .

For  $L80 + L54 + L29 + L43 - L121$  added to the Series (or Theor. II.) gives the Logarithm of  $211^2$ , half which is the Logarithm of 211. Lastly, the

Fraction  $\frac{1082119441}{1082119440} = \frac{211^4}{60 \times 28 \times 53 \times 113 \times 197} = \frac{b}{a}$ , where

$d=1$ , and  $s=3964238881$ , converges to that Degree that the first Step of the Series quotes the Logarithm of the Fraction to 29 Places, to which add the Logarithms of the 5 Numbers in the *Denominator*, and it gives the Logarithm of  $211^4$ , then  $\frac{1}{4}$  of that is the Logarithm of 211, as before.

14. The greatest Difficulty consists in finding out proper Numbers for producing such Fractions as aforesaid; and the best Method of this is by *prudent Tryals*. An Example of which is here subjoin'd. Suppose I would procure a convenient Fraction for the Logarithm of 223, I make tryal thus;

$$\text{First } 223 \begin{cases} \times 7 = 1561 \\ \times 8 = 1784 \end{cases}$$

$$\text{Then } 223 \begin{cases} \times 87 = 19401 \\ \times 2 = 446 \end{cases}$$

$$\text{Therefore } 223 \times 387 = 64001$$

Having thus obtain'd the Term 64001, 'tis easy to observe the other may be 64000, wherefore the Fraction is  $\frac{64001}{64000} = \frac{287 \times 223}{64000}$ , and finds the Logarithm of



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223 with good Dispatch. Or thus, to find a Fraction of *larger Terms*; suppose I assume the Numerator  $159000 = 1000 \times 53 \times 3$ , then to find the other Term as near this as may be, I try thus;

$$223 \begin{cases} \times 3 = 669 \\ \times 1 = 223 \end{cases}$$

$$223 \begin{cases} \times 13 = 2899 \\ \times 7 = 1561 \end{cases}$$

Therefore  $223 \times 713 = 158999$ , which is within Unity as great as the other Term 159000, and consequently the Fraction  $\frac{159000}{158999}$  is that required, and thus you proceed to raise the Terms of any other.

15. Let the Terms of any Fraction be represented by  $a$ =Least, and  $b$ =Greatest. Then if the *Ratio* be *increasing* it will be  $\frac{a}{b}$ , but if *decreasing*,  $\frac{b}{a}$ ; let that

Term, in which the Number sought is *ingredient*, be express'd by the Product  $cx$ , where  $c$ =the Number (or Product of Numbers) whose Logarithm is known, and  $x$ =the Number whose Logarithm is sought.

If  $a=cx$ , then  $\frac{cx}{b} = \frac{a}{b}$ , or  $\frac{b}{cx}$ ; but if  $b=cx$ , then

$\frac{a}{cx}$ , or  $\frac{cx}{a}$ ; also let there be put the 2d Theorem

$$\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5} + \frac{2d^7}{7s^7}, \text{ \&c.} = z;$$

then if the unknown Number  $x$  be in the *Denominator* of the *increasing Ratio*  $\frac{a}{b}$ , viz.  $\frac{a}{cx}$ ; or in the *Numerator* of

the *decreasing Ratio*  $\frac{b}{a}$ , viz.  $\frac{cx}{a}$ ; then it will be

$Z + La - Lc = Lx$ . And, *vice versa*, if it be  $\frac{cx}{b}$  or

$\frac{b}{cx}$ , the Theorem will be  $Z + Lb - Lc = Lx$ . From

hence the Operations in the foregoing Articles for making Logarithms have their Grounds and Reason; and

and every thing there asserted is from this Process very evident.

16. To find the natural Number of any Logarithm proposed; this is best done by the Theorem in Art. 17. Chap. VII.

$$\text{viz. } \left\{ \begin{array}{l} a \\ b \end{array} \right\} \times 1 \pm nE + \frac{1}{2}n^2E^2 \pm \frac{1}{6}n^3E^3 + \frac{1}{24}n^4E^4 \pm \frac{1}{120}n^5E^5 \\ \text{&c.} = N.$$

For Example, let it be required to find the Interest of one Pound for one Day, at the rate of 6 *l. per Cent. per Annum, Compound Interest*; which is to extract the Root of 1.06 taken as the 365th Power; thus the Logarithm of 1.06 = 0.0253058652647702408-46731186351, &c.

Which divide by 365, }  $L = 0.0000693311377116-$   
the Quotient is } 99289991044346, &c.

The next nearest }  $b = 1.00016 = 0.0000694815587-$   
Log. & its N<sup>o</sup>. } 28037517724712696, &c.

Their Difference is  $E = 0.000000150421016338227-$   
733668350, &c.

Mult. this by  $n = 2.3025850929940456840179914-$   
54684, &c.

The Product is

$$\left\{ \begin{array}{l} nE = 0.000000346357189893416971322305 \\ n^2E^2 = \dots\dots\dots 119963302990864503 \\ n^3E^3 = \dots\dots\dots 41550152514 \\ n^4E^4 = \dots\dots\dots 14391 \end{array} \right.$$

The Powers of  $nE$ . Then

$$1 + \frac{1}{2}n^2E^2 = 1.0000000000000059981651495432251$$

$$\frac{1}{24}n^4E^4 = \dots\dots\dots 599$$

$$\text{The Sum } 1 + \frac{1}{2}n^2E^2 + \frac{1}{24}n^4E^4 = \left\{ \begin{array}{l} 1.000000000000005998165- \\ 1495432851 = X. \end{array} \right.$$

The odd Powers

$$\left\{ \begin{array}{l} nE = 0.000000346357189893416971322305 \\ \frac{1}{6}n^3E^3 = \dots\dots\dots 6925025419 \end{array} \right.$$

$$\text{Sum } nE + \frac{1}{6}n^3E^3 = 0.00000034635718989342389634-$$

$$7724 = Z.$$

Then

Then the Value of the Series is

$$X-z=0.999999653642870088227599085126.$$

Which multiply by  $b=1.00016$ , produces

$\sqrt[365]{1.06}=1.000159653587452947441715500980$ ,  
 $\&c.=N$ , the Number sought of the given Logarithm  $L$ , and that to 30 Places of Figures. The same Number may be seen produced to 60 Places in Mr. Sherwin's Mathematical Tables.



## CHAP. IX.

*Of the \*Logarithmic \*Spiral; and the Nature and Construction of the Table of \*Meridional \*Parts, or the Nautical \*Meridian \*Line, deduced therefrom.*

1. **I**F any Right-Line  $pW$  be moved with an *equable Motion* about the fix'd Point  $p$ , and at the same time the Point  $W$  be mov'd towards the Point  $p$ , with a Velocity such that the Radii  $pW, pV, pS, \&c.$  form'd thereby, be in a *Geometrical Ratio* decreasing, then the Curve  $WVSQ, \&c.$  is called the *Logarithmic Spiral*; and that for the same Reason as the Logarithmic Curve before describ'd received its Appellation. See Chap. III. Art. 8, 9, 10.

2. For suppose the Arches  $AC=CE=EG, \&c.$  and therefore in *Arithmetical Progression*; and since, from the *Generation* of the *Spiral*, the Radius  $pB:pD::pD:pF::pF:pH, \&c.$  'tis evident the Arches  $AC, AE, AG, \&c.$  are the *Exponents* of the *Ratio's* of the Radii  $Dp, Fp, Hp, \&c.$  to the first  $pB$ ; and so those *Arches* are in respect of the Radii, as *Logarithms* in respect of *Numbers*; as is sufficiently manifest from the preceding *Theory of Logarithms*. Wherefore



fore if  $Bp$  be = 1, 10, 100, &c. and  $PW$  (the 10th Proportional from  $pB$ ) be = 10, 100, 1000, &c. then shall the Arch  $AC$  =, 1000000,  $AE$  =, 2000000,  $AG$  =, 3000000, &c.  $AW$  = 1,0000000; be the Logarithms of the Numbers  $pD$  = 1,259, &c.  $pF$  = 1,585, &c.  $pH$  = 1,996, &c.  $pW$  = 10; of Mr. Briggs's Form.

3. This Spiral is also called the *Equiangular Spiral*; because it intersects all the Radii  $pW$ ,  $pQ$ ,  $pB$ , at equal Angles. For suppose the Arches  $NP$ ,  $TW$ , infinitely small, and equal to each other, then may the Parts of the Spiral  $OQ$  and  $VW$ , be esteemed Right-Lines; and so since in the Triangles  $pOQ$ ,  $pVW$ , the Sides are proportional, viz.  $Op : pQ :: Vp : pW$ , and the Angle  $OpQ = VpW$ , those Triangles are similar; and consequently the Angle  $pOQ = pVW$ , or  $pQO = pWV$ ; and thus it will be every where.

4. Now let the whole Scheme be considered as the *Stereographic Projection* of one Quarter of a parallel Hemisphere, then shall  $p$  be the Pole;  $WLA$ , a Quadrantal Arch of the Equator; the Radii  $pW$ ,  $pT$ ,  $pR$ , &c. the several Meridians projected on the Plane of the Equator. And since 'tis the Property of every Rumb Line to make equal Angles with every Meridian on the Globe, and the Angles contained between circular Arches on the Globe, are equal to the Angles between the same Arches in this Projection, therefore the *Logarithmic Spiral*  $WQB$  is the Projection of a Rumb Line; since it has the same Property on the Projection, as the Rumb on the Globe, as was proved Art. 3. hereof.

5. Moreover, since all Right Circles, such as are the Meridians in this Case, are projected into Right Lines equal to the Tangents of half the Arches, the Lines  $pB$ ,  $pD$ ,  $pF$ ,  $pH$ , &c. will here represent the Tangents of half the Complements of the Latitudes  $AB$ ,  $CD$ ,  $EF$ ,  $GH$ , &c. And since the Arches in the Equator  $AC$ ,  $AE$ ,  $AG$ , &c. are the Differences

K

of

of *Longitude* made by sailing from the Latitude B to the Latitudes D, F, H, &c. on the *Rumb* or *Spiral* BOW; and it has been shewn that those Arches are the *Logarithms* of the *Radii* pD, pF, pH, &c. therefore the Difference of Longitude is the *Logarithm* of the *Tangent* of half the *Complement* of Latitude, reckoning from the Meridian Ap whence the Logarithms begin.

6. Therefore the Difference of Longitude RT, made by sailing from the Latitude S to the Latitude V, is equal to the Difference of the Logarithms (AT—AR) of the Tangents of the half Complements (Sp, Vp) of the Latitudes TV, RS. And since the Ratio of the *Progression*, or of pW to pV, may be infinitely varied, 'tis plain the infinite Number of Rumbs in a Quadrant of the Compass determine so many *Scales* of *Logarithms* in the Equator of the Tangents of the half Complements of the Latitudes proper to those Rumbs.

7. Since then every different Rumb is a *Logarithmic Spiral*, or determines a peculiar Scale of *Logarithms* for the Tangents of the Half-Complements of its Latitudes, therefore any Canon or Table of Logarithm-Tangents, whether of *Neper's*, *Briggs's*, or any other Form whatsoever, is the Scale of the Differences of Longitude on some determinate Rumb or other. And consequently if this Rumb be investigated for the Canon of *Briggs's Logarithms* (now in common Use,) the said Canon may be made to answer all the Purposes of the *Nautical Meridian Line*, in Propositions of *Navigation by Mercator's Chart*.

8. In order to this it must be considered, that the *Meridian Line* is a Table or Scale of Longitudes to every Degree of Latitude on the Rumb which makes an Angle of 45 Degrees with the Meridian; since in this Case the Differences of Longitude are always equal to the *Meridional*, or enlarg'd Differences of Latitude. And since there is a certain Rumb on which

which *Neper's* or *Briggs's* Logarithm-Tangents are the *Differences* of Longitude, and the Differences of Longitude on different Rumbs are to one another as the Tangents of the Angles of those Rumbs with the Meridian; therefore by having given the Difference of Longitude on the Rumb of  $45^\circ$ , in Logarithms of *Neper's* Form, and the Length of the Arch of one Minute or Degree in Parts of the Radius, we can thence find the Angle of that Rumb which determines that *Species of Logarithms*.

9. Now the *Momentary Augment* or *Fluxion* of the Tangent-Line of  $45^\circ$ , is exactly double to the *Fluxion* of the Arch of the Circle (as is easily proved), and the *Tangent* of  $45^\circ$  being equal to *Radius*, the *Fluxion* also of the *Logarithm-Tangent* will be double to that of the Arch, if the Logarithm be of *Neper's Form*; but for *Briggs's Form*, it will be as the same double Arch multiplied into  $\frac{1}{n} = 0.43429$ , &c. or divided by  $n = 2.30258$ , &c. the Index for *Briggs's Logarithms*. See Chap. VI. Art. 7.

10. Now since the Radius of a Circle being put  $=1$ , the *Periphery* thereof will be  $6.2831853$ , &c. therefore  $360)6.2831853$ , &c.  $(0.01745329$ , &c. = the Length of the Arch of *one Degree*. Also  $60)0.01745329$ , &c.  $(0.0002908882$ , &c. = the Length of an Arch of *one Minute*, in Parts of the Radius. If one Minute be supposed Unity, then the Proportion for finding the Angle of the Rumb required for *Neper's Logarithms*, will be, as  $1 : 2.908882$ , &c. :: Radius  $= 1000000$ , &c. : the Tangent  $= 2908882$ , &c. of the Angle  $71^\circ 1' 42''$ , whose Logarithm is  $10.463726117$ , &c. and under that Angle is the *Meridian* intersected by that *Rumb Line*, on which the Differences of *Neper's Logarithm-Tangents* of the Complements of the Latitudes are the true Differences of Longitude, estimated in Minutes and Parts, taking the first 4 Figures for Integers.



11. But since *Neper's* Logarithms are to those of *Mr. Briggs's* Form, as 2.302585, &c. is to 1.000000, &c. therefore to find the Angle of the Rumb for the Logarithms of *Briggs's* Form; this must be the Analogy, As 2302585, &c. : 2908882, &c. :: 1000000 = Radius : 12633114, &c. = the Tangent of the Angle  $51^{\circ} 38' 9''$ , whose Logarithm is 10.101510428, &c. Wherefore in the *Rumb* Line that makes an Angle of  $51^{\circ} 38' 9''$  with the *Meridian*, the *common* (viz. *Briggs's*) Logarithm-Tangents are the true Differences of Longitude.

12. But if a Table or Scale of *Logarithm-Tangents* be made by Extraction of the Root of the *infinitest Power*, whose *Index* is the Length of the Arch you put for *Unity* in the said Scale; then such a Scale of *Logarithm-Tangents* shall be the true *Meridian Line* required. If then the *Radius* or Tangent of  $45^{\circ}$ , be put = 1; and the Difference between Radius and any other Tangent T, be called t; so that it be  $R \pm t = T$ ; the Logarithm of the *Ratio* of Radius to such *Tangent* will be

$\frac{1}{n} \times t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5}, \text{ \&c.} = t =$  the Logarithm of the Tangent T, when it is  $R + t = T$ .

Or  $\frac{1}{n} \times t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{4} + \frac{t^5}{5}, \text{ \&c.}$  when it is  $R - t = T$ . All which is evident from Chap. VI. Art. 6.

13. According to the same Doctrine (Art. 9. of the same Chap.) if T be any given Tangent, and t the Difference thereof from another Tangent; then the Logarithm of their Ratio will be  $\frac{1}{n} \times \frac{t}{T} - \frac{t^2}{2T^2} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \text{ \&c.}$  when T is the lesser Term. But

$\frac{1}{n} \times \frac{t}{T} + \frac{t^2}{2T^2} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \text{ \&c.}$  when T is the greater Term.

14. Again, it was shewn in the same Chap. VI. Art. 10, and 11. that this Series may be made to converge twice as swift, omitting all the even Powers, by putting  $T$  = the *Sum* of the Tangents, and  $t$  = the *Difference*, as above. For thus the Logarithm will be  $\frac{2}{n} \times \frac{t}{T} + \frac{t^3}{3T^3} + \frac{t^5}{5T^5} + \frac{t^7}{7T^7}$ , &c. = the Logarithm of the Ratio of those two Tangents.

15. But the Ratio of  $T$  to  $t$ , or of the *Sum* of two Tangents to their *Difference* is the same as that of the Sine of the Sum of those Arches to the Sine of their Difference; that is, again, as the Ratio of the *Co-Sine* of *middle Latitude* (or *half Sum* of the Arches) to the Sine of half the Difference. Therefore putting  $S$  = Sine-Complement of *middle Latitude*; and  $s$  for the Sine of half the Difference of Latitudes; then

$\frac{t}{T} = \frac{s}{S}$ ; and so the Series will become  $\frac{2}{n} \times \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7}$ , &c. wherein as the Differences of Latitude are smaller, fewer Steps will suffice.

16. So that, if the *Equator* be put for *Middle-Latitude*, then shall  $S$  = Radius, and  $s$  = Sine of the Latitude; then the Meridional Parts reckon'd from the Equator will be  $\frac{2}{n} \times \frac{s}{r} + \frac{s^3}{3r^3} + \frac{s^5}{5r^5} + \frac{s^7}{7r^7}$ , &c.

Here because  $r=1$ , therefore  $\frac{2}{n} \times s + \frac{s^3}{3} + \frac{s^5}{5} + \frac{s^7}{7}$ , &c. the half of which is  $\frac{s}{n} + \frac{s^3}{3n} + \frac{s^5}{5n} + \frac{s^7}{7n}$ , &c. = half the Logarithm of the Ratio of  $r + s$  to  $r - s$ ; that is, of the *versed Sines* of the Distances from both Poles. See Chap. VI. Art. 11.

17. I shall exemplify this Series by shewing how the *Meridional Parts* answering to  $30^\circ$  Latitude, are to be found thereby, and that by the Logarithms, as follows.

The

The Logarithm of the }  
Sine of 30 Degrees is }  $s = 0,5000000 = 9.6989700$

Multiply by .....  $\frac{3}{3}$

The Logarithm of .....  $s^3 = 9.0969100$

Subtract the Logarithm of .....  $3 = 0.4771213$

There remains the }  
Logarithm of }  $\frac{s^3}{3} = 0.0416667 = 8.6197887$

And proceeding thus, you'll find the other Steps of the Series by their Logarithms, as here set down.

$$\left\{ \begin{array}{l} \frac{s^5}{5} = 0.0062500 = 7.7958800 \\ \frac{s^7}{7} = 0.0011160 = 7.0476920 \\ \frac{s^9}{9} = 0.0002171 = 6.3364875 \\ \frac{s^{11}}{11} = 0.0000444 = 5.6472773 \end{array} \right.$$

The Sum of  $s + \frac{s^3}{3} + \frac{s^5}{5} + \frac{s^7}{7} + \frac{s^9}{9} + \frac{s^{11}}{11}$ , &c. is }  $= 0.5492942 = 9.7398051$

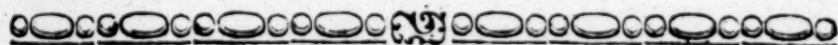
To which add }  
the Log. of }  $n = 3437.74677078, \&c. = 3.5362739$

The Sum is the }  
Log. of the Meridional Parts }  
for the Arch }  $\text{viz. } 1888,334, \&c. = 3.2760790$   
of  $30^\circ$

18. And thus you may proceed to find the Length of any other Arch, or the Distance from the Meridian of its Parallel of Latitude; and so the *Meridian Line* may be constructed *de novo*, if any one thinks it worth while. But tho' it may be done with greater Accuracy and Exactness by these *infinite Series* than what we have by the *common Method*; yet the Table of *Meridional Parts*, or *Nautical Line* made from thence, now in Use, is abundantly sufficient for all the Purposes of Sailing; and consequently



quently renders a new Calculation thereof unnecessary, and a Matter of mere Curiosity. And indeed, since it has been shewn above (Art. 9, 10, 11.) the *Meridian Line* is no other than a Scale of the Logarithm-Tangents of the Half-Complements of the Latitude on the Rumb of  $51^{\circ} 38' 9''$ , the Propositions of Sailing by this Method are resolvable by only the Canon of Logarithm-Tangents at the End of this Treatise; so that where this Canon is at hand, neither Meridional Table or Line can be necessary, as will appear by a Chapter particularly on this Subject, in the *Practical Part*. They who would see the Theory of this Branch of the Art, may peruse N<sup>o</sup>. 219. of the *Philosophical Transactions*, where they will find a most learned Tract on this Subject, wrote by Dr. *Halley*; from whence the Substance of this Chapter is taken.



# CHAP. X.

*Of the Construction of a Large \* Logarithmic Scale, exhibiting by Inspection a distinct \* Idea of the Nature and Agreement of \* Numbers and their \* Logarithms.*

1. **I**T is an Observation of the earliest Antiquity, that we have no *Ideas* in the *Mind* which were not *first* in the *Senses*; or that the *Senses* of the *Body* are the only *Inlets* or *Entries* by which the *Idea's* of *Objects* present themselves to the *Mind*. It follows then, that the *Idea's* must needs be so much the more *clear* and *distinct* in the *Mind*, and consequently be the better understood by it, by how much the more *fully*, *compleatly*, and *obviously* they first of all affect our *Senses*. Single uniform Objects easily in-

sinuate

sinuate themselves, and make strong and clear Impressions on the *Mind*, while those which involve *Multiplicity* and *Variety* in their Nature, are proportionally more difficultly apprehended by the Senses, and consequently affect the Mind with *imperfect, confused, and slight Impressions*, which therefore must produce a more *perplex'd, obscure, and uncertain Notion or Conception* of the Things themselves.

2. From this Consideration we may easily learn the Reason why, of all the vast number of Persons who understand the *practical Use* of Logarithms, so very few of them know any thing of the *Nature and Construction* of them. The Use of Logarithms is very obvious to the Senses by easy Examples, but their Nature and Construction lead the Mind too much upon the Contemplation of *Infinities* both of *Quantity* and *Variety*, which are Subjects too vastly *abstruse* and *remote* from *Sense*, ever to be very commonly understood; unless some *Expedients* be contrived, which may help to facilitate so difficult an Affair.

3. And as there are principally but three Ways, whereby the Nature of Logarithms are explained, *viz.* by *Numbers, Species, and Lines*, the *Expedient* aforesaid must be sought in one of these *three Methods*. But *Numbers*, of all things else, exhibit the most *complex* and *various Idea*, therefore it cannot be hoped for from them. *Species*, on the other hand, are too *simple and concise a Representation* of so *vast and various Idea's* as are those of *Logarithms*, and have nothing of their *Resemblance* in their *Form*; this Expedient therefore is not to be expected from this Head. It remains then, that *Right or Curve Lines* be used for the Purpose of explaining the Nature of Logarithms, by making the whole Matter obvious to the Senses. And here indeed we shall find all that can be desired, or is necessary to the Purpose.

4. For Example, let the complex *Idea* of a *thousand Units* be to be exprefs'd most advantageously to a Mind unexercis'd about such *complex'd Notions*; if you do it by *Numbers* or *Figures*, it must be by this Expression 1000; but there are but *four Characters* to form an *Idea* of a *thousand* separate Objects in the Mind. In Species, this *great* and *complex Idea* is often represented by one Character alone, as  $x$ ; or two, as  $1+x$ ; which are still more obscure and absolutely unintelligible without some *Pre-Notions* of the Matter. But a Line may be taken of a Length sufficient, that by proper Divisions, all the thousand Units may be render'd distinct and obvious to the Sense, in any variety of Magnitude almost, but especially if they are equal to each other, as in the Case of Logarithms before us. Wherefore, since by *Lines* such *great* and almost *inconceivably complex Ideas* are capable of being represented to the Senses, *distinct* and separate in their *proper Parts*, and the *Doctrine of Logarithms* depending entirely on such *Ideas*, 'tis evident that by Means of Lines of a sufficient Length, the *Nature* and *Properties* of Logarithms, and the *Operations* thereby, may be render'd more *apparent* and *complete* to the Senses, and so be better understood in the Mind, than by any other Means whatsoever.

5. The Consequence of all which, is, that the young Tyro, and all who would have a *true Notion* and most *clear Understanding* of this *abstruse* and *mysterious Doctrine*, should be assisted with such a *large lineal Construction of the Logarithms*, as hath been hinted at. And this, I hope, I have effected in the *large Diagram* on the Copper-Plate before you, with considerable Exactness, which I call the *Logarithmic Scale*. Wherein all that has been said in the *General Theory* aforegoing, or may follow in the *practical Operations of Logarithms*, is evident even to Sense it self, to a very wonderful degree, by a bare

L

Inspection,



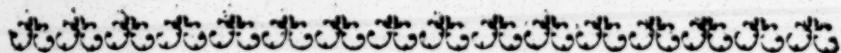
Inspection, or a Glance of the *Eye* only ; and therefore cannot but conduce to form a very distinct and agreeable *Idea* or Notion both of the *Theory* and *Praxis* of this admirable Art.

6. The Scale consists of *three principal Lines* which bound it ; the *first* is AB on the Side, which is 22 *Inches* in Length, and is divided into 1000 equal Parts which represent the *natural Numbers* from 1 to 1000, all which are visible and distinct to the *naked Eye* ; which Numbers therefore are affixed to every 10th Division. The second is AC at the Bottom, divided into 300 *equal Parts*, (as being but  $16\frac{1}{10}$  *Inches* long.) These represent the *Logarithms* ; if each of these *equal Parts* be supposed to represent 10, or 100, the *Logarithms*, then, for all Numbers under 1000, will be exhibited by Lines only to 4 or 5 Places of Figures, including the *Indices*. The *third* principal Line is the *Logarithmic Curve* eDEFB, in which all the Lines of Numbers and *Logarithms* terminate, and whose *Genesis* and *Properties* have been before described. See Chap. III, IV, V, &c.

7. The Scale consists (or is made up) of Lines of Numbers, and others which are the *Complements* of the *Logarithms* to 3.0000. The first are *perpendicular* to the *Logarithmic Line* AC in its several Divisions, and increase in Length in a *Geometrical Ratio* ; as hath been observed : thus dividing AC into 3 equal Parts  $CG=GH,=HA$ , if GD be the 10th *proportional* Term from  $Ce=1$ , or Unity, then shall  $GD=10Ce=10$ , and  $HE=10GD=100Ce=100$ . Lastly,  $AB=10HE=100GD=1000Ce=1000$  ; as is evident from the *Nature* of the Curve, and by *Inspection*. The *Complements* of the *Logarithms* are the *Lines* which run across the *Diagram*, parallel to the *Line* of *Logarithms* AC ; these at the Curve refer the *Numbers* to their proper *Logarithms*, and by means of those Lines thus crossing each other

in every Part of the Scheme, the *Logarithms* of Number, and the *Numbers* of *Logarithms* are most easily and obviously found, for the Extent of the Scale, by *Inspection* only.

8. It is not pretended that this (or any other) *Instrument* is capable of any great Exactness in *practical Operations*; 'tis sufficient for my Design, if it only *illustrates* and *proves the Truth* of every part of the *Doctrine* of *Logarithms* to the Sense, and thereby renders it easier to the *Intellects* of young *Learners*. If the *Theory* before deliver'd be examined by this *Scale*, it will be found to agree with it to a *sensible Exactness*; it being as it were but the same thing at large. In the following Part, I shall shew the *Correspondence* and *mutual Agreement* between the *fundamental Operations* by *Logarithms* wrote by *Numbers*, and the same performed on this *Scale*; than which nothing more, that I know of, can be said or expected.



## CHAP. XI.

*Of the Construction of the Artificial LINES of NUMBERS, SINES, and TANGENTS, by means of the LOGARITHMS.*

1. **T**HE *Canon* of *Logarithms* being compleated and orderly digested in *Books*, tho' this was a greater Advantage than the *Mathematicians* of any former Age enjoy'd, yet not content to have a bulky *Book* of *Logarithms*, fit to be used in *Studies* and with the *Pen* only, the restless and unsatisfy'd Faculty of *Invention* in Men put them upon Contrivances to new-model and reduce the *veluminous Art*

to *Miniature*, that so it might be render'd more easily manageable, and more *universally* useful.

2. In the Pursuit of this Design they also very well succeeded; for since Numbers of any kind are capable of being represented by *Right-Lines*, they were not long unappriz'd that the *whole Body of the Canon of Logarithms* might be laid down and express'd in the Divisions of one *strait Line*. Mr. Gunter, Professor of Geometry at *Gresham-College*, was the first who took this matter in hand, and constructed such an *artificial Line* of Logarithms; which therefore from him was called (ever since) *Gunter's Line*, or simply, the *Gunter*. The same Person also constructed *artificial Lines* of *Sines* and *Tangents*; and all those Lines, with some others laid down on a Scale, make what we commonly call *Gunter's Scale*.

3. The *Construction* of those *artificial Lines* is easy to be understood, and is as follows. Draw the *Right-Line* AB (Fig. VII.) which divide into 10 great equal Parts, as is there denoted by 1, 2, 3, 4, &c. and each of these into 10 others, and so on. Conceive these *several Divisions*, or *equal Parts*, to represent the Logarithms in the Canon for the natural Numbers. Now suppose the whole Length  $AB=10$ , then the first grand Divisions will be 1, 2, 3, 4, &c. But if  $AB=100$ , then the *first Divisions* will be 10, 20, 30, &c. and the *second Divisions* 1, 2, 3, &c. Again if  $AB=1000$ , the *prime Divisions* will be 100, 200, 300, &c. and the *secondary Divisions*, 10, 20, 30, &c. Suppose the latter Case, viz.  $AB=1000$ ; then draw another *Right-Line* CD equal and parallel to AB, the *natural Line* of Logarithms.

4. Now in the Line CD, such Divisions are to be made as may represent the Places of the natural Numbers 1, 2, 3, &c. or 10, 20, 30, &c. or 100, 200, 300, &c. But neglecting the *Indices* of Logarithms,



rithms, 'tis plain, since the Logarithms of the Numbers 1, 10, 100; 2, 20, 200; 3, 30, 300; &c. are the same, the Distances of those Numbers will be the same on the Line or Scale CD. And therefore since the Logarithm of 1, is  $= 0$ , the Number 1 must be placed at the very Beginning of the Line CD, from whence the Logarithms begin in the Line AB. Then because the whole Line  $AB = 1000 =$  Logarithm of 10  $= CD$ , therefore against the Logarithm of 2, which is  $= 301$  in the Line AB, make a Division in the Line CD, and by it place the Number 2. Again, because the Logarithm of 3 is  $= 477$  in AB, therefore correspondent to the Point in AB, make another Division in CD, and by it place the Number 3. The Logarithm of 4 is 602, therefore from 602 in AB make a Division in CD, by which you must place the Number 4; and thus you proceed to find the Divisions for the other Numbers to 10 in the Line CD, by the Logarithms of those Numbers in the Line AB.

5. If the Divisions in the Line CD now found for the Numbers 1, 2, 3, 4, &c. be suppos'd, instead of them, to be for the Numbers 10, 20, 30, 40, &c. then each of those Divisions may be subdivided into 10 others, by the *Logarithmic Parts* in the Line AB. Thus, because the Logarithms of 11, 12, 13, 14, &c. are 41, 79, 113, 146, &c. therefore against these latter Numbers in the Line AB, make Divisions in the Line CD, so shall the first grand Division from 1 to 2 be divided again into 10 others. Again, because the Logarithms of 21, 22, 23, 24, &c. are 322, 342, 361, 380, &c. therefore Divisions made in CD against these Numbers in AB will finish the Subdivisions of the Space from 2 to 3, in the said Line CD. And thus proceeding, you may subdivide all the other *grand* or *prime Divisions*, to the End of the Line.

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6. If your Lines be of so great Length, that these last Subdivisions in CD, are still of a Length capable of another *tenfold Division*; then the first grand Divisions must be reputed 100, 200, 300, &c. and so since the Logarithms of the Numbers 101, 102, &c. 201, 202, &c. 301, 302, &c. are 4, 8, &c. 303, 305, &c. 478, 480, &c. therefore if against these Parts in AB, you make Divisions in CD, there will ensue a triple Division of the said Line CD, which is more than is necessary for *Instrumental Uses*, and indeed cannot be done but only for the two or three first Divisions.

7. Thus have you seen the Construction of the *Artificial, Logarithmic, or Gunter's Line*, so famous in all Parts of the Mathematics. A Line which performs the Business of the whole Logarithmic Canon; since the Divisions of this Line have all the same Properties with regard to the natural Numbers on it, as the Logarithms of the Table have to the Numbers corresponding to them. 'Tis plain the Divisions and Relation of these two Lines AB, CD, are the same as At, and AT, in Fig. IV. and III. Consequently what has been said of those Lines heretofore will help to illustrate the *Theory* and Construction of the Lines now under Consideration. But since in Use the *Gunter* CD is supposed to be divided into an 100 Parts at least, therefore you always (or mostly) observe it of a double Length of that which is expressed in Fig. VII. which Length is commonly called *Radius*; and so the *Gunter* in common Use is said to be of a *double Radius*; because else the Divisions for the *nine Digits* would be wanting, since the Distance from 1 to 10 is equal to that from 10 to 100, as is evident from the foregoing Construction, and from the *Theory* of Logarithms.

8. Having thus shewn the *Construction* of the *Line of Numbers*, the *Construction* of the Lines of artificial *Sines* and *Tangents* easily follows; since, as before observed,

observed, the Logarithms of *Sines* and *Tangents* are nothing more than *common Logarithms* of such Numbers as express the *Sines* and *Tangents* of each *Minute* of the *Quadrant*.

Deg.	N. Sine.	Log.	N. Tang.	Log.
1	17	2418	17	2419
2	34	5428	35	5430
3	52	7188	52	7194
4	69	8345	70	8446
5	87	9402	87	9419
6	104	1.0192	105	1.0216
7	121	1.0858	122	1.0891
8	139	1.1435	140	1.1478
9	156	1.1943	158	1.1997
10	173	1.2396	176	1.2463
20	342	1.5340	363	1.5610
30	500	1.6989	577	1.7614
40	642	1.8080	839	1.9238
50	766	1.8842	1191	2.0761
60	866	1.9375	1732	2.2385
70	939	1.9729	2747	2.4389
80	984	1.9933	5671	2.7536
90	1000	2.0000	Infin.	Infin.

In the little Table above, the first Column contains the *Degrees*, the 2d and 4th the *Natural Sines* and *Tangents*, and the 3d and 5th Columns contain the *Logarithms* of those natural *Sines* and *Tangents*, the *Indices* being omitted, and the *Radius* supposed = 10000.

9. Let three Lines be drawn, and let L. = Line of *Logarithms*, or *double Radius* of 20000 equal Parts; S = a Line for *Sines*; and T = Line for *Tangents*; the two latter must be drawn equal and parallel to the first; as in the Construction of the Line of *Numbers*. Then having graduated the Line L into 20000 equal Parts,



## 80 *Construction of Artificial Lines*

Parts, if against such of those Parts as are express'd by the Numbers in the 3d Column, you make Divisions in the Line S, and by those Divisions you place the Numbers in the first Column, you will then have the *artificial Line of Sines S* graduated for the first great Divisions of 1, 2, 3, 4, 5, 6, &c. 10, 20, 30, &c. Degrees: After the same manner by the Table of *Logarithmic Sines* you find Numbers, from whence in the Line L you find Divisions in the Line S for *Minutes*, and *Parts of Minutes*. And thus the Line of *artificial or Logarithmic Sines* is finished.

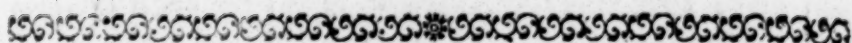
10. Again, from the same Parts of the graduated Line L, as are found in the fifth Column of the *Tablet*, you make Divisions in the Line T, and by them place the Numbers of the first Column, the Line T shall be the *artificial Line of Tangents* graduated for the first great Divisions of 1, 2, 3, 4, 5, &c. 10, 20, 30, &c. Degrees. And the Subdivisions for Minutes will be found as before directed. But tho' the *double Radius* on the Line L suffices for graduating the Line of Sines S, to the whole Length of 90 Degrees, because all *Sines* are less than the *Radius* of a Circle, which is the greatest Sine; yet because the *Radius of a Circle* and the *Tangent of 45 Degrees* are equal; therefore 'tis evident the *Logarithms* of all Tangents greater than 45 Degrees, will exceed the Length of the Line L, as is plain from the fifth Column of the foregoing Tablet.

11. But since *Radius* is a *mean Proportional* between the *Tangent* of any *Arch*, and the *Tangent* of that *Arch's* Complement, it follows, that the *natural Tangents* in the *Geometric Ratio or Scale* are equally distant on each Side from the *Radius or Tangent of 45 Degrees*: and therefore the *Logarithms* of those *natural Tangents*, which are *equidistant* on each Side the *Radius* or *Logarithm of 45°*, are also *equidifferent*; that is, their *Differences* are *equal*. Thus the Difference of 44° and 46°, from *Radius* is the same; and the

the Differences of  $40^\circ$  and  $50^\circ$ ,  $30^\circ$  and  $60^\circ$ ,  $20^\circ$  and  $70^\circ$ , &c. are respectively equal to each other; and consequently the first great Divisions from 1 to  $45^\circ$  on the Line T of *artificial Tangents*, will likewise serve for the *Co-Tangents* of those Degrees, that is, for all the *Tangents* from  $45^\circ$  to  $90^\circ$ , reckon'd back again to the beginning of the Line T. And this is the Reason why on those Lines of *Tangents*, you see the Numbers placed at each 10th Division, thus 10 | 30, 20 | 70, 30 | 60, 40 | 50, 45, at the End. For otherwise the said Line of *Tangents*, must be continued out to *double the Length* it now is, which would not be near so convenient.

12. The Numbers in the 2d and 4th Columns are the Divisions on the *Gunter*, which correspond to the *similar Divisions* on the *Lines of Sines and Tangents*; wherefore the *former* being already made, the *two latter* may also easily be constructed by means of that. And these things are all I judge necessary to be said here concerning the Construction of those excellent *Lines of artificial Numbers, Sines, and Tangents*; and as to their Uses, that will be a Subject to be treated of after the *practical Use* of the *Logarithms* themselves is first explain'd and inculcated; for then the Use of these Instruments will be much better conceived and understood.





## C H A P. XII.

*Of the MANNER of using the TABLES of LOGARITHMS in PRACTICE ; and of the Pre-requisites thereto.*

1. **T**HE *Logarithms* being made for *natural Numbers* by some of the Methods before-going, the next thing necessary was to dispose them into some convenient *Order* or *Form* for *practical Uses*. And such a *Digest* or Collection of *Logarithms*, we call the *Logarithmic Canon* or *Tables*.

2. These *Tables* are of *two Sorts* ; the first contains the *Logarithms* of all *natural Numbers* from *Unity* or 1 to 10000, or 101000 (as those *large Tables* of Mr. *Sherwin*.) In these, the general *Manner* or *Form* is such as here express'd in the Margin, which consists of *two Columns* ; in the first are placed the *Numbers*, in the second the *Logarithms* corresponding thereto, with their *Indices*. And three of these *double Columns* fill a *Page* in common *Books* of this *Form* ; and this, of all others, is the most obvious and easy as to its *Use*, which therefore can need no *Explanation*. For by *Inspection* only is seen what *Logarithm* belongs to *any Number* within the *Compass* of the *Table*.

Numb.	Logarithms.
997	2.9986951
998	2.9991305
999	2.9995655
1000	3.0000000
1001	3.0004341
1002	3.0008677
1003	3.0013009

3. But tho' the *aforesaid Form* be the most *natural* and *obvious*, yet it is not the most *artful* and *comprehensive* ; therefore another *Form* or *Disposition* of the *Tables* for *natural Numbers* has been contrived

more



more concise, or which takes up less room, and is yet as perfect and useful as the other; a *Specimen* of this Form I have here annexed.

N <sup>o</sup> .	Logarithms.				
	0	1	2	3	4
173	238046	238297	238548	238799	239049
174	240549	240799	241048	241297	241546
175	243038	243286	243534	243782	244030

In which the natural Numbers are placed in the *Side-Column* to the *left hand*, all but the *Units Place*, or *first Figure* of the Numbers, which is found in a parallel Column on the top of the Page, in the Order 0, 1, 2, 3 4, &c. as you see in the Specimen. By this, means *one Column of Numbers* suffices for *one Page*, whereas in the other Form there are three such Columns, the whole Page itself consisting entirely of the Logarithms, which in this Case admits of five Columns; but the *Indices* are here omitted as not being necessary, since they are known by the Numbers. The manner of using this Form is yet very easy. For Example, to find the Logarithm of the Number 1742; against 174 in the Side-Column, and under the Units Place 2 at the top, I find the Logarithm 241048; and since the Number has four Places, the *Index* must be 3; where 3.241048 is the Logarithm compleat for the Number 1742. Thus the Logarithm of 1753 = 3.243782; and so for others.

4. But the Tables of Logarithms are yet capable of a further, and much more curious Improvement with regard to their Contraction or Conciseness; for since the Differences of Logarithms decrease as the Numbers increase, 'tis plain those will grow very small as these become very large; and consequently the *two* or *three first Figures* of the Logarithms to the left will be the same for *divers large Numbers* together in

the Canon. Thus for Instance, the Logarithms of all the Numbers between 4168 and 4266, have the two first Figures to the left the same in every one, viz. 62. So likewise all the Logarithms between the Numbers 9954 and 9977 have their first three Figures the same, viz. 998, the Difference of the Logarithms being only in the remaining Figures. The Figures of the Logarithms therefore may be reckoned of two sorts, viz. such as are *permanent* or the same, for certain Intervals; and such as are *variable* or always altering. In this Form of the Canon now under Consideration, these permanent Figures are printed but once for their respective Intervals, and that in the first Column of Logarithms next the Numbers the *variable* Figures *only* pertaining to each Logarithm make the Substance, or fill the whole Face of each Page, as in the Specimen here subjoin'd.

N <sup>o</sup> .	Logarithms.				
	0	1	2	3	4
132	12.0574	0903	1231	1560	1888
133	3852	4178	4504	4830	5156
134	7105	7429	7752	8076	8399
135	13.0334	0655	0977	1298	1619
136	3539	3858	4177	4496	4814
137	6721	7037	7354	7670	7987
138	9879	*0194	0508	0822	1136
139	14.3015	3327	3639	3951	4263
945	975.432	478	524	570	616
946	.891	937	983	*029	075
947	976.350	396	442	487	533
948	808	854	900	946	991
949	977.266	312	358	403	449

5. The Numbers here, as in the last *Form*, are, for the three first Figures to the left, found in the Side-

Side-Column, the other Figure at top. The Logarithms in this Specimen are 65, yet but six of them need be expressed at length, viz. those for the Numbers 1320, 1350, 1390, 9450, 9470, 9490. The Logarithm for 1320 is thus wrote, 12.0574, to denote the two first Figures 12 (separated by a Dot) are permanent thro' the Interval between 1320 and 1350, that is, they belong to the Logarithms of all the intermediate Numbers between those two, and therefore need be express'd only for the first; the other Part of the said Logarithm which is variable, is express'd alone in all the rest. The *upper* Part of this *Specimen* or *Tariff* consists of Logarithms having two Figures in the *permanent Part*; the lower part is an Example of Logarithms having three Figures in the *permanent Part*. To find a Logarithm therefore to any given Number, will also be very easy in this Form. Thus, suppose the Logarithm be sought for in the Number 1323, 'tis found in this Manner. Take the permanent Part either *against* or *next above* the three first Figures 132, which here is 12, then against 132 and under 3 at the top, you find the *variable Part* 1560, to which prefix the *permanent Part* 12, and you have the Logarithm 121560, which with the *Index*, is 3.121560. Again, to find the Logarithm of 1374, take 13 the *permanent Part* next above the three first Figures 137, then against 137, and under 4 at top, you find the *variable Part* 7987, which annexed to the former Part 13, make 137987, and with the *Index*, 3.137987, the Logarithm sought.

6. Thus also you proceed when the permanent Part consisteth of *three Figures*, where the *Intervals* are much shorter. One caution only is necessary, and that is, that you observe in those Lines where there is found an *Asterism* \*, to join all the *variable Parts* in that Line *before* the \*, to the *permanent Part next above*; and all *after* it, to that *next below*.

Thus



Thus in the 7th Line, and 2d Column, you see an *Asterism* \*, therefore the *variable Part* 9879 before it must be join'd to 13 the *permanent Part* next above it, to form the Logarithm 3.139879 for the Number 1380: But the *variable Parts* 0194, 0508, &c. following it, are to be annexed to 14 the *permanent Part* next below, to form the Logarithms 3.140194, 3.140508, &c. for the Numbers 1381, 1382, &c. The Reason of which I presume must be self-evident to every Reader.

7. I have been the more prolix on this *last Form*, lest any *Obscurity* or *Uncertainty* should remain to discourage or prejudice the *young Tyro* against so rare and so *advantageous* a Contrivance. I say, *so rare*; because I have never seen (amongst many) above one, *viz.* *Sherwin's Canon*, in this *Form*; and that, by reason of its *great Price*, is not very common. Tho' that Gentleman says in his Preface, he has in this *excellent Method* followed Dr. *John Newton* in his *Trigonometria Britannica*, a Book which I have not seen. The Advantage also of this Abbreviation is next to that of the Invention it self; for hereby the *prolix* and *unwieldy Tables* (in their original Form) are reduced or abridg'd to one half the Bulk nearly; all the *superfluous Part* being omitted, and nothing but what was necessary retained in this Canon.

8. According to this most excellent Abridgment therefore, I have first of all, that I know of, published the common Canon of Logarithms for Numbers from 1 to 10000; having taken the no small Pains of transcribing the whole with my own hand from the aforementioned large Work of Mr. *Hen. Sherwin*, which is the most correct of any extant.

9. In this Form or Disposition of the Canon, I have also published the Logarithms of Sines and Tangents; which thing hath not been done before in any Work great or small, that I have ever seen or heard of. This makes the 2d Part of the Logarithmic Tables,

Tables, as mentioned Art. 2. And since they are here in a different Form from all others, it may not be unnecessary to hint to the young Learner, that the Numbers express'd in the Side-Column are the Degrees, and every 10th Minute, and the Numbers in the parallel Column at top are the Minutes between the 10ths; see the following Tariff of the Logarithms of Sines in this *Form* for the Minutes from 72 Degrees to 73.

D.		0	1	2	3	4
72.	0	978.206	247	288	329	370
	10	615	655	696	736	777
	20	979.019	059	100	140	180
	30	420	460	500	539	579
	40	816	855	895	934	973
	50	980.208	247	286	325	364

To give an Example, let the Logarithm be sought for the Sine of  $72^{\circ} 43'$ . Seek in the Side-Column  $72^{\circ} 40'$ , next above, which is the *permanent Part* of the Logarithm 979 in the first Column of Logarithms; then against  $72^{\circ} 40$ , and under  $3'$  at top you find the variable Part 934, which annexed to the other makes 979934; to which-prefix the Index (which is set at the top of each Page) 9, and the Logarithm is compleat, *viz.* 9.979934 for the Sine of  $72^{\circ} 43'$ ; and thus you proceed for any other.

10. I have contracted the Logarithms to six Places of Figures only, as being sufficient in common Use; the *natural Sines* and *Tangents* are not here inserted, since when their Logarithms can be used, they themselves are useless. Besides, whenever they are required, they may be immediately had from their Logarithms. For Example, suppose I would know the natural Sine and Tangent for  $38^{\circ} 47'$ , the Logarithm, Sine and Tangent of this Arch, are 9.796836,  
and

and 9.905009. To these Logarithms (neglecting their *Indices*) find the natural Numbers, by the first Part of the Canon, they will be 626377 and 803542, the natural Sine and Tangent sought. (See the Method below, Art. 15. for finding the Number of a given Logarithm.)

11. The Logarithm of the *Secant* of any Arch, as of  $38^{\circ} 47'$ , is thus easily obtain'd :

From the double Radius . . . . . 20.000000

Subtract the Co-Sine of  $38^{\circ} 47' = \dots 9.891827$

There remains the Log. Secant of  $38^{\circ} 47' = 10.108173$

And thus the Logarithm of any other Secant may be found, and consequently the *natural Secant*, or *natural Number* belonging thereto.

12. The Reason why the *Indices*, on the top of the Pages, of the Logarithms of Sines and Tangents, are so large, viz. 7, 8, 9, 10, 11, &c. is because the *Radius* of the Circle was supposed to consist of 10000000000 equal Parts, whose Logarithm therefore is 10.000000; wherefore a Number of such *equal Parts* expressing the Sine of one Minute  $1'$ , will consist of 7 Places, whose Logarithm then will have its *Index* 6. The other Sines will consist of 8, 9, and 10 Places, and so the *Indices* of their Logarithms will be 7, 8, 9, as in the Tables; thus also the Numbers expressing the *Tangents* in such *equal Parts* will consist of 7, 8, 9, 10, 11, 12, 13, and 14 Places of Figures, whence the *Indices* of their Logarithms must be 6, 7, 8, 9, 10, 11, 12, 13, according to Art. 11th and 12th of Chap. I. But since, as before said, the first six Places of the Logarithms to the left are sufficient, the rest are rejected as superfluous.

13. In future Operations there will be frequent Occasion for what is called the *Arithmetical Complement* of a Logarithm, which is nothing but the *Difference*



ference between that Logarithm and Logarithm-Radius 10.000000.

Thus if from . . . . . 10.000000

You subduct the Logarithm . . . . . 4.877026

There will remain the *Arithmet. Comp.* = 5.122974

And this is done *mentally* in an Instant, by taking every Figure from 9, except the first, which you take from 10.

*Note.* If the Logarithm be of any Sine or Tangent, add 10 to the *Index* of the Arithmetical Complement, and it will be the Logarithm of the Co-Secant of the same Arch. For Example,

Suppose the Log. Sine of  $3^{\circ} 48'$  = . . . 8.821342

The Arithmetical Comp. thereof is . . . 1.178658

to which add . . . . . 10,

The Sum is the Log. Secant of  $86^{\circ} 12'$  = 11.178658

Which is evidently the *same Operation* as that in Art. 11. hereof.

14. From the *Theory of Logarithms* we learn, that the Differences of great Numbers are proportional to the Differences of their Logarithms. (See Chap. V. Art. 6, 7.) Therefore tho' the Canon of Logarithms goes no farther than the Number 10000, it may by this means be extended to the Number 10000000, or the Logarithm of any Number under 10000000 may be found by the present Canon, according to the following Rule.

First; find the Logarithm of the *four first Figures* of the given Number, by the *first Part* of the Canon.

Secondly; subtract this Logarithm from the Logarithm *next greater or next following* in the Table; and reserve the *Difference*.

Thirdly; multiply the *Difference*, by the *remaining Figures* of the given Number; and from the *Product* cut off to the *Right hand*, so many Figures as there were *remaining* in the given Number.

N

Fourthly;

Fourthly ; add the *Remainder* of the *Product* to the Logarithm first found, the *Sum* shall be the Logarithm sought.

For Example, let the Logarithm of the Number 127053 be sought.

The Logarithm of the } . . 127000 = 5.103804  
first 4 Figures

The next greater Log. is of . . 127100 = 5.104146

The *Differences* . . . . . 100 342

Wherefore say, as 100 : 342 :: 53 : 181,26

$$\begin{array}{r} 53 \\ 1026 \\ 1710 \\ \hline 181|26 \end{array}$$

Add the first Logarithm 5.103804

The Sum is . . . . . 5.103985 = the Logarithm sought for the Number 127053.

*Example 2.* Required the Logarithm of the Number 3567894 ?

The Logarithm of the } . . 3567000 = 6.552303  
first 4 Figures

The Log. next following is of 3568000 = 6.552425

The Diff. of Numb. and Log. 1000 122

Then say, as 1000 : 122 :: 894 : 109,068

Add the first Logarithm . . 6.552303

the Sum is the Log. 3567894 = 6.552412, as was required. And thus you proceed for the Logarithm of any other greater Number than those in the Canon.

15. By a Method *reverse* to the foregoing, you find the *Number* corresponding to a *given Logarithm*; thus, suppose the given Logarithm be 3.567026, and you would know the Number thereof. Seek this Logarithm in the Table, and because you there find it *exactly*, the Number 3690 corresponding thereto, is the Number sought. But if the given Logarithm

## Chap. XII. *Tables of* LOGARITHMS. 91

Logarithm be *not exactly contain'd* in the Table, and more than 4 Figures be required, proceed as follows.

First; seek in the Table a Logarithm the *next less* to the given one, for the four first Figures of the Number sought.

Secondly; subtract this Logarithm from the *given one*, and annex to the Remainder, *so many Cyphers* as you seek Figures more than four.

Thirdly; take the *Difference* between the Logarithm just found and the *next greater*, by which divide the said *augmented Remainder*, the *Quotient* annexed to the four first Figures shall compleat the *Number sought*.

*Example 1.* Let there be sought the Number to the Logarithm 5.103985, to six Places of Figures.

The Log. next less is of . . . 127000 = 5.103804

The given Logarithm . . . . . 5.103985

The Difference or Remainder 181

The next greater Logarithm 127100 = 5.104146

Diff. between the *greatest* and *least* Log. 342

Since the Places in the Number sought are 6, augment the first Remainder 181 with *two Cyphers*, and it will be 18100; then 342)18100(=53, which annex'd to the four Figures 1270 before found, make the Number 127053 required.

Thus also to the given Logarithm 6.552412, you may find its proper Number 3567894, and so for others.

16. In the same manner you proceed to find any Decimal Number to a Logarithm given, only in this Case the *Indices* of the Logarithms are neglected till the Operation is finished, and then so many Figures are to be cut off from the Number found for Decimals, as the Index of the given Logarithm shall indicate; what is here said relates to *plain* or *terminate Decimals* only; but there are other sorts of Decimals which *circulate* or *perpetually repeat* one or



more Figures *ad infinitum*: And those Figures which thus circulate are *Repetends*, as in these Numbers, viz. 235,2222, &c. 27,83333, &c. 2.383838, &c. 702,6026026, &c. 0,2672326723, &c. Now these *Repetends* need be wrote but once if we slur the first and last Figures in each, to denote them such, as thus 235,2 ; 27,83 ; 2,38 ; 702,60 ; 0,26723, &c. See more in my *Universal System, or Body of Decimal Arithmetic*, printed for Mr. Noon.

17. The Logarithms for the Repeating 9 Digits are made by adding the *Arithmetical Complement* of the Logarithm of 9, to the Logarithms of the said Digits, and are such as here annexed. The Logarithms of pure compound *Repetends* are made by adding the *Arithmetical Complement* of so many 9's, as there are Figures in the *Repetend*, to the Logarithms of those Numbers considered as *terminate*. Thus the Logarithm of the *Repetend* 36,5 is found as follows:

To the Logarithm of . . . . .	36.5	=	1.562293
Add the <i>Arith.</i> Complement of . .	999	=	0.000434
The Sum is the Log. of the <i>Repetend</i> 36,5		=	1.562727

and thus proceed for others.

18. If the *Repetend* have any prefix'd terminate Part; then from such a *mixed Repetend* subtract its *terminate Part*, and to the Logarithm of the *Remainder* add the *Arithmetical Complement* of the Logarithm of as many 9's as there are Figures in the *Repetend*. For Example, suppose you would find the Logarithm of the *mix'd Repetend* 26892,7, proceed thus:

From

From the *Repetend* . . . . . 26892,7  
 Subtract the *terminate Part* 26  
 To the Log of the Rem. . . 26890,1=4.429592  
 add the *Arithm. Complem.* of 9999=0.000043  
 The Sum is the Logarithm of 26892,7=4.429635

*Note*, the *Indices* of the *Arithmetical Complements* are here (as in these Cases they always must be) omitted.

### C H A P. XIII.

#### *Of the Origin and Construction of SHAKERLY'S and STREET'S LOGISTICAL LOGARITHMS.*

I. **T**HE Use of *Logistical Logarithms* is in *Astronomical Calculations*, or *Sexagesimal Arithmetical*; but this sort of *Arithmetic*, which taught the Rules of *Addition*, *Subtraction*, *Multiplication*, *Division*, &c. of *Sexagesimal Fractions*, viz. *Degrees*, *Minutes*, and *Seconds* of *Motion* or *Time*, was, in former times, called *Logistical Arithmetic*. And since the Invention of the common Canon of *Logarithms*, Mr. *Jeremiah Shakerly*, in his *Tabulæ Britannicæ*, first contrived from them a sort of *Logarithms* adapted to the Rules of *Logistical Arithmetic*; and therefore gave them the Name of *Logistical Logarithms*. And since him Mr. *Thomas Street*, in his *Astronomia Carolina*, has invented another and more convenient Form of *Logistical Logarithms*, than *Shakerly's*. And since Tables of both these sorts of *Logistical Logarithms* are extant, 'tis proper to acquaint the Reader with the *Construction* of both, which is as follows.

2. Since *Logistical Logarithms* are altogether concerned in working Proportions of *Degrees, Minutes, and Seconds*, and more especially of *Minutes and Seconds*, together with *Integers*, 'tis evident, if those Sexagesimal Fractions were reduced into the *lowest Denomination*, viz. of *Seconds*, &c. they might then be work'd with the *Logarithms of common Numbers*. Thus suppose the Proportion be  $60' 00'' : 3' 47'' :: 51^{\circ} 29'' : 3' 15''$ ; if these *fractional Numbers* be reduced to *Seconds*, they will stand thus,  $3600'' : 227'' :: 3089'' : 195''$ ; wherefore 'tis plain, the Proportion in this Case may be wrought by the *common Canon of Logarithms*, as will be hereafter shewn. But then as here is no *Radius*, there will arise *double Trouble* in the Work by Logarithms in common Use, in first *adding* the Logarithms of the two *middle Terms*, and then *subtracting* the Logarithm of the *first* from that *Sum*, in order to have the Logarithm of the *fourth Term sought*; or else the *Complement Arithmetical* of the Logarithm of the *first Term* must be taken to perform all by *Addition only*. To avoid therefore the Trouble attending perpetually either of these Methods,

3. Mr. *Shakerly* makes this Proportion, as  $3600'' : 227'' :: 100000, \&c. : 0.06305, \&c.$  or with the Logarithms, thus;

As the Logarithm of  $3600'' = 3.556302$   
 to the Logarithm of  $227'' = 2.356025$   
 so the Logarithm of Radius  $1.00000, \&c. = 10.000000$   
 to the Logarithm of  $0.06305, \&c. = .8.799723$

Now 'tis plain, the *two last Logarithms* perform the same as the *two first*, their Properties being the same; but the first of the two latter Logarithms is Radius  $10.000000$ , which therefore he called the *Logistical Logarithm* of  $3600''$  or one Degree, and consequently the Logarithm  $.8.799723$  is the *Logistical Logarithm* of  $227'' = 3' 47''$ . And thus the *Logistical Logarithm*

of



of any other *Degrees, Minutes and Seconds* may be found; *viz.* by reducing them to *Seconds*, and then by taking from the *Logarithm* of those *Seconds* the *constant Logarithm* 3.556302 of *one Degree in Seconds*.

4. Thus from the Log. of . . .  $1' = 60'' = 1.778151$   
 Take the Log. of one Degree  $= 3600'' = 3.556302$   
 there remains the Logist. Log. of  $1' = 60'' = .8.221849$

Again, from the } . .  $1^\circ 3' 00'' = 3780'' = 3.577492$   
 Log. of }  
 take the *constant* Log.  $1^\circ = 3600'' = 3.556302$   
 there remains the }  $1^\circ 3' 00'' = 3780'' = 10.021190$   
 Logist. Log. of }

By these Examples you easily perceive how the *Logistical Logarithm* may be found for any Number of *Degrees, Minutes and Seconds*, in *Shakerly's Form*. And since there are 60' or 3600'' in *one Hour*, as well as in *one Degree*, therefore a Table of these *Logistical Logarithms* serves equally as well in the *Computation of Time as Motion*.

5. But if the Time of a whole Day or 24 Hours be the *Integer*, since there are but 1440' therein; and  $3600'' : 1440' :: 2^{\frac{1}{2}} : 1'$ ; also since  $60' : 24H^\circ :: 2^{\frac{1}{2}} : 1H^\circ$ ; therefore if, thro' the course of these Tables of *Logistical Logarithms*, against every  $2^{\frac{1}{2}}$  you place the *Hours*, and against every  $2^{\frac{1}{2}}$  (or rather every 3d and 5th Second) you place the *Minutes* of an *Hour*; the Table of *Logistical Logarithms* for *Motion and Time* will be compleated; a *Specimen* of which, in this Form of Mr. *Sha-*

°	Motion	Time.
I	45	H°.
II	2700	XVIII.
0	987506	0
1	987522	
2	987538	
3	987554	1
4	987570	
5	987586	2
6	987602	
7	987618	
8	987634	3
9	987651	
10	987667	4

*kerley's,*

kerley's, I have before annexed. The first Column contains the Degrees, Minutes or Seconds; the second Column the *Logistical Logarithms* thereof; and the third Column contains the Minutes of Time, the Hour being express'd at the top, viz. XVIII. answering to the Motion of  $45'$ , or  $2700''$ .

6. But because these *Logistical Logarithms* of Mr. Shakerley, consist of many Figures throughout the Table, it minister'd occasion to Mr. Thomas Street to contrive a more compendious and convenient Form of these Logarithms; and such he invented, which tho' large at the beginning of the Table, yet immediately lessen very fast, and so continue to the End of the Table, or  $60'$  or  $3600''$ ; whose *Logistical Logarithm* is  $=0$ .

7. The Reason of which is manifest from the Manner of their Construction, which is as follows. Suppose any Proportion of *Sexagesimal Numbers*, as that before made use of, Art. 2. viz.  $60' : 3' 47'' :: 51' 29'' : 3' 15''$ , which reduced to Seconds, stands thus,  $3600'' : 227'' :: 3089'' : 195''$ . Now in order to obtain a *vacant Term* in this Analogy, Mr. Street (instead of Shakerly's Analogy  $3600'' : 227'' :: 100000 : \&c.$ ) inverts the first Ratio, as thus;  $227'' : 3600'' :: \text{Unity} ;$  to a fourth Number whose Logarithm is reputed the *Logistical Logarithm* of the first Term  $227''$ , as the Logarithm of Unity is of the second  $3600''$ . See the Work.

The Logarithm of . . . .  $3' 47'' = 227'' = 2.356026$

The Logarithm of . .  $1^0 = 60' = 3600'' = 3.556302$

The Logarithm of Unity . . . . .  $0.000000$

The Logist. Log. of . . .  $3' 47'' = 227'' = 1200276$

8. Whence it evidently appears, that to find the *Logistical Logarithm* of any Number of Seconds, you need only subtract the common Logarithm of the Number of Seconds from the constant common *Logarithm* of  $3600''$ , for the Remainder or Difference will

will be the *Logistical Logarithm* required, in *Street's* Form.

Thus from the Logarithm of  $\dots 3600'' = 3.556302$   
 Subtract the Log. of  $51' 29'' = 3089 = 3.489818$   
 There remains the Logist. Log. of  $51' 29'' = \dots 66484$

Again from the said Log. of  $\dots 3600'' = 3.556302$   
 Subduct the Logarithm of  $3' 15'' = 195 = 2.290035$   
 There remains the Logist. Log. of  $3' 15'' = 1266267$

9. From whence 'tis evident, that the greater the Number of Seconds is, the less will be the Logistical Logarithm thereof; till you come to the Number  $3600''$ , whose Logistical Logarithm is nothing at all, as before said. And thus it appears that the Logarithms of this kind of Mr. *Street's* Form, have in them fewer Places of Figures, and are therefore more convenient for Use, by much, than those of *Shakerly's* Form, before describ'd. And for that Reason I have chose to give the Reader a Table of *Street's* Logarithms rather than the other; and tho' Mr. *Leadbetter* has given us Tables of both sorts, yet I think it intirely needless; since all the principal Uses of *Shakerly's*, are much better perform'd in *Street's* Logistical Logarithms.

10. In shewing the Manner of making these Logarithms from the common ones, I have expressed them at large, viz. 1200276, 66484, 1266267, the Logistical Logarithms of  $3' 47''$ ,  $51' 29''$ , and  $3' 15''$ , as per Art. 7, 8. Yet here two things are to be observ'd: First, that the Index is not distinguished from Logarithm itself, with a *Point*, as in the common sort; but the remaining Figures, both of Logarithms and *Indices*, be they more or less, are reputed together, the Logistical Logarithm. Secondly, that two Places of Figures to the Right Hand in the Examples, are struck off in the Table; the other being fully sufficient for all the Purposes thereof. So

O

that



that in the Table you will find the said *Logistical Logarithms* wrote 12003, 665, 12663, &c.

11. Mr. *Leadbetter* has taken the pains to continue the Table of *Logistical Logarithms* in *Street's* Form, to 120', or 2 Degrees; but as there is little occasion for any more than the *Logistical Logarithms* of 60' or 1 Degree; and when there is, the same *Logarithms* are capable of answering it, I have therefore continued them no farther than the Inventor did, viz. to 60' or 3600".

12. As to the Form of the Table, 'tis very easy to be understood, especially if those of the common *Logarithms* of *Sines* and *Tangents* before described are: The first Column of the Tables contains the *Degrees* or *Minutes*, or *Minutes* and *Seconds* in *Sexagesimals*, as in the common Tables, in the Order of *Denaries* or 10's, the nine Digits running along on the top of the Table, under which, in the several Columns, are the *Logistical Logarithms* abbreviated in the same manner as those of *Sines* and *Tangents*; and are to be taken out according to the Directions there given, which see. In the last Column are contained the Numbers of the first reduced to *Minutes* or *Seconds*; and are to be compleated likewise with the *Digits* on the top of the Table. These are referred to when the *Logistical Logarithm* of any integral Number is sought. An Example or two will render all easy.

13. Let it be required to find the *Logistical Logarithm* of  $2^{\circ} 48'$  or  $2' 48''$ . First seek 2 40 in the first Column, and against it and under 8 at top, you find 310, which annexed to its proper permanent Part 13 in the first Column of *Logarithms* makes 13310, the *Logistical Logarithm* of 2 48, as required. Thus the *Logistical Logarithm* of  $37' 59''$  is found to be 1986; and of  $54' 40''$  to be 404; and of  $58' 37''$  to be 101; and of  $59' 53''$  to be 8. And thus the *Logistical*

# Chap. XIII. Tables of LOGARITHMS. 99

*Logistical Logarithm* of 60', is = 0; and so will be a vacant Term in all Analogies for Operation.

14. Let it be required to find the *Logistical Logarithm* of the Number 584. First seek 580 in the last or right-hand Column, and against it and under 4 at top you see \*99, which shews it must be join'd to the following permanent Part 78 in the first Column of *Logarithms*, and therewith makes 7899, the *Logarithm* sought. Thus the *Logistical Logarithm* of 1000, is found 5563; and of 3359, to be 301; and of 3596, to be 5; and that of 3600 is nothing.

15. This Table also equally serves for Time, whether for a Day and *Minutes*, or *Minutes* and *Seconds*; by help of the little Table at the End of the *Logistical Logarithms*, which shews what Parts of Motion in *Degrees* and *Minutes* correspond to Time in *Hours* and *Minutes*. Thus against 13' in Time, you see 33' of *Motion*; against 1 Hour is 2° 30'; and the Motion answering to VII<sup>h</sup> 43', is 17° 30', + 1° 48' = 19° 18'; and that answering XXI<sup>h</sup> 21' is 52° 30', + 0° 53' = 53° 23'; and so for other Parts of Time; consequently

H°. Time.							
The Logistical Logarithm of	0	13'	} is {	20378	} as being the same for the Motion of	0°	33'
	1	00		13802		2	30
	7	43		4926		19	18
	21	21		507		53	23
	24	00		0		60	00

The Reason of all which is evident, from Art. 5. hereof.

16. These *Logistical Logarithms* may in like manner be render'd applicable to Computations of *Money*, *Weights*, *Measures*, &c. Thus since 60° : 20° :: 3 : 1, therefore the Shillings in a Pound correspond to each 3d Degree or Minute of Motion. And again, since 960<sup>rs</sup> : 3600<sup>rs</sup> :: 1 : 3<sup>1/3</sup>; therefore to 1, 2, 3, 4 &c. Farthings there answers 3', 7', 11', 15', &c. Minutes

100 *Of Shakerly's and Street's Tables, &c.*

of *Motion*; whence a Table may be form'd to shew the Logistical Logarithm for any Number of Farthings under 960, or one Pound. And thus you may proceed to frame a Table to render these *Logistical Logarithms* for Motion useful for the *Ounces* and *Pounds* in an *hundred Weight*; and that with great *Ease*, since 1792, the Number of Ounces in an *hundred Weight* Averdupoise, is nearly half the Number 3600", the Seconds of the Table. Consequently, the *Logistical Logarithm* of the *Double* of any Number of Ounces, is that required for the *Pounds* and *Ounces* equal thereto.

17. Or, lastly, 'tis easy for any one who shall think it worth while, to calculate Tables of *Logistical Logarithms*, for any Species of Computation, peculiar to it self. Thus with respect to Money, if the common Logarithms of all the Farthings under 960, be subducted from the constant Logarithm of 960, the *Remainders* will be the Logistical Logarithms of the Farthings in a Pound. The common Logarithms of all the Numbers of *Ounces* under 1792 subtracted from the constant Logarithm of the said Number 1792, will leave the *Logistical Logarithms* of all the Ounces in an hundred Weight. And thus you proceed to make *Logistical Logarithms* of any Kind you please, or may have occasion for. Which is too easy a matter to require an Example, besides those above, Art. 7, and 8.







# LOGARITHMOLOGY.

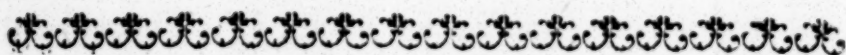
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## PART II.

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### *The PRAXIS of LOGARITHMS, Common and Logistical:*

*With its APPLICATION to Vulgar and Duodecimal Arithmetic, Plain and Spherical Trigonometry, Navigation, Mensuration of Surfaces and Solids, Gauging, Timber-Measure, Astronomy, perform'd Numerically and Instrumentally.*



### CHAP. I.

*Of the Rules of ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION of the Indices of LOGARITHMS.*

1. **A**S the *Theory of Logarithms* has been largely premised, the *Rules for a Practical Use and Management* of those *artificial Numbers* will from thence be easily understood, and the *Rationale of every Operation* be apparent to the intelligent Reader. Before we proceed to the Use of Logarithms as applied to *common Arithmetic, &c.* we must

must first consider some *previous Rules and Methods* which regard the *due ordering and working* those Numbers themselves, on account of the *Indices*, which admit of divers particular Cases in the *Rules of Addition, Subtraction, Multiplication, and Division*: which therefore must be *exemplified and illustrated* as in the *Sequel* of this Chapter.

### ADDITION of LOGARITHMS.

2. As in all *Species of Arithmetic* the first *fundamental Rule* is *Addition*, so in this of *Logarithms*, the *Rules* which require this *primary Operation* come first to be consider'd, and they are as follow.

(*Note*, I shall in this Place call the *Indices* of *Logarithms* of *whole Numbers*, *Integral Indices*; and those of the *Logarithms* of *Decimal or Fractional Numbers*, *Decimal Indices*.)

*Rule I.* If the *Indices* be both *Integral*, add them together for the *Sum* required.

*Rule II.* If the *Indices* are both *Decimal*, add them as before; and observe, (1.) if the *Sum* be *above* or *just* 10 or 100 ( $=tA$ ; see Chap. IV. Theory) cast away 10 or 100. (2.) If the *Sum* be under 10 or 100 ( $=tA$ ) add 10 or 100 thereto; and both the *Sum* in the *latter Case* and the *Remainder* in the *former*, will be *Decimal*.

*Rule III.* If the *Indices* be of *different Kinds*, viz. *Integral* with *Decimal*, the *Sum*, if under 10 or 100, is *Decimal*; if *just* 10 or 100, or *above*, cast away 10 or 100, the *Remainder* is *Integral*.

*Rule IV.* In case the *Sum* of *two or more decimal Indices* be less than  $10=tA$ , the best way will be to use the larger *decimal Indices*, where  $tA=100$ ; and then their *Sum* will be greater than 100, and so the *Reason* of the *Operation* will be more evident.

3. All these *Rules* relate to the *Indices* of the *Logarithms* only, and are exemplified as follow:

Ex-

Chap. I. *Indices of LOGARITHMS.* 103

Exam. I. To	3.513217	Exam. II.	2.317227
add	2.303196		0.850891

Rule I. Sum =	<u>5.816413</u>	Sum =	<u>3.168118</u>
---------------	-----------------	-------	-----------------

Exam. III. To	.9.849235	Exam. IV.	.97.237406
add	.7.786822		.95.072607

Rule II. } Part I. }	Sum =	<u>.7.636057</u>	Sum =	<u>.92.310013</u>
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Exam. V. To	.4.273760	Exam. VI.	.62.346174
add	.3.067247		.21.300725

Rule II. } Part 2. }	S. =	<u>.17.341007</u>	Sum =	<u>.183.646899</u>
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Exam. VII. To	6.372458	Exam. VIII.	.88.426703
add	2.673842		5.268402

Rule III. Sum =	<u>9.046300</u>	Sum =	<u>.93.695105</u>
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Exam. IX. To	5.206737	Exam. X.	8.426735
add	.8.312046		.92.105374

Rule III. Sum =	<u>3.518783</u>	Sum =	<u>0.532109</u>
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Exam. XI. } add }	.4.203106 .1.312729 .2.070346	} { Or rather thus, per Rule IV. Ex. XII. .94.203106 .91.312729 .92.070346 <u>.77.586181</u>
Rule II. } Part 2. }	S. = <u>.27.586181</u>	

'Tis possible these two last Examples may seem somewhat *obscure*; but the *Reason* and *Truth* of each will appear, if they are wrought *at twice*, as follows:



To .4.203106		And .94.203106	
add .1.312729		.91.312729	
Rule II. } Part 2. }	S. = .15.515835	Rule II. } Part 1. }	S. = .85.515835
to which add .2.070346		.92.070346	
Rule II. } Part 2. }	S. = .27.586181	Rule II. } Part 1. }	S. = .77.586181

From whence appears the Reason why, in these Cases, the *larger Indices* are preferable to the *smaller ones*.

### SUBTRACTION of LOGARITHMS.

4. In the *Subtraction* of *Indices* the following *Rules* are to be observ'd, viz.

*Rule I.* If they are both *Integral*, and the *higher* one the *greater*, the *Remainder* will be *Integral*. But if the *lower* one be the *greater*, then add 10 to the *higher* one, and *subtract*; the *Remainder* will be *Decimal*.

*Rule II.* If both *Indices* are *Decimal*; and the *higher* be the *greater*; the *Remainder* is *Integral*; if not, add 10 or 100 to the *higher* one, and subduct the *lower*, the *Remainder* will be *Decimal*.

*Rule III.* If the *Indices* are of *different* sorts, viz. one *Integral* the other *Decimal*; then if the *higher* be *Integral*, add 10 or 100 to it, and *subtract*; the *Remainder* will be *Integral*. But if the *higher* be *Decimal* and the *greater*, the *Remainder* will be *Decimal*; if *lesser*, the *larger Indices* must be used.

5. The *Rules* also are easily deriv'd from the foregoing *Theory*, and are illustrated by the following *Examples*.

Exam. I. From 5.816413  
Subtract 3.513217

Rule I. Rem. = 2.303196

Ex. II. 3.168118  
2.317227

Rem. = 0.850891

Ex.

# Chap. I. *Indices of* LOGARITHMS. 105

Ex. III. From 3.513217 Ex. IV. 0.850891  
sub. 5.816413 3.168118

Rule I. Rem. = 7.696804 Rem. = 7.682773

Ex. V. From .7.503617 Ex. VI. .94.420345  
sub. .3.467306 .92.673457

Rule II. Rem. = 4.026311 Rem. = 1.746888

Ex. VII. From .3.467306 Ex. VIII. .92.673457  
sub. .7.503617 .94.420345

Rule II. Rem. = 5.963689 Rem. = .98.253112

Ex. IX. From 5.816413 Ex. X. 6.2067347  
sub. .8.132700 .94.1535012

Rule III. Re. = 7.683713 Rem. = 12.0532335

Ex. XI. From .8.132700 Ex. XII. .94.278769  
sub. 5.816413 6.165348

Rule III. } Rule III. }  
Part 2. } Re. = .2.316287 Part 2. } R. = .88.113421

## MULTIPLICATION of LOGARITHMS.

6. In Chap. IV. Art. 19th and 20th of the *Theory*, the Rules for multiplying the *Indices of Logarithms* (of *Pure Fractions* especially) are demonstrated; and are,

Rule I. If the *Index* be *Integral*, multiply as usual; the Product shall be *Integral*.

Rule II. If the *Index* be *Decimal*, make the Logarithm of *Unity*, or  $tA=100$ , then shall the *Index* be

P

be

Ex.

be of the *larger Sort*, which in this Case, will be more convenient for Use ; and then according as you multiply by 2, 3, 4, 5, 6, &c. you must reject 100, 200, 300, 400, 500, &c. from the *Product*, the *Remainder* thereof will be *Decimal*.

$$\begin{array}{rcl} \text{Ex. I. Mult.} & 3.420673 & \text{Ex. II.} \\ \text{by} & 2 & 5.700672 \\ \hline & & 4 \end{array}$$

$$\begin{array}{rcl} \text{Rule I. Prod.} & = 6.841346 & \text{Product} = 22.802688 \\ \hline & & \hline \end{array}$$

$$\begin{array}{rcl} \text{Ex. III. Mult.} & .96.130126 & \text{Ex. IV.} \\ \text{by} & 2 & .91.034106 \\ \hline & & 3 \end{array}$$

$$\begin{array}{rcl} \text{Rule II. Pr.} & = .92.260252 & \text{Prod.} = .73.102318 \\ \hline & & \hline \end{array}$$

$$\begin{array}{rcl} \text{Ex. V. Mult.} & .84.034121 & \text{Ex. VI.} \\ \text{by} & 4 & .70.061050 \\ \hline & & 5 \end{array}$$

$$\begin{array}{rcl} \text{Rule II. Pr.} & = .36.136484 & \text{Prod.} = .50.305250 \\ \hline & & \hline \end{array}$$

In this last Example, the *Index*  $.70. \times 5 = 350$  ; but since from 350 you cannot reject 400, as *per* Rule II; therefore it must be  $400 - 350 = .50$ . which subtracted from 99, leaves 49 ; which shews that 49 *Cyphers* above 100, that is, 149 *Cyphers* are to be prefix'd ; and this you are to understand in all Cases where  $xL$  is less than  $x - 1 \times tA$ . See Chap. IV. Art. 20th, of *Theory*. Or thus, in general let  $tA = 10$  or 100 ; then when  $xL$  is *greater* than  $x - 1 \times tA$ , then it will be  $xL - x - 1 \times tA - 9$  or 99, is equal to the Number of *Cyphers* to be prefix'd. But if  $x - 1 \times tA$  is *greater* than  $xL$ , then it will be  $x - 1 \times tA - xL - 9$  or 99, = to the said Number of *Cyphers*.



## DIVISION of LOGARITHMS.

7. This is but the *Reverse* of the foregoing Operation; and the Rule for decimal *Indices* the reverse to that; which is also derived from Chap. IV, Art. 21st, and 22d of the *Theory*, which see.

*Rule I.* If the *Index* be *Integral*, divide as usual; and the *Quotient-Index* will be *Integral*.

*Rule II.* If the *Index* be *Decimal*, use the larger *Sort*; and then adding to the said *Index* 100, 200, 300, &c. divide by 2, 3, 4, &c. the *Quotient-Index* will be *Decimal*.

Ex. I. Divide 6.841348 Ex. II. 22.802688  
by 2 4

Rule I. Quot. = 3.420673 Quotient = 5.700672

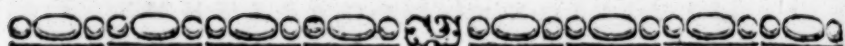
Ex. III. Div. .92.260252 Ex. IV. .83.102300  
by 2 3

Rule II. Qu. = .96.130126 Quot. = .94.367433

Ex. V. Divide .36.136484 Ex. VI. .30.305250  
by 4 5

Rule II. Qu. = .84.034121 Quot. = .86.061050





## C H A P. II.

Of MULTIPLICATION and DIVISION of  
WHOLE NUMBERS and DECIMALS by LO-  
GARITHMS.

1. **H**AVING before shewn the Method of finding the *Logarithms* of all kinds of *Numbers*, both *Integers* and *Decimals*, and also of fitting and adjusting proper *Indices* thereto ; and in the foregoing Chapter, the *Arithmetical Management* thereof in *all Varieties* : I shall now apply the *Use* of those excellent *Numbers* in the *Rules of Arithmetic* ; and first in the *Multiplication* and *Division of Whole Numbers* and *Decimals*.

2. From the foregoing Theory (see Chap. I. Art. 10 ; and Chap. III. Art. 12.) we obtain this easy and obvious Rule for the *Multiplication* of all kind of *Numbers* by *Logarithms*,

viz. { To the *Logarithm* of the *Multiplicand*,  
Add the *Logarithm* of the *Multiplier* ;  
The *Sum* is the *Logarithm* of the *Product*.

*Examples of INTEGERS.*

		Logarithms.
3. Example I. Multiply . . . .	12	= 1.079181
	by . . . . .	8 = 0.903090
		<hr/>
	Product . . . .	96 = 1.982271
		<hr/>

Logarithms.

Example II. Multiply . . . . . 127 = 2.103804  
 by . . . . . 12 = 1.079181  


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 Product . . . . . 1524 = 3.182985

Example III. Multiply . . . . . 526 = 2.720986  
 by . . . . . 100 = 2.000000  


---

  
 Product . . . . . 52600 = 4.720986

Example IV. Multiply . . . . . 9876 = 3.994581  
 by . . . . . 517 = 2.713490  


---

  
 Product . . . . . 5105892 = 6.708071

Example V. Multiply . . . . . 987600 = 5.994581  
 by . . . . . 517000 = 5.713590  


---

  
 Product 510589200000 = 11.708071

4. *Examples of MIX'D NUMBERS.*

Example VI. Multiply . . . . . 7,5 = 0.875061  
 by . . . . . 10 = 1,000000  


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 Product . . . . . 75 = 1.875061

Example VII. Multiply . . . . . 12,4 = 1.093422  
 by . . . . . 3,6 = 0.556302  


---

  
 Product . . . . . 44,64 = 1.649724

Ex-



		Logarithms.
Example VIII. Multiply . . 0,762	=	.9.881955
by . . . . . 570	=	2.755875
Product . . 434,34	=	2.637830

Example IX. Multiply . . . . 36,5	=	1.562293
by . . . . . 0,00019	=	.6.278754
Product 0,006935	=	.7.841047

Example X. Multiply . . . . . 473	=	.9.674861
by . . . . . 63	=	1.803705
Product . . . . . 30.1	=	1.478566

Example XI. Multiply . . . . . 6	=	0.823909
by . . . . . ,8	=	.9.744727
Product 3,78	=	0.568636

Example XII. Multiply . . . 21,23	=	1.326541
by . . . . . 47,8	=	1.623458
Product 900,718	=	2.949999

5. *Examples of PURE DECIMALS.*

Example XIII. Multiply . . . . ,12	=	.9.079181
by . . . . . ,8	=	.9.903090
Product . . . . ,096	=	.8.982271

Ex-

	Logarithms.
Example XIV. Multiply . . ,0097	= .97.986772
by . . . . ,00021	= .96.322219
Product 000002037	= .94.308991

Example XV. Multiply ,00426	= .97.623458
by . . . ,00008	= .95.948847
Product ,0000003735	= .93.572305

Example XVI. Mult. ,00000085	= .93.929419
by ,0000012	= .94.079181
Product ,00000000000102	= .88.008600

Ex. XVII. Mult. ,00000000075	= .90.875061
by ,000006	= .94.778151
Product ,0000000000000045	= .85.653212

## 6. DIVISION by LOGARITHMS.

In the same Part of the *Theory* referr'd to (Art. 2.) for the Rule by which *Multiplication* is performed by *Logarithms*, you will likewise find the Demonstration of the following Rule of *Division* of Numbers by *Logarithms*;

viz. { From the Logarithm of the *Dividend*,  
Subtract the Logarithm of the *Divisor*;  
The Remainder is the Logarithm of the  
*Quotient*.

Ex.

*Examples of INTEGERS.*

Logarithms.

$$\begin{array}{rcl}
 7. \text{ Example I. Divide } & \dots\dots & 96 = 1.982271 \\
 & \text{by } \dots\dots & 12 = 1.079181 \\
 & & \hline
 \text{Quotient } \dots\dots & 8 = & 0.903090 \\
 & & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Example II. Divide } & \dots\dots\dots & 1524 = 3.182985 \\
 & \text{by } \dots\dots\dots & 172 = 2.103804 \\
 & & \hline
 \text{Quotient } \dots\dots\dots & 12 = & 1.079181 \\
 & & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Example III. Divide } & \dots\dots & 52600 = 4.720986 \\
 & \text{by } \dots\dots\dots & 526 = 2.720986 \\
 & & \hline
 \text{Quotient } \dots\dots & 100 = & 2.000000 \\
 & & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Example IV. Divide } & \dots\dots & 5105892 = 6.708071 \\
 & \text{by } \dots\dots\dots & 517 = 2.713490 \\
 & & \hline
 \text{Quotient } \dots\dots & 9876 = & 3.994581 \\
 & & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Exam. V. Divide } & 510589200000 & = 11.708071 \\
 & \text{by } \dots\dots\dots & 987600 = 5.994581 \\
 & & \hline
 \text{Quotient } & 517600 = & 5.713490 \\
 & & \hline
 \end{array}$$

8. *Examples in MIX'D NUMBERS.*

$$\begin{array}{rcl}
 \text{Example VI. Divide } & \dots\dots\dots & 75 = 1.875061 \\
 & \text{by } \dots\dots\dots & 7.5 = 0.875061 \\
 & & \hline
 \text{Quotient } \dots\dots\dots & 10 = & 1.000000 \\
 & & \hline
 \end{array}$$



Example VII. Divide . . . . . 44,64 = 1.649724  
by . . . . . 12,4 = 1.093422  


---

Quotient . . . 3,6 = 0.556302  


---

Example VIII. Divide . . . . 434,34 = 2.637830  
by . . . . . 570 = 2.755875  


---

Quotient . . . ,762 = .9.881955  


---

Example IX. Divide . . . ,006935 = .7.841046  
by . . . . . 36,5 = 1.562293  


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Quotient . . . ,00019 = .6.278753  


---

Example X. Divide . . . . . 30,1 = 1.478566  
by . . . . . 63 = 1.803705  


---

Quotient . . . . ,473 = .9.674861  


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Example XI. Divide . . . . . 3,70 = 0.568636  
by . . . . . 6 = 0.823909  


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Quotient . . . . . 8 = .9.744727  


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Example XII. Divide . . . 800,718 = 2.949999  
by . . . . . 42,0 = 1.623458  


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Quotient . . . 21,23 = 1.326541  


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Q

9. Ex-

9. *Examples in PURE DECIMALS.*

	Logarithms.
Example XIII. Divide . . . . ,096	= .8.982271
by . . . . . ,12	= .9.079181
Quotient . . . . ,8	= .9.903090

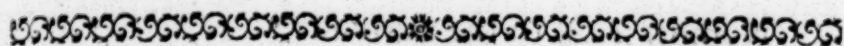
Exam XIV. Divide ,000002037	= .94.308991
by . . . . ,0097	= .97.986772
Quotient ,00021	= .96.322219

Ex. XV. Divide . . ,0000003735	= .93.572305
by . . . . . 00008	= .95.948847
Quotient ,00470	= .97.623458

Ex. XVI. Div. ,000000000000102	= .88.008600
by . . ,00000085	= .93.929419
Quotient ,0000012	= .94.079181

Ex. XVII. Div. ,0000000000000045	= .85.653212
by . . ,000006	= .94.778151
Quotient ,00000000075	= .90.875061

10. I think these Examples in the *Multiplication* and *Division* of Numbers by *Logarithms*, are sufficient to instruct any *docible Genius* in his Practice herein; and as the latter are but the *Converse* of the former, so they mutually illustrate and prove the Truth of each other respectively.



# C H A P. III.

## *Of raising* POWERS, and the EXTRACTION of ROOTS by LOGARITHMS.

1. **F**ROM the Theory of Logarithms (Chap. I. Art. 12. and Chap. III. Art. 13.) we have an evident Rule for the *Involution of Numbers*, or *raising* them to any proposed *Power* by means of Logarithms; which is this,

*viz.* { Multiply the *Logarithm* of the *given Number*  
by the Index of the Power, *viz.* 2, 3, 4, 5,  
&c. the *Product* shall be the *Logarithm* of  
the Power, *viz.* the *Square*, *Cube*, *Biqua-*  
*drate*, *Sur-solid*, &c. *Power* of the said *given*  
*Number*.

### *Examples in* INVOLUTION.

2. Example I. What is the Square of the Number 32?

Multiply the Logarithm of . . . . . 32=1.505150  
by the Index of the Power . . . . . 2

The Prod. is the Log. of the Square 1024=3.010300

Ex. II. Required the Square of . . . . . 3.2=0.505150

Multiply by . . . . . 2

The Prod. is the Log. of the Square 10,24=1.010300

Ex. III. Required the Square of . . . ,32=.9.505150

Multiply by . . . . . 2

The Product is the Answer . . . ,1024=.9.010300

Q 2

Ex-



Example IV. Required the several Powers of the Number 1.05 to the Surfsolid?

$$\begin{array}{r} 1. \text{ The Logarithm of } \dots\dots\dots 1.05 = 0.021189 \\ \text{Multiply by } \dots\dots\dots 2 \end{array}$$

$$\text{The Product is the Square } \dots\dots\dots 1,1025 = 0.042378$$

$$\begin{array}{r} 2. \text{ Multiply the Logarithm of } 1.05 = 0.021189 \\ \text{by } \dots\dots\dots 3 \end{array}$$

$$\text{The Product is the Cube } 1,157625 = 0.063567$$

$$\begin{array}{r} 3. \text{ Multiply the Logarithm of } 1.05 = 0.021189 \\ \text{by } \dots\dots\dots 4 \end{array}$$

$$\text{Product is the Biquadrate } 1.21650625 = 0.084756$$

$$\begin{array}{r} 4. \text{ Lastly, Multiply the same } \dots\dots\dots 1.05 = 0.021189 \\ \text{by } \dots\dots\dots 5 \end{array}$$

$$\text{Product is the Surfsolid } 1.2773315625 = 0.105945$$

Example V. Required the *surfsolid* Power of the Number ,0006?

$$\begin{array}{r} \text{Multiply the Logarithm of } ,0006 = .96.778151 \\ \text{By the Index of the Power } \dots\dots\dots 5 \end{array}$$

$$\text{The Surfsol. } ,0000000000000000007776 = .83.890755$$

Example VI. What is the *Cubo-Cube* Power of ,08?

$$\begin{array}{r} \text{Multiply the Logarithm of } \dots\dots\dots ,08 = .98.903090 \\ \text{by the Index of the Power } \dots\dots\dots 6 \end{array}$$

$$\text{The Cubo-Cube Power } ,000000262144 = .93.418540$$

Example VII. What is the 57th Power of the Number ,99?

Multiply

Multiply the Logarithm of . . . ,99=.9.995635  
by the Index of the Power . . .

$$\begin{array}{r} 57 \\ \hline 69969445 \\ 49978175 \\ \hline \end{array}$$

The 57th Power is . . . ,56389, &c.=.9.751195

3. There is another way of raising the Powers of *Decimal Numbers* by Logarithms, and it is thus ;

viz. { Multiply the *Arithmetical Complement* of the  
Logarithm of the given Fraction by the  
Index of the Power, the *Arithmetical Com-*  
*plement* of the Product is the Logarithm of  
the Power sought.

And this in many Cases, (as when the *Index* of the *Power* is a *mix'd Number* or *pure Decimal*) will be found most certain and ready. Thus in the last Example this way ;

Example VIII. What is the 57th Power of the Number ,99 ?

The Logarithm of . . . . . ,99=.9.995635  
The *Arithmetical Complement* 0.004365  
which multiply by . . . 57

$$\begin{array}{r} 30555 \\ 21825 \\ \hline \end{array}$$

The Product . . . 0.248805  
The Arithmet. Comp. is }  
the Log. of the Power } ,56389, &c.=.9.751195

Example IX. What is the 4th or 25th Power of ,2 ?

The

The Logarithm of . . .  $,2 = .9.301030$   
 The *Arithmetical Comp.* thereof  $.0.698970$   
 which multiply by the Index  $,25$

---

3494850  
 1397940

---

The Product . . .  $0,17474250$   
 The Arithmet. Comp. is }  $,66874, \&c. = .9.825258$   
 the Log. of the Power }

Example X. What is the 6,25th Power of ,0032?

The Logarithm of . . .  $,0032 = .7.505150$   
 The Arithmetical Complement  $2.494850$   
 which multiply by . . .  $6.25$

---

The Product is . . .  $15.5928125$   
 The Arithmetical Complem. of which is  $.84.4071875$   
 And the Number answering thereto, viz.  
 ,000000000000000025538 is the 6.25th Power of  
 ,0032.

#### 4. EVOLUTION or EXTRACTION of ROOTS by LOGARITHMS.

This is done by a Rule, the *converse* of that for *Invo-*  
*lution*, in Art. 1st;

viz. { Divide the Logarithm of the Power by the  
 Index of the Root, the Quotient shall be  
 the Logarithm of the Root sought.

#### Examples in EVOLUTION.

Example I. What is the *Square Root* of 1024?

Divide the Logarithm of . . .  $1024 = 3.010300$   
 By the Index of the Root  $2$

---

The Quot. is the Log. of the square Root  $32 = 1.505150$

Ex.



Example II. Required the *Cube Root* of 1,157626?

Divide the Logarithm of 1,157625 = 0.063567  
By the Index of the Root 3

The Qu. is the Log. of the *Cube R.* 1,05 = 0.021189

Example III. What is the *sur-solid Root* of the Power  
,00000000000000007776?

The Logarithm thereof is . . . .83.890755  
which divide by the Index 5

The Quot. is the Log. of the }  
Root sought ,0006 = .96.778151

Example IV. What is the *Cubo-Cube Root* of the  
Power ,000000262144?

The Log. of . . . ,000000262144 = .93.418540  
Which divide by the Index 6

The Root sought is . . . ,08 = .98.903090

Example V. What is the 57th Root of the Power  
,56389, &c.?

The Logarithm of . . . ,56389, &c. .9.751195  
which divide by . . . 57

The Root required is . . . ,99 = .9.995635

5. Another different way to extract the Root of  
Decimal Numbers is the converse of that in Art. 3d,  
hereof.

viz. { Divide the *Arithmetical Complement* of the Lo-  
garithm of the *Decimal* given by the Index  
of the Root required, the *Arithmetical Com-*  
plement of the Quotient is the Logarithm  
of the Root sought.

Example VI. What is the ,25th Root of the Power  
 ,66874, &c.?

The Logarithm thereof is . . .	.9.825258
The Arithmetical Complement is	0.174742
which divide by . . .	,25

The Quotient . . .	0.698970
--------------------	----------

The Arith. Comp. is the Log. } . . ,2=	.9.301030
of the Root	

Example VII. Required the 6.25th Root of the  
 Power ,00000000000000025538 ?

The Logarithm thereof is . . .	.84.407187
The Arith. Comp. of which is	15.592813
which divide by . . .	6.25

The Quotient is . . .	2.494850
-----------------------	----------

the Arith. Complem. thereof is . . .	.7.505150
--------------------------------------	-----------

the Logarithm of ,0032 the Root sought.

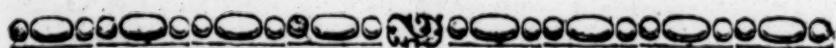
Example VIII. What is the *Cubo-Cube* or 6th Root  
 of the Power ,1 ?

The Logarithm of . . .	,1=.9.000000
the Arith. Comp. thereof . .	.0.999999
which divide by the Index	6

The Quotient . . .	0.166666
--------------------	----------

The Arith. Comp. is the } ,68129, &c. =	.9.833333
Log. of the Root	

6. Thus you see the great Use of Logarithms in extracting the Roots of a given Power, which tho' a thing so very difficult by the Rules of common Arithmetic, is yet render'd most easily practicable by this excellent Art; yea 'tis easy to make it appear, that the *Extraction of Roots* is not only most expeditiously performed, but hath a more universal Perfection in this Method, than in any other.



# CH A P. IV.

*Of the various RULES of PROPORTION, and of finding MEAN PROPORTIONALS by LOGARITHMS.*

1. **F**ROM the *Theory* 'tis evident, that the *golden Rule*, or *Rule of Proportionals*, is wrought in Logarithms by only the *Addition* and *Subtraction* of the Logarithms of the Terms of the Proportion. And if the Proportion be direct, the Rule is thus;

*viz.* { Add the Logarithms of the *second* and *third* Terms, from that *Sum* subtract the Logarithm of the *first*; the Remainder is the Logarithm of the *fourth* sought.

*Examples in the GOLDEN RULE Direct.*

Example I. If 12 Pounds cost 1 *l.* 15 *s.* 9 *d.* what will 173 Pounds cost?

The Logarithm of . . . . . 12 = 1.079181

To the Log. of 1 *l.* 15 *s.* 9 *d.* = 1.7875 = 0.252246

Add the Logarithm of . . . . . 173 = 2.238046

The Sum . . . . . 2.490292

Subtract the first, there remains 25,77 *l.* = 1.411111

Wherefore the Answer is 25,77 *l.* = 25 *l.* 15 *s.* 4 *d.* 4.

2. But since if you *divide* by any Number, or *multiply* by its *Reciprocal*, the Effect is the same; and also since the Arithmetical Complement of any Number, is but the Logarithm of the *Reciprocal* of  
R
that



that Number; therefore it follows, that where the *Subtraction* of a Logarithm is required in any Operation, if you take the *Arithmetical Complement* of that Logarithm, the whole may be performed by *Addition* only. Thus in the foregoing Example.

Add  $\left\{ \begin{array}{l} \text{the Ar. Comp. of the Log. of } 12 = 8.920819 \\ \text{the Log. of } \dots\dots\dots 1,78751 = 0.252246 \\ \text{the Log. of } \dots\dots\dots 173 = 2.238046 \end{array} \right.$

The Sum is the Answer  $\dots\dots 25,771 = 1411111$   
the same as before.

Example II. If 2 C. 1 q. 21 l. 14 oz. cost 5 l. 17 s. 8 d.  
what will 31 C. 2 q. 26 l. 15 oz. cost?

Then 2 C. 1 q. 21 l. 14 oz. = 2.4453 A. Com. .9.611668  
The Log. of 5 l. 17 s. 8 d.  $\frac{1}{4} = 5.8844 = 0.769702$   
The Log. of 31 C. 2 q. 26 l. 15 oz. }  $\dots\dots = 1.501614$   
 $= 31,7405$

The Sum is the Logarithm of 76,381 l. = 1.882984  
Therefore 76,381 l. = 76 l. 7 s. 7 d.  $\frac{1}{4}$  is the Answer.

### 3. Of the Rule of Three Inverse.

In this Case you must take the *Arithmetical Complement* of the *third* Term, and add it with the Logarithms of the other two as before; so shall the Sum be the Logarithm of the Answer.

Example. Suppose a Field feeds 18 Horses for 7 Weeks, how long will it feed 42, at that rate?

Add  $\left\{ \begin{array}{l} \text{the Logarithm of } \dots\dots\dots 18 = 1.255272 \\ \text{the Logarithm of } \dots\dots\dots 7 = 0.845098 \\ \text{the Arith. Comp. of the Log. of } 42 = 8.376751 \end{array} \right.$

The Sum is the Logarithm of Answer 3 = 0.477121

4. Of the double Rule of Three, or Rule of Five Numbers.

As in Questions of this sort, there are always *three conditional or supposed Terms*; the first of which is the *principal Cause of Gain, Loss, Action, &c.* the *second* denotes the *Time, Distance, &c.* and the third is the *Gain, Loss, or Action, &c.* So let these three Terms be denoted by the Capitals P, T, G. Also there are *three other Terms* (similar to the three former) which make the Question to be resolv'd; and let these be represented by the small Letters p, t, g. Two of which are always given, and the other is sought. But since  $P : G :: p : \frac{Gp}{P}$ ; and again, since  $T : \frac{Gp}{P} :: t : g$ ; therefore  $Tg = \frac{tGp}{P}$ , and consequently  $PTg = tpG$ ; from which general Theorem we can easily find p, t, or g. Thus, I.  $\frac{PTg}{Gt} = p$ ; and II.  $\frac{PTg}{Gp} = t$ ; and III.  $\frac{tpG}{Pt} = g$ . The Contrivance of these excellent Theorems we owe to the late Mr. Ward, of Chester.

5. I shall exemplify Questions in this Rule by Examples, as follow.

Example I. If 100 l. in 12 Months gain 6 l. what will 350 l. gain in 9 Months?

Here  $P=100$ ,  $T=12$ ,  $G=6$ ; also  $p=350$ ,  $t=9$ , to find g.

Add {	the Logarithm of . . . . .	$G=6=0.778151$
	the Logarithm of . . . . .	$p=350=2.544068$
	the Logarithm of . . . . .	$t=9=0.954242$

The Sum is the Logarithm of . . . .  $Gpt=4.276461$

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Add { the Logarithm of . . . . .  $P=100=2.000000$   
 { the Logarithm of . . . . .  $T=12=1.079181$

The Sum is the Logarithm of . . . . .  $PT=3.079181$

Then from the Logarithm of . . . . .  $Gp=4.276461$

Subduct the Logarithm of . . . . .  $PT=3.079181$

There rem. the Log. of  $\frac{Gp}{PT}=g=15.75=1.197280$

Wherefore the Answer is  $15.75^l.=15^l. 15s.$

6. Example II. If 36 Bushels will serve 24 Horses  
 48 Days, how long will 126 Bushels serve  
 96 Horses?

Here  $P=24$ ,  $T=48$ ,  $G=36$ ; also  $p=96$ , and  
 $g=126$ ; to find  $t$ .

Add { the Logarithm of  $P=24=1.380211$   
 { the Logarithm of  $T=48=1.681241$   
 { the Logarithm of  $g=126=2.100370$

The Sum is the Log. of . .  $PTg=5.161822$  from

Add { the Logarithm of  $G=36=1.556302$   
 { the Logarithm of  $p=96=1.982271$

The Sum is the Log. of . . . .  $Gp=3.538573$  subf.

Therefore the Log. of  $\frac{PTg}{Gp}=t=42=1.623249$

The Answer, viz. 42 Weeks.

7. Example III. At the Rate of  $4\frac{1}{2}^l.$  per Cent. per  
*Ann.* what Principal will produce  $35^l. 15s.$   
 in 7,5 Months?

Here  $P=100$ ,  $T=12$ ,  $G=4.5$ ; also  $t=7.5$ , and  
 $g=35.75$ ; to find  $p$ .

Add



Add { the Logarithm of  $P=100=2.000000$   
the Logarithm of  $T=12=1.079181$   
the Logarithm of  $g=35.75=1.553276$

The Sum is the Log. of . . .  $PTg=4.632457$  from

Add { the Logarithm of  $G=4.5=0.653212$   
the Logarithm of  $t=7.5=0.875061$

The Sum is the Log. of . . . .  $Gt=1.528273$  subf.)

Whence the Log. of  $\frac{PTg}{Gt}=p=1271,114=3.104184$ .

Consequently,  $1271,114^l.=1271^l. 2s. 3d.^l$  is the Answer.

*Note, These Theorems give the Answer absolutely, without regarding whether the Proportion be Direct or Inverse, or both together, as in Art. 6. Exam. II.*

### 8. Of the Method of finding Mean Proportionals.

In order that a clear Notion of this most useful Problem may be had, I shall premise the following things.

1. Between two square Numbers  $AA$  and  $BB$ , there will fall but one Mean Proportional  $AB$ ; that is,  $A^2 : AB :: AB : B^2$ . See *Eucl.* 8. 11.

2. Between two Cubic Numbers  $A^3$  and  $B^3$ , there will fall two Mean Proportionals  $A^2B$  and  $AB^2$ ; that is,  $A^3 : A^2B :: AB^2 : B^3$ . See *Eucl.* 8. 12.

3. Between two Biquadrate Numbers  $A^4$  and  $B^4$ , there will fall three Mean Proportionals  $A^3B$ ,  $A^2B^2$ , and  $AB^3$ ; that is,  $A^4 : A^3B :: A^3B : A^2B^2 :: A^2B^2 : AB^3 :: AB^3 : B^4$ .

4. Again, between two sursolid Numbers  $A^5$  and  $B^5$ , there will fall four Means, viz.  $A^4B$ ,  $A^3B^2$ ,  $A^2B^3$ , and  $AB^4$ ; that is,  $A^5, A^4B, A^3B^2, A^2B^3, AB^4, B^5$ , will be Proportionals. And thus the

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*Number of Mean Proportionals* will be always less by one than the *Index* of the Power of the given Extremes.

9. But the *common Ratio* of all such Series is  $\frac{B}{A}$

For  $AA \times \frac{B}{A} = AB$ ; and  $AB \times \frac{B}{A} = BB$ .

And  $A^3 \times \frac{B}{A} = A^2B$ ; and  $A^2B \times \frac{B}{A} = AB^2$ ; and  $AB^2 \times \frac{B}{A} = B^3$ , and so in the others. Here I have

supposed *A* to be the least Number, and *B* the greatest, and the Series to begin from  $A^2, A^3, \&c.$  But if *B* be less than *A*, and the Series begin from  $B^2, B^3, \&c.$  then the *Ratio* of the Series will be  $\frac{A}{B}$ .

Now  $\frac{B}{A}$  is Root of  $\frac{B^2}{A^2}, \frac{B^3}{A^3}, \frac{B^4}{A^4}, \&c.$  and therefore from these Premises well understood, 'tis easy to conceive the Reason of the following Rule for finding Mean Proportionals, *viz.*

Rule { Subtract the Logarithm of the *least Term* from the Logarithm of the *greatest*, and divide the Remainder by a Number greater by one than the Number of Means desired; then add the Quotient to the Logarithm of the least Term (or subtract it from the Logarithm of the greatest) continually, and it will give the Logarithms of all the *Mean Proportionals* desired.

10. Example I. Let two Mean Proportionals be sought between 8 ( $=A^3$ ) and 28 ( $=B^3$ .)

The

The Logarithm of . . . . .  $B^3=28=1.447158$

The Logarithm of . . . . .  $A^3=8=0.903090$

The Difference is . . . . .  $\frac{B^3}{A^3} = 0.544068$

which divide by . . . . .  $3$

The Quotient is . . . . .  $\frac{B}{A} = 0.181356$

To which add the Log. of . . .  $A^3=8=0.903090$

The Sum is the Log. of }  $A^2B=12,14=1.084446$   
1st Mean

To which add again . . . . .  $\frac{B}{A} = 0.181356$

The Log. of the 2d Mean  $AB^2=18,44=1.265802$

Wherefore the Proportional- }  $8 : 12,14 :: 18.44 : 28$   
tionals are }  $A^3 : A^2B :: AB^2 : B^3$

11. Example II. Between 16 and 64 find five Mean Proportionals.

The Logarithm of . . . . .  $B^6=64=1.806180$

Subduct the Logarithm of . .  $A^6=16=1.204120$

There remains . . . . .  $\frac{B^6}{A^6} = 0.602060$

$\frac{1}{6}$  of which is . . . . .  $\frac{B}{A} = 0.100343$

To which add the Log. of . . . .  $A^6=16=1.204120$

The Sum is the Log. of }  $A^5B=20.158=1.304463$   
the 1st Mean

To which add again . . . . .  $\frac{B}{A} = 0.100343$

The Log. of the 2d Mean  $A^4B^2=25.398=1.404806$

And



And thus you produce the Logarithms of

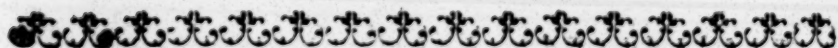
$$\begin{cases} 3^{\text{d}} \text{ Mean} & \dots\dots\dots A^3B^3=32=1.505150 \\ 4^{\text{th}} \text{ Mean} & \dots\dots\dots A^4B^4=40.317=1.605493 \\ 5^{\text{th}} \text{ Mean} & \dots\dots\dots AB^5=50.796=1.705836 \end{cases}$$

The Series therefore is this,

$$\begin{cases} 16. & 20,158. & 25,398. & 32. & 40,317. & 50,796. & 64. \\ A^6. & A^5B. & A^4B^2. & A^3B^3. & A^2B^4. & AB^5. & B^6. \end{cases}$$

12. If it were required to find 364 *Mean Proportionals* between 0 and 1.06; or 0 and 1.05; or 0 and 1.04, &c. then  $A^{365}=0$ , and  $B^{365}=1.06$ ; 1.05, &c. and so  $\frac{B}{A} = B = \sqrt[365]{1.06}$ , or  $\sqrt[365]{1.05}$ , &c.

Wherefore if you multiply the Logarithm of B, by 2, 3, 4, 5, &c. to 365, you will thereby obtain the *Mean Proportionals* required. And these will be the several *Amounts* of 1*l.* and its *Interest* for each *Day* of the Year, at the Rates of 6, 5, 4*l.* &c. *per Cent. per Annum, Compound Interest.* But more will be said of this hereafter.



## CH A P. V.

### SIMPLE INTEREST by LOGARITHMS.

1. **M**Y Design being only to acquaint the Reader with the *Theory* and *practical Uses* of *Logarithms*, and not to treat of the Theory of any other Art or Branch of Mathematical Science; it will be sufficient for me barely to mention the *Theorems* or *Rules*, on which the divers Parts of Learning (I shall treat of) depend, and shew how they are most conveniently wrote by *Logarithms*.

Of

Of SIMPLE INTEREST.

2. I have more than once serv'd myself with those excellent Theorems of Interest contrived by the late ingenious Mr. *Ward*; and shall once again make them subservient to my Design in this Place. In order to which

Let  $\begin{cases} P = \text{any Principal or Sum put to Interest.} \\ R = \text{the Ratio of the Rate, per Cent. per Ann.} \\ T = \text{the Time the Principal continues at Interest.} \\ A = \text{the Amount of the Principal and Interest.} \end{cases}$

3. Then any three of these being given, the other may be found by the following Theorems.

Theor. I.  $TRP + P = A$ . Theor. II.  $\frac{A}{TR+1} = P$ .

Theor. III.  $\frac{A-P}{TP} = R$ . Theor. IV.  $\frac{A-P}{RP} = T$ .

Quest. I. What will 275*l.* 15*s.* amount to in 3½ Years, at 4½*l.* per Cent. per Annum?

Here  $P=275,75$ ,  $T=3,5$ ,  $R=0.045$ , to find  $A$ .  
Theor. I.

Add  $\begin{cases} \text{the Logarithm of } \dots P=275,75=2.440515 \\ \text{the Logarithm of } \dots T=3,5=0.544068 \\ \text{the Logarithm of } \dots R=0.045=.8.653212 \end{cases}$

The Sum is the Log. of  $PTR=43.431=1.637795$

To which add  $\dots P=275,75$

The Sum is }  $PTR + P = A = 319,181 = 319*l.* 3*s.* 7*d.*$   
the Amount }

Quest. II. What Principal or Sum being put to Interest, will amount to, or raise a Stock of 319*l.* 3*s.* 7*d.* in 3½ Years, at the rate of 4½*l.* per Cent. per Annum?

S

The

The Log. of the *Amount*  $A = l. 319,181 = 2.504036$

Add { the Log. of the Time . . .  $T = 3,5 = 0.544068$   
 { the Log. of the Rate  $R = 0.045 = 8.653212$

The Sum is the Log. of . .  $TR = 0.1575 = 9.197280$

Then the Log. of . . .  $TR + 1 = 1.1575 = 0.063521$   
 Subtracted from the Log. }  $P = 275,75 = 2.440515$   
 of the Amount leaves

Wherefore the *Principal* }  $l. 275.75 = 275 l. 15 s.$   
 required is

Quest. III. At what Rate *per Cent.* &c. will  $275 l. 15 s.$   
 amount to  $319 l. 3 s. 7 d.$  in  $3\frac{1}{2}$  Years?

Here  $A - P = 319,181 - 275,75 = 43.431$ .

The Logarithm . . .  $A - P = 43.431 = 1.637800$

Add { the Logarithm of . . .  $P = 275,75 = 2.440515$   
 { the Logarithm of . . .  $T = 3,5 = 0.544068$

The Sum is the Logarithm of  $PT$  . . .  $= 2.984583$

Which subf. from the Log. }  $R = 0,045 = 8.653218$   
 $A - P$ , there will remain

Quest. IV. In what Time will  $275 l. 15 s.$  raise a  
 Stock of  $319 l. 3 s. 7 d.$  at the Rate of  $4\frac{1}{2} l.$   
*per Cent. per Annum*?

Add { the Log. of the Prin.  $P = 275,75 = 2.440515$   
 { the Log. of the Ratio  $R = 0.045 = 8.653212$

The Sum is the Log. of  $PR = 1.093727$

Which subtract from }  $A - P = 43,431 = 1.637800$   
 the Log. of

There



There will remain the }  $\frac{A-P}{RP} = T = 3,5 = 0.544073$   
 Log. of }  
 Therefore the Answer is  $3\frac{1}{2}$  Years.

4. Of ANNUITIES, &c. in ARREARS.

Put {  $U =$  the Annuity, Pension, or yearly Rent.  
 $T =$  the Time of its Continuance unpaid.  
 $R =$  the Ratio of the Rate of Interest.  
 $A =$  the Amount of the Annuity and its Interest.  
 $\frac{2}{R} - 1 = x.$

Then the Theorems for finding each of those Particulars, are as follows.

Theor. I.  $\frac{TTU - TU}{2} \times R : + TU = A.$

Theor. II.  $\frac{2A}{TTR - TR + 2T} = U.$

Theor. III.  $\frac{2A - 2TU}{TTU - TU} = R.$

Theor. IV.  $\sqrt{\frac{2A}{RU}} + \frac{xx}{4} : - \frac{1}{2}x = T.$

Question I. If 250*l.* yearly Rent (Pension, &c.) be forborn or unpaid 7 Years, what will it amount to in that time at 6*l.* per Cent. for each Payment as it becomes due?

Here  $U = 250$ ,  $R = 0.06$ ,  $T = 7$ ; to find  $A$ , per

Theor. I.

Add { the Logarithm of . . . .  $U = 250 = 2.397940$   
 { the Logarithm of . . . .  $T = 7 = 0.845098$

The Sum is the Log. of . .  $TU = 1750 = 3.243030$   
 add the Logarithm of . . .  $T = 7 = 0.845098$

The Sum is the Log. of  $TTU = 12250 = 4.088128$

Then the Log. of  $\frac{TTU-TU}{2} = 5250 = 3.720159$   
 to which add the Log. of  $R = 0.06 = .8.778151$

The Sum is the Log. of  $\frac{TTU-TU}{2} \times R = 315 = 2.498310$

To which add . . . . .  $TU = 1750$

The Sum is  $\frac{TTU-TU}{2} \times R : + TU = A =$  } . . . . . 2065*l.* the Anf.

*Note*, if these Payments be made

$\left\{ \begin{array}{l} \text{Quarterly,} \\ \text{Half-yearly,} \\ \text{Three-quarterly,} \end{array} \right\}$  then use  $\left\{ \begin{array}{l} \frac{1}{4}R, \frac{1}{4}U, \text{ and } 4T. \\ \frac{1}{2}R, \frac{1}{2}U, \text{ and } 2T. \\ \frac{3}{4}R, \frac{3}{4}U, \text{ and } \frac{4}{3}T. \end{array} \right.$

Quest. II. What *Year-Rent, Pension, &c.* being forborn or unpaid seven Years, will raise a Stock of 2065*l.* allowing 6 *per Cent. per Annum*, for each Payment as it becomes due ?

Here  $A = 2065$ ,  $T = 7$ ,  $R = 0.06$ ; to find  $U$ , *per Theor. II.*

Add  $\left\{ \begin{array}{l} \text{the Logarithm of } \dots T = 7 = 0.845098 \\ \text{the Logarithm of } \dots R = 0.06 = .8.778151 \end{array} \right.$

The Sum is the Log. of . . .  $TR = 0.42 = .9.623249$   
 to which add the Log. of  $T = 7 = 0.845098$

The Sum is the Log. of . .  $TTR = 2.94 = 0.468337$   
 From which subduct . . .  $TR = 0.42$

There remains . .  $TTR - TR = 2.52$   
 to which add . . . . .  $2T = 14$

the Sum is  $TTR - TR + 2T = 16.52 = 1.218010$   
 the Logarithm of . . . . .  $2A = 4130 = 3.615950$

The

The Difference of these Logarithms is  $U=250l. = 2.397940$ , which is the Annuity sought.

Quest. III. If 250*l.* Yearly Rent, &c. being forborn 7 years, will amount to 2065*l.* allowing simple Interest for each Payment as it becomes due, what must the Rate of Interest be *per Cent. per Annum*?

Add { the Logarithm of . . . .  $U=250=2.397940$   
 { the Logarithm of . . . .  $T=7=0.845098$

The Sum is the Log. of . . .  $UT=1750=3.243030$   
 to which add the Log. of . . .  $T=7=0.845098$

The Sum is the Log. of  $TTU=12250=4.088128$   
 from which subduct . . . .  $TU=1750$

there remains  $TTU-TU=10500=4.021189$   
 the Logarithm of  $2A-2TU=630=2.799340$

Subduct the former from the }  
 latter, there will remain }  $R=0.06=.8.778151$

Wherefore, as 1*l.* : 0.06*l.* :: 100*l.* : 6*l.* the Rate required.

Quest. IV. In what time will 250*l.* Yearly-Rent, raise a Stock of 2065*l.* allowing 6*l. per Cent.* &c. for the Forbearance of each Payment as it becomes due?

Here  $U=250$ ,  $A=2065$ ,  $R=0.06$ ,  $\frac{2}{R} - 1 = \frac{2}{0.06}$   
 $-1=32,3=x$ ; to find  $T$ , *per* Theor. IV.

Add { the Logarithm of . . . .  $U=250=2.397940$   
 { the Logarithm of . . . .  $R=0.06=.8.778151$

The



The Sum is the Logarithm of  $RU=15=1.176091$   
 which subf. from the Log. of  $2A=4130=3.615950$

there remains . . . . .  $\frac{2A}{RU} = 275,3 = 2.439859$

Add { the Logarithm of . . . . .  $x=32,3=1.509658$   
 { the Logarithm of . . . . .  $\frac{1}{4}x=8.083=0.907594$

The Sum is the Log. of  $\frac{xx}{4} = 261,3605 = 2.417252$

to which add . . . . .  $\frac{2A}{RU} = 275,3333$

the Sum is . .  $\frac{2A}{RU} + \frac{xx}{4} = 536,6938 = 2.729724$

Half that Log. }  $\sqrt{\frac{2A}{RU} + \frac{xx}{4}} = 23,1666 = 1.364862$   
 is the Log. }

from which deduct . . . . .  $\frac{1}{2}x = 16,1666$

there will remain  $\sqrt{\frac{2A}{RU} + \frac{xx}{4}} : - \frac{1}{2}x = T = 7$ , the  
 Number of Years required.

### 5. Of the PRESENT WORTH of ANNUITIES, PENSIONS, &c.

Here U, P, R, T, are used to denote the *Annuity*,  
*present Worth*, *Ratio of the Rate of Interest*, and  
*Time*, as in the former Articles ; here also let  $\frac{2}{R} -$   
 $\frac{2P}{U} - 1 = x$  ; then the Theorems for Operation are  
 as follows ;

Theor. I.  $\frac{TTR - TR + 2T}{2TR + 2} \times U = P = \text{Present Worth.}$

Theor. II.  $\frac{TR + 1}{TTR - TR + 2T} \times 2P = U, = \text{Annuity, \&c.}$

Theor.

Theor. III.  $\frac{2P-2TU}{TTU-TU-2PT} = R =$ , Ratio of the Rate.

Theor. IV.  $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} \pm \frac{1}{2}x = T$ , = the Time.

Quest. I. What is the *present Worth* of 75 *l.* Yearly Rent, to continue 9 Years, at 6 *per Cent.* &c.

Here  $U=75$ ,  $T=9$ ,  $R=0.06$ , to find  $P$ , *per* Theor. I.

Add { the Logarithm of . . . . .  $T=9=0.954242$   
the Logarithm of . . . .  $R=0.06=.8.778151$

The Sum is the Log. of . . . .  $TR=,54=.9.732393$

Also the Logarithm of . .  $TTR=4,86=0.686635$   
subtract . . . . .  $TR=,54$

there remains . . . .  $TTR-TR=4.32$   
to which add . . . . .  $2T=18.$

The Sum is  $TTR-TR+2T=22,32=1.348694$   
the Logarithm of . . .  $2TR+2=3.08=0.488551$

the Difference is . . . . .  $=0.860143$   
To which add the Log. of . . .  $U=75=1.875061$

The Sum is the Logarithm of

$\frac{TTR-TR+2T}{2TR+2} \times U = P = 543,506 = 2.735204$

Wherefore the present Worth is 543 *l.* 10 *s.* 1 *d.*  $\frac{1}{2}$ .

Quest. II. What Annuity, Pension, &c. may be purchased for 543 *l.* 10 *s.* 1 *d.*  $\frac{1}{2}$ , to continue 9 Years, allowing to the Purchaser 6 *per Cent.* *per An.* simple Interest?

Here  $P=543,506$ ;  $T=9$ ;  $R=0,06$ ; to find  $U$ , *per* Theor. II.

From

From the Log. of . . . .  $TR+1=1,54=0.187521$   
 subduct the }  
 Log. of }  $TTR-TR+2T=22.32=1.348694$

to the Difference . . . . .  $.8.838827$   
 add the Logarithm of  $2P=1087,012=3.036234$

The Sum is the Logarithm of

$$\frac{TR+1}{TTR-TR+2T} \times 2P = U = 75 = 1.875061$$

That is, 75*l. per Annum*, is the Annuity, &c. sought.

Quest. III. If 543*l. 10s. 1d.* ready Money, will purchase an Annuity, Lease, &c. of 75*l. per Annum*, to continue 9 Years; *Quære* the Rate of Interest *per Cent.* &c.?

Here  $P=543,506$ ;  $U=75$ ;  $T=9$ ; to find  $R$ , *per Theor. III.*

From the Log. of  $2P-2TU=262,988=2.419936$   
 subd. the }  
 Log. of }  $TTU-TU-2PT=4383,108=3.641782$

the Difference is the Log. of  $R=0.06=.8.778154$

Wherefore, as 1*l.* : 0,06*l.* :: 100*l.* : 6*l.* the Rate required.

Quest. IV. If for 543*l. 10s. 1d.* I purchase an Annuity, Pension, &c. of 75*l. per Annum*; *Quære*, how long I may enjoy it, at the Rate of 6 *per Cent.* &c. Interest?

Here  $P=543,506$ ,  $U=75$ ,  $R=0,06$ ; to find  $T$ , *per Theor. IV.*

From the Log. of . . .  $2P=1087,012=3.036234$   
 subduct the Log. of . . . . .  $U=75=1.875061$

the Diff. is the Log. of . .  $\frac{2P}{U}=14,493=1.161173$

Which



which subtract from  $\dots \frac{2}{R} - 1 = 32,338$

there remains  $\frac{2}{R} - \frac{2P}{U} - 1 = x = 17,840 = 1.251395$

add the Log. of  $\dots \frac{1}{4}x = 4.46 = 0.649335$

the Sum is the Log. of  $\dots \frac{1}{4}xx = 79,5664 = 1.900730$

to which add  $\dots \frac{2P}{RU} = 241,558$

the Sum is  $\frac{2P}{RU} + \frac{xx}{4} = 321,1244 = 2.506659$

half which is }  $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} = 17,919 = 1.253329$   
 the Log. of }

subtract  $\dots \frac{1}{2}x = 8,92$

there remains  $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} - \frac{1}{2}x = T = 8,999 = 9,$   
 the Years fought.

6. If the Question be of Annuities, &c. in Reversion, you must find the *Amount* of the *Purchase-Money* to the time of Commencement (together with its Interest) by Quest. I. Art. 3. and make that *Amount* the Sum for the Purchase; and then proceed as in the Questions of this last Article. These are all the *fundamental* or *original* Cases of *simple Interest*.



## C H A P. VI.

## COMPOUND INTEREST by LOGARITHMS.

1. **T**HE former Questions of *simple Interest* might be resolved by the Rules of *vulgar Arithmetic*; and I have only there shewn, they may also be (and that in many Cases, *most conveniently*) wrought by *Logarithms*. But in the present Affair of *compound Interest*, the Use of *Logarithms* is *absolutely necessary*; no other Method of the Solution of Questions in compound Interest being equal to it in Extent and Perfection. And consequently the young Student in Arithmetic is under an indispensable Obligation to be acquainted with this most excellent and useful Branch of the Science.

2. I shall here also proceed according to the Theorems of Mr. *Ward*; and therefore

Put  $\left\{ \begin{array}{l} P = \text{the Principal put to Interest.} \\ t = \text{the Time of its Continuance.} \\ A = \text{the Amount of the Principal and Interest.} \\ R = \text{the Amount of 1 l. and its Interest for one Year, at any given Rate.} \end{array} \right.$

Note, you find  $\left\{ \begin{array}{l} 100 : 106 :: 1 : 1,06 = R, \text{ at 6 per Ct.} \\ R \text{ thus, } 100 : 105 :: 1 : 1,05 = R, \text{ at 5 per Ct.} \end{array} \right.$

The Amounts of 1 l.  $\left\{ \begin{array}{l} 1. \ 2. \ 3. \ 4. \ 5, \ \&c. \text{ Years.} \\ \text{in several Years } \{ R, R^2, R^3, R^4, R^5, \ \&c. \text{ Am. of 1 l.} \end{array} \right.$

Therefore  $R^t =$  the Amount of 1 l. in the Time  $t$  for the Rate agreed on; this being premised, the Theorems for *Compound Interest* are as follow.

3. Theor. I.  $PR^t = A =$  the Amount.

Theor.

Theor. II.  $\frac{A}{R^t} = P$  = the Principal.

Theor. III.  $\frac{A}{P} = R^t \begin{cases} = \text{the Time.} \\ = \text{the Amount of 1 l.} \end{cases}$

By these Theorems the several Questions of Compound Interest are answered most expeditiously by the *Logarithms* in the Manner following.

4. Quest. I. What will 275*l.* 15*s.* amount to in  $3\frac{1}{2}$  Years, at  $4\frac{1}{2}$  *l. per Cent. per Annum*, Compound Interest?

Here  $P=275,75$  ;  $R=1,045$  ;  $t=3,5$  ; to find  $A$ ,  
per Theor. I.

The Logarithm of . . . . .  $R=1,045=0.019116$   
multiply by the Time . . .  $t=3,5$

The Prod. is the Log. of  $R^t=1,1665=0.066906$   
To which add the Log. of  $P=275,75=2.440515$

The Sum is the Log. of  $PR^t=A=321,68=2.507421$

So the Amount sought is 321*l.* 13*s.* 7*d.* which is more than the Amount by simple Interest by 2*l.* 10*s.*  
See Quest. I. Art. 3. of the foregoing Chapter.

Quest. II. What Principal or Sum being put to Use at  $4\frac{1}{2}$  *l. per Cent.* Compound Interest, will amount to 321*l.* 13*s.* 7*d.* in  $3\frac{1}{2}$  Years?

Here  $A=321,68$  ;  $R=1,045$  ;  $t=3,5$  ; to find  $P$ ,  
per Theor. II.

From the Logarithm of  $A=321,68=2.507421$   
Subduct the Logarithm of  $R^t=1.1665=0.066906$

The Diff. is the Log. of  $\frac{A}{R^t}=P=275,75=2.440515$   
therefore the *Principal* sought is 275*l.* 15*s.*



Quest. III. At what rate *per Cent.* &c. Compound Interest, will 275*l.* 15*s.* raise a Stock, or amount to 321*l.* 13*s.* 7*d.* in 3½ Years?

Here  $A=321,68$  ;  $P=275,75$  ;  $t=3,5$  ; to find  $R$ ,  
*per Theor. III.*

From the Logarithm of  $A=321,68=2.507421$   
Subtract the Log. of . . .  $P=275,75=2.440515$

The Difference is the Log. of  $R^t$  0.066906  
which divide by the Time . . .  $t=3,5$

The Product is the Log. of  $R=1.045=0.019116$

Then as  $1 : 0,045 :: 100 : 4,5=4\frac{1}{2}$  *l. per Cent.* the  
Rate required.

Quest. IV. In what Time will 275*l.* 15*s.* raise a Stock of 321*l.* 13*s.* 7*d.* at the rate of 4½*l. per Cent.* Compound Interest?

Here  $P=275,75$  ;  $A=321,68$  ;  $R=1.045$  ; to find  
 $t$ , *per Theor. IV.*

From the Logarithm of . .  $A=321,68=2.507421$   
Subduct the Logarithm of  $P=275,75=2.440515$

The Diff. is the Log. of . . . .  $R^t=1,045^t=0.66906$

Then the Log. of  $1.045=0.019116$  )  $0.66906(3,5=t.$   
57348

95580

95580

.....

Thus the Answer is 3 Years and 6 Months.

*Note,* As the *Amount* of any *Sum* and its *Interest* is greater at *Compound Interest* than at *Simple Interest* for any *Time* above a *Year*; so it is *less* at *Compound*

pound than *Simple Interest* for any *Time less* than a Year, as the Learner may easily prove by the Theorems beforegoing.

5. Of ANNUITIES, &c. in ARREARS.

Theor. I.  $\frac{UR^t - U}{R - 1} = A = \text{the Amount of any Sum.}$

Theor. II.  $\frac{RA - A}{R^t - 1} = U = \text{the Annuity, Pension, \&c.}$

Theor. III.  $\frac{RA + U - A}{U} = R^t = \text{the Time.}$

Theor. IV.  $\frac{A - U}{U} = \frac{A}{U} R - R^t = \text{the Amount of 1 l.}$

6. Quest. I. If 30 l. yearly Rent be forborn 9 Years, what will it amount to at 5 l. per Cent. &c. Compound Interest?

Here  $U=30$ ,  $t=9$ ,  $R=1,05$ ; to find  $A$ ; per Theor. I.

The Logarithm of . . . .  $R=1,05=0.021189$   
multiply by the Index . . . 9

The Product is . . . . .  $R^t=1.516=0.180701$   
To which add the Log. of . . .  $U=30=1.477121$

The Sum is the Log. of  $UR^t=45,48=1.657822$

Subduct . . . .  $U=30$

There remains . . .  $UR^t - U=15,48=1.189771$   
Subtract the Log. of  $R-1=0.05=.8698970$

The Diff. is the }  $\frac{UR^t - U}{R - 1} = A = 309,6=2.490801$   
Log. of }

The Amount therefore is 309 l. 12 s. for Answer.

Quest.

Quest. II. What Annuity, &c. will raise a Stock (or amount to) 309*l.* 12*s.* being forborn or unpaid 9 Years, at 5*l.* per Cent. &c. Compound Interest?

Here  $A=309,6$ ;  $t=9$ ;  $R=1,05$ ; to find  $U$ , per Theor. II.

Add  $\left\{ \begin{array}{l} \text{the Logarithm of } \dots A=309,6=2.490801 \\ \text{the Logarithm of } \dots R=1,05=0.021189 \end{array} \right.$

The Sum is the Log. of  $\dots AR=325,08=2.511990$

Therefore the Log. of  $AR-A=15,48=1.189771$

Subduct the Log. of  $\dots R-1=0.516=0.971265$

The Diff. is the  $\left. \begin{array}{l} \text{Log. of} \end{array} \right\} \dots \frac{AR-A}{R-1} = U = 30 = 1.477121$

That is 30*l.* per Annum is the Annuity sought.

Quest. III. In what time will 30*l.* per Annum amount to 309*l.* 12*s.* at 5*l.* per Cent. Compound Interest?

Here  $U=30$ ;  $A=309,6$ ;  $R=1,05$ ; to find  $t$ , per Theor. III.

Add  $\left\{ \begin{array}{l} \text{the Logarithm of } \dots A=309,6=2.490801 \\ \text{the Logarithm of } \dots R=1,05=0.021189 \end{array} \right.$

The Sum is the Log. of  $AR=325,08=2.511990$

Subtract  $\dots A-U=279,6$

There remains  $AR+U-A=45,48=1.657822$

Subtract the Log. of  $U=30=1.477121$

The Diff. is the Log. of  $\dots R-1=0.516=0.971265$

Then the Log. of  $1,050.02=1.189771$  (9= $t$ ).  
180701

Answer, 9 Years.

Quest.



Quest. IV. At what Rate *per Cent.* Compound Interest will 30*l.* Yearly Rent amount to 309*l.* 12*s.* in 9 Years?

Here  $U=30$ ;  $A=309,6$ ;  $t=9$ ; to find  $R$ , *per* Theor. IV.

From the Log. of . . .  $A-U=279,6=2.446537$   
Subduct the Logarithm of . . .  $U=30=1.477121$

The Diff. is the Log. of  $\frac{A-U}{U}=9,32=0.969416$

Again from the Log. of . . .  $A=309,6=2.490801$   
Subtract the Logarithm of . . .  $U=30=1.477121$

The Diff. is the Log. of  $\frac{A}{U}=10,32=1.013680$

Therefore the said Theorem IV. is reduced to this Equation, *viz.*  $9.32=10,32R-R^9$ . Now this Equation may be easily resolv'd by a *converging Series*; or still easier by the Tables of the Amount of 1*l.* for several Years successively; for therein  $R^9$ , or the Amount of 1*l.* in 9 Years may be tried for the several Rates *per Cent.* in the Tables. For instance, suppose I pitch upon the Table at 5*l. per Cent.* Compound Interest, then I find  $R^9=1.551328$ , &c. Wherefore  $9.32+1.551328=10.871328=10.32R$ ; therefore  $10.32)10.871328(=1,05$ . Now since 1*l.*:0,05*l.* :: 100*l.*:5*l.* the very Rate *per Cent.* assumed in the Tables, it will follow, that is the true Rate sought.

*Note,* This Question may be resolved by the Rule of *false Position*; which I leave to the Learner's Exercise.

7. Of the PRESENT WORTH of ANNUITIES,  
&c. at Compound Interest.

Here P denotes the *present Worth* of any Annuity, Lease, &c. and the rest of the Letters as before, Art. 6. Then the Theorems are as follows.

Theor. I.  $\frac{U - \frac{U}{R^t}}{R - 1} = P = \text{the present Worth.}$

Theor. II.  $\frac{PRR^t - PR}{R^t - 1} = U = \text{the Annuity, \&c.}$

Theor. III.  $\frac{U}{P - U - PR} = R^t = \text{the Time.}$

Theor. IV.  $\frac{U}{P} = \frac{U}{P} R^t + R^t - R^t + 1 = \text{the Value of } R.$

Quest. I. What is 30*l.* per Annum to continue 9 Years, worth in ready Money, abating 5*l.* per Cent. &c. Compound Interest to the Purchaser?

Here  $U=30$ ,  $t=9$ ;  $R=1,05$ ; to find P, per Theor. I.

The Logarithm of . . . . .  $R=1,05=0.021189$   
Multiply by the Time . . . . . 9

The Prod. is the Log. of  $R^9=1.516=0.180701$   
which subtract from the Log. of  $U=30=1.477121$

The Diff. is the Log. of  $\frac{U}{R^t}=19,788=1.296420$   
which take from . . .  $U=30$

there remains . . .  $U - \frac{U}{R^t}=10.212=1.009111$   
Subduct the Log. of . . .  $R-1=0.05=.8.698970$

The Diff. is the Log. of  $P=204,24=2.310141$   
The present Worth is 204*l.* 4*s.* 9*d.*  $\frac{1}{2}$ , the Answer.

Quest.

Quest. II. What Annuity, &c. to continue 9 Years  
may one purchase for 204*l.* 4*s.* 9*d.*  $\frac{1}{2}$ : abating  
5*l.* *per Cent.* &c. to the Purchaser?

Here  $P=204,24$ ;  $t=9$ ;  $R=1,05$ ; to find  $U$ , *per*  
Theor. II.

Add { the Logarithm of . . .  $R'=1,516=0,180701$   
the Logarithm of . . .  $R=1,05=0,021189$   
the Logarithm of . .  $P=204,24=2,310141$

The Sum is the Log. of  $PRR'=325,11=2,512031$   
Then from . . .  $PRR'=325,11$   
Subtract . . . . .  $PR'=309,628$

there remains  $PRR'-PR'=15,482=1,189799$   
Subduct the Log. of  $R'-1=0,516=.9712650$

The Diff. is the Log. of . . .  $U=30=1,477149$   
Therefore the Annuity sought is 30*l.* *per Annum.*

Quest. III. What time may one enjoy an Annuity,  
&c. of 30*l.* *per Annum*, for 204*l.* 4*s.* 9*d.*  $\frac{1}{2}$ .  
ready Money, abating to the Purchaser 5*l.*  
*per Cent.* &c.?

Here  $P=204,24$ ;  $U=30$ ;  $R=1,05$ ; to find  $t$ ,  
*per* Theor III.

Add { the Logarithm of . .  $P=204,24=2,310141$   
the Logarithm of . . .  $R=1,05=0,021189$

The Sum is the Log. of  $PR=214,452=2,331330$

Then . . . . .  $P \pm U=234,24$

The Diff. . . .  $P+U-PR=19,788=1,296402$   
which subtract from the Log. of  $U=30=1,477121$

The Diff. is the Log. of  $R'=1,05'=1,516=0,180719$   
 $U$  Then



Then the Log. of  $1.05 = 0.021189$   $0.180719 (= 9 = t.$   
 $\underline{180701}$

Answer, 9 Years.

.... 18

Quest. IV. Suppose an Annuity,  $\text{£}c.$  of 30*l.* per Annum, to continue 9 Years, be sold for 204*l.* 4*s.* 9*d.*  $\frac{1}{2}$ . ready Money, what rate of Interest hath the Purchaser allow'd per Cent. &c. for his Money?

Here  $P = 204.24$  ;  $U = 30$  ;  $t = 9$  ; to find  $R$ , per Theor. IV.

From the Log. of the Annuity  $U = 30 = 1.477121$   
 Subduct the Log. of . . . .  $P = 204.24 = 2.310141$   
 $\underline{\hspace{1.5cm}}$

The Diff. is the Log. of  $\frac{U}{P} = 0.14679 = .9.166980$

Then the said Theor. IV. is reduced to this Equation, viz.  $0.14679 = 0.14679R^9 + R^9 - R^{10}$  ; which is to be solv'd by the Method of Infinite Series. But for those who understand not that Method, this Question is much better answer'd by the *Rule of False* ; or yet easier by the Tables of Compound Interest. See a compleat Set, with their Uses, in my *System of Decimal Arithmetic*, published for Mr. Noon, at the *White Hart*, in *Cheapside*, London.

#### 8.    *Of Purchasing FREEHOLD ESTATES.*

*Estates in Fee Simple* (which are such as we commonly call *Freehold* or *Real Estates*) being purchased for ever, or without *Reversion* ; 'tis plain, that in the foregoing Theorems for finding the present Worth of Annuities, &c. if those Terms wherein  $t$  (= the *Time*) is found, be made to vanish, as being *Infinite* ; the said Theorems will be reduced

reduced to such as suit the *present Case*, and are as follows.

Theor. I.  $\frac{U}{R-1} = P$ , the *Present Worth*, or *Purchase-Money*.

Theor. II.  $PR - P = U$ , the *Annuity*, or *Estate per Annum*,

Theor. III.  $\frac{P+U}{P} = R$ , the *Amount of 1 l. at the given Rate*.

Question. What must be given for a *Freehold Estate* of 50 l. *per Annum*, allowing the Buyer 6 l. *per Cent.* Compound Interest for his Money?

Here  $U=50$ ;  $R=1,06$ ; to find  $P$ , *per Theor. I.*

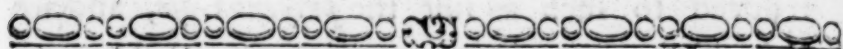
The Logarithm of . . .  $U=50=1.698970$   
 Subduct the Log. of  $R+1=0,06=.8778151$

The Diff. is the Log. of  $\frac{U}{R-1} = P = 833,3 = 2.920819$

The *Purchase-Money*, then, is 833 l. 6 s. 8 d. If it had been at the Rate of 5 l. *per Cent.* Compound Interest, the said Estate would be worth 1000 l. *present Money*; which is equal to twenty times the *Yearly Rent*: and therefore the general Method of buying *Freehold Estates*, is by paying 20 or 25 times the *Yearly Rent*; which is commonly called 20 or 25 Years Purchase.

The other two Theorems are wrought in the same Manner for  $U$  and  $R$ ; which, being so very easy, need no Example.





## C H A P. VII.

## Of VULGAR FRACTIONS by LOGARITHMS.

1. **T**O find the *Logarithm* of a *Vulgar Fraction*, this is the

Rule { From the *Logarithm* of the *Numerator*  
 Subtract the *Logarithm* of the *Denominator*;  
 the Remainder is the *Log.* of the *Fraction*.

Example I. What is the *Log.* of the Fraction  $\frac{51}{73}$ ?

From the *Logarithm* of . . .  $51 = 1.707570$   
 Subtract the *Logarithm* of . . .  $73 = 1.863323$

there remains the *Log.* of  $\frac{51}{73} = 0.69862 = .9.844247$

Exam. II. What is the *Logarithm* of  $\frac{1}{17.56}$ ?

From the *Logarithm* of . . . .  $1 = 0.000000$   
 Subtract the *Logarithm* of . . .  $1756 = 3.244524$

there remains the }  $\therefore \frac{1}{1756} = ,00056947 = .6.755476$   
 Log. of

Exam. III. What is the *Logarithm* of  $\frac{100}{1357}$ ?

From the *Logarithm* of . . . .  $100 = 2.000000$   
 Take the *Logarithm* of . . .  $1357 = 3.132580$

there remains the *Log* of  $\frac{100}{1357} = ,073692 = .8.867420$

Exam. IV. What is the *Logarithm* of  $\frac{5973}{1000000}$ ?

The



The Logarithm of . . . . .  $5973 = 3.776192$

From which take the Log. of  $1000000 = 6.000000$

there rem. the Log. of  $\frac{5973}{1000000} = .005973 = .7.776192$

2. If the Fraction be a *mix'd one*, it must be reduced to an *improper Fraction*; and then proceed with it as before.

Example I. What is the Logarithm of  $13\frac{2}{7}$ ?

Here  $13\frac{2}{7} = \frac{96}{7}$ ;

Therefore from the Logarithm of  $96 = 1.982271$

take the Logarithm of . . . . .  $7 = 0.845098$

there remains the Log. of  $13\frac{2}{7} = 13.714 = 1.137173$

Exam. II. What is the Logarithm of  $193\frac{51}{73}$ ?

Here  $193\frac{51}{73} = \frac{14140}{73}$ ;

Then from the Logarithm of  $14140 = 4.150449$

take the Logarithm of . . . . .  $73 = 1.863323$

there rem. the Log. of  $193\frac{51}{73} = 193.69862 = 2.287126$

3. If the *mix'd Fraction* consists of *large Numbers*, it may be most easily reduc'd by Logarithms, thus; supposing the Example be  $2145\frac{57}{89}$ ;

To the Log. of the *Integral Part*  $2145 = 3.331427$

Add the Log. of the Denominator  $89 = 2.770115$

The Sum is the Log. of . .  $1263405 = 6.101542$   
to which add the Numerator . .  $57$

Then you have  $\left\{ \begin{array}{l} \text{the new Num. } 1263462 \\ \text{the Denomin. } 57 \end{array} \right\}$  the Fract. sought.

4. To multiply *Vulgar Fractions* by Logarithms, add the Logarithms of the Numerators for the Logarithm of a new Numerator; and the Logarithms of the Denominators for the Logarithm of a new Denominator.

Example. What is the Product of  $\frac{35}{173}$  into  $\frac{7}{9}$ ?

Add the Log. of the Numerators {  $35=1.544068$   
 $7=0.845098$

The Log. of the new Numerator  $245=2.389166$

Add the Log. of the Denominat. {  $173=2.238046$   
 $9=0.954242$

The Log. of the new Denomin.  $1557=3.192288$

Therefore  $\frac{35}{173} \times \frac{7}{9} = \frac{245}{1557}$ , the fractional Product required.

5. The Logarithm of the Product of several Fractions multiplied into one another is thus obtain'd; viz. Add the Logarithms of all the Numerators and the Arithmetical Complements of the Logarithms of all the Denominators together; the Sum is the Logarithm required.

Example. What is the Logarithm of  $\frac{87}{97} \times \frac{33}{46} \times \frac{7}{9}$ ?

Add together { the Logarithms of {  $87=1.939519$   
 $33=1.518514$   
 $3=0.477121$   
the Arith. Comp. of the {  $97=.8014228$   
Logarithms of {  $46=.8337242$   
 $7=.9154902$

The Log. of  $\frac{87}{97} \times \frac{33}{46} \times \frac{7}{9} = \frac{8613}{31234} = .27639 = .9441526$

6. To divide Vulgar Fractions by Logarithms, do thus; Add the Logarithm of the *Denominator* of the *Divisor* to the Logarithm of the *Numerator* of the *Dividend*; the Sum is the Logarithm of a *new Numerator*; and the Sum of the Logarithms of the other *two Factors*, is the Logarithm of the new *Denominator* of the Quotient required.

Example. Divide  $\frac{245}{1557}$  by  $\frac{7}{9}$ .

Add the Logarithms of { . . . . .  $9=0.954242$   
 $245=2.389166$

The Log. of the new Numerator  $2205=3.343408$

Add the Logarithms of { . . .  $7=0.845098$   
 $1557=3.192283$

The Log. of the new Denom.  $10899=4.037386$

Therefore  $\frac{7}{9}) \frac{245}{1557} (= \frac{2205}{10899} = \frac{35}{173}$ . See Art. 4.

7. The Logarithm of this Quotient may be found by one Addition, in like manner as directed in Art. 5. thus;

Add together { the Logarithm of {  $9=0.954242$   
 $245=2.389166$   
 { the Arith. Comp. of {  $7=.9.154902$   
 { the Logarithms of {  $1557=.6.807712$

The Log. of . .  $\frac{7}{9}) \frac{245}{1557} (= \frac{35}{173} = .20231 = .9.306022$

8. The *Extraction* of the *Roots* of *Vulgar Fractions* by Logarithms is thus performed. Divide the Logarithm both of the Numerator and Denominator of the given Fraction by the *Index* of the *Root*; the *Quotients* shall be the Logarithms of the *Numerator* and *Denominator* of the *Fractional Root* required.

Examp.



Examp. I. What is the *Square Root* of the Fraction  $\frac{1849}{10201}$ ?

Divide the Logarithm of . . . . . 1849 = 3.266937  
by the Index of the Root . . . . . 2

The Log. is the Num. of the Root 43 = 1.633468

Again, divide the Log. of . . . 10201 = 4.008643  
by . . . . . 2

The Log. of the Denom. of the Root 101 = 2.004321

Therefore  $\sqrt{\frac{1849}{10201}} = \frac{43}{101}$  the Root required.

Example II. What is the *Cube Root* of  $86\frac{291}{343}$ ?

This reduced to an improper Fraction, is  $\frac{29791}{343}$ ;

Therefore the Logarithm of 29791 = 4.474085  
 $\frac{1}{3}$  thereof is the Log. of the *new Num.* 31 = 1.491362

Again, the Logarithm of . . . . . 343 = 2.535294  
 $\frac{1}{3}$  of which is the Log. of the *new Denom.* 7 = 0.845098

Consequently  $\sqrt[3]{86\frac{291}{343}} = \frac{31}{7} = 4\frac{3}{7}$ , the Cube Root sought.

9. To find the *Logarithm* of the *Root* of any Fraction; add the Logarithm of the *Numerator* to the *Arithmetical Complement* of the Logarithm of the *Denominator*, and divide that Sum by the *Index* of the *Root*; the Quotient shall be the Logarithm sought.

Example I. What is the Logarithm of the *Square Root* of the Fraction  $\frac{1849}{10201}$ ?

The

The Log. of the Numerator . . . 1849 = 3.266937  
 Comp. Arith. of the Log. } .. 10201 = 5.991357  
 of the Denom.

The Sum is . . . . . 9.258284  
 which divide by the Index of the Root 2

The Quot. is }  $\sqrt{\frac{1849}{10201}} = \frac{47}{101} = .42574 = .9.629147$   
 the Log. of

Exam. II. What is the Logarithm of the *Cube Root*  
 of the mix'd Fraction  $86\frac{291}{343}$ , or its equal  
 $\frac{29791}{343}$ ?

Add } the Logarithm of . . . . 29791 = 4.474085  
 the Ar. Com. of the Log. of 343 = 7.464706

The Sum is the Log. of . . . . .  $68\frac{291}{343} = 1.938791$   
 which divide by the *Index* of the Root ., 3

The Quot. is }  $\sqrt[3]{68\frac{291}{343}} = 4\frac{1}{3} = 4.4285 = 0.646263$   
 the Log. of

## C H A P. VIII.

### DUODECIMAL ARITHMETIC *performed by* LOGARITHMS.

i. **S**INCE this kind of Arithmetic is so very *com-*  
*mon*, and yet in the common Way so very *dif-*  
*fcult*; I hope 'twill not be unacceptable to the *young*  
*Artificer* to be convinced with how much more Ease  
 and Pleasure he may compute his Dimensions in this  
 Way by the help of Logarithms. And as it is pro-  
 per to reduce them first to *Decimals*, I have here sub-  
 joined

joined a Table shewing by Inspection the *Decimal Parts* of a Foot (in this Case the *Integer*) answering to any Number of *Primes*, *Seconds*, and *Thirds*; which are the Parts or Divisions of a Foot made use of in this kind of Mensuration.

2.

The Duodecimal Table.

Duodecimals.	Decimal Parts.		
	Primes <sup>i</sup>	Seconds <sup>ii</sup>	Thirds <sup>iii</sup>
1	,083333	,006944	,000578
2	,166666	,013888	,001157
3	,25	,020833	,001736
4	,333333	,027777	,002314
5	,416666	,034722	,002893
6	,5	,041666	,003472
7	,583333	,048611	,004051
8	,666666	,055555	,004629
9	,75	,0625	,005208
10	,833333	,069444	,005787
11	,916666	,076388	,006365

3. The foregoing Table is too easy to need Description, I mean for any Person concern'd in *Duodecimal Mensuration*; and therefore I shall proceed to exemplify and illustrate the Operations this Way by Logarithms in all the Rules of *Multiplication*, *Division*, *Involution*, and *Extraction* of Roots.

4. *Multiplication* of DUODECIMALS by LOGARITHMS.

Examp. I. Suppose a Plane be 9<sup>f</sup> 10<sup>i</sup> in Length, and 8<sup>f</sup> 8<sup>i</sup> in Breadth; *Quere* the Content or Area?

Add { the Log. . . of 9<sup>f</sup> 10<sup>i</sup>. = 9.8333 = 0.992704  
 { the Log. of . . 8<sup>f</sup> 8<sup>i</sup>. = 8.6666 = 0.937849

The Sum is the Log. of . . . 85,222 = 1.930553  
 That is, 85,222 Feet, = 85<sup>f</sup> 2<sup>i</sup> 8<sup>ii</sup>. the Area required.

Examp.



Examp. II. What is the Product of  $40^f. 9'. 10''$  by  $11'. 9''$ ?

$$\text{Add} \begin{cases} \text{the L. of } 40^f. 9'. 10'' = 40.8194 = 1.610866 \\ \text{the Log. of } 11'. 9'' = 0.97916 = .9.990854 \end{cases}$$

The Sum is the Log. of . . . .  $39,969 = 1.601720$

Therefore  $39,969 = 39^f. 11'. 7''. 6'''$ . the *Product*,  
or *Area* required,

Examp. III. What is 175 Feet  $00'. 04''$ . by  $8'''$ ?

$$\text{Add} \begin{cases} \text{the L. of } 175^f. 0'. 4'' = 175.027 = 2.243107 \\ \text{the Log. of } 8''' = 0.04629 = .7.665487 \end{cases}$$

The Sum is the Log. of . . ,  $81111 = .9.908594$

Therefore  $0,81111 = 0^f. 9'. 8''. 9'''$  = the *Area*  
sought.

Examp. IV. What is  $17^f. 9'. 2''. 6'''$  by  $6^f$ ?

$$\text{Add} \begin{cases} \text{the L. of } 17^f. 9'. 2''. 6''' = 17,76736 = 1.247417 \\ \text{the Log. of } 6 = 0.778151 \end{cases}$$

The Sum is the Log. of . .  $106,60416 = 2.025568$

Thus  $106.60416 = 106^f. 7' 3''$ . the *Area* sought.

Examp. V. What Number of *solid Feet* is in a *Cellar*  
 $21^f. 2'$ . long,  $11^f. 10'. 8''$ . broad, and  $7^f. 3'$ .  
deep?

$$\text{Add} \begin{cases} \text{the Log. of } 21^f. 2' = 21,1666 = 1.325659 \\ \text{the Log. of } 11^f. 10'. 8'' = 11,888 = 1.074109 \\ \text{the Log. of } 7^f. 3' = 7,25 = 0.860338 \end{cases}$$

The Sum is the Log. of . . .  $1820,19 = 3.260106$

Therefore  $1820,19 = 1820^f. 2'. 3''. 4'''$ , the *Solidity*  
required.

## 5. DIVISION of DUODECIMALS by LOGARITHMS.

Examp. I. What is  $85^f. 2'. 8''$ . divide by  $8^f. 8'$ ?

$$\begin{array}{r} \text{The Log. of } \dots 85^f. 2'. 8'' = 85,72 = 1.930553 \\ \text{Subduct the Log. of } 8^f. 8' = 8,666 = 0.937849 \\ \hline \end{array}$$

The Diff. is the Log. of  $\dots 9.8333 = 0.992704$

So that  $9.8333 = 9^f. 10'$ , the Answer.

Examp. II. What is  $9'. 8''. 9'''$ . divided by  $8'''$ ?

$$\begin{array}{r} \text{From the Log. of } 9'. 8''. 9''' = 0.81111 = 9.908594 \\ \text{Subtract the Log. of } 8''' = ,004629 = 7.665487 \\ \hline \end{array}$$

The Diff. is the Log. of  $\dots 175,027 = 2.243107$

Therefore the Answer is  $175,027 = 175^f. 0'. 4''$ .

Examp. III. Divide  $39^f. 11'. 7''. 6'''$ . by  $40^f. 9'. 10''$ .

$$\begin{array}{r} \text{From the } \left. \begin{array}{l} \text{Log. of } \dots 39^f. 11'. 7''. 6''' = 39.9687 = 1.601720 \\ \text{Subd. the Log. of } 40^f. 9'. 10'' = 40.8194 = 1.610866 \end{array} \right\} \\ \hline \end{array}$$

The Diff. is the Log.  $\dots$  of  $0,97916 = 9.990854$

But  $,97916 = 11'. 9''$ . the Quotient required.

Examp. IV. Divide  $106^f. 7'. 3''$ . by 6.

$$\begin{array}{r} \text{From the L. of } 106^f. 7'. 3'' = 106,60416 = 2.205568 \\ \text{Subtract the Log. of } 6 = 0.878151 \\ \hline \end{array}$$

The Diff. is the Log. of  $17,76736 = 1.247417$

Thus  $17,76736 = 17^f. 9'. 2''. 6'''$ . the Quotient sought.

6. INVOLUTION of DUODECIMALS by LOGARITHMS.

Examp. I. What is the Area of that *Square* whose Side is  $12^f. 9'. 7''. 10'''$ ?

The Log. of  $12^f. 9'. 7''. 10''' = 12.80439 = 1.107359$   
Multiply by ... 2

The Product is the Log. of  $163.9524 = 2.214718$

Therefore  $163.9524 = 163^f. 11'. 5''. 1'''$ . the *Area* required.

Examp. II. What is the Solidity of a *Cube* whose Side is  $1^f. 2'. 9''. 11'''$ ?

The Log. of  $1^f. 2'. 9''. 11''' = 1.23553 = 0.091854$   
which multiply by ... 3

The Product is the Log. of  $1.8774 = 0.273562$

Therefore  $1.8774 = 1^f. 10'. 6''. 4'''$ . the Solidity sought.

7. EXTRACTION of ROOTS of DUODECIMALS by LOGARITHMS.

Examp. I. What is the Side of that *Square* whose Area is  $163^f. 11'. 5''. 1'''$ ?

The L. of  $163. 11'. 5''. 1''' = 163.9524 = 2.214718$   
Divide by ... 2

The Quotient is the Log. of  $12.80439 = 1.107359$

Thus  $12.80439 = 12^f. 9'. 7''. 10'''$ . the Side sought.

Examp. II. What is the *Side* of that *Cube*, whose Solidity is  $1^f. 10'. 6''. 4'''$ ?

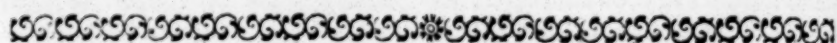
The



The Log. of  $1. 10'. 6''. 4''' = 1.8774 = 0.273562$   
 Divide by . . . . 3

The Quotient is the Log. of  $1.23553 = 0.091854$   
 But  $1.23553 = 1^f. 2'. 9''. 11'''$ . the *Side* of the *Cube*  
 required.

These few Examples abundantly shew with how much more *Ease*, *Brevity* and *Expedition* the Operations of *Duodecimals* are performed by *Logarithms*, than by the *ordinary Method*.



## C H A P. IX.

*The OPERATION of the common RULES of ARITHMETIC by INSTRUMENTS; viz. the LOGARITHMIC SCALE; and GUNTER'S LINE, with the COMPASSES, and on the SLIDING-RULE.*

I. **H**AVING in the *Theory* shewn the *Nature* and *Construction* of the *Logarithmic Scale*, and *Gunter's Line*; I shall here briefly exemplify their *Uses* in the Operation of the common Rules of Arithmetic thereby; and in doing of this I shall observe this Method; first, to give the Operation by *Logarithms* in *Numbers*. Secondly, to perform the same by the *Logarithmic Scale*. Thirdly, to work the same Case on the *Gunter* with the *Compasses*; and fourthly, to do the same thing on the *Sliding-Gunter*. In this Method, the *Analogy* or *Agreement* between the *Numerical* and *Instrumental Operations* will more easily appear; and the *Nature* and *Reason* of the latter be much better understood by young *Learners*.

2. MULTIPLICATION.

Examp. I. Multiply 9 by 7.

First, by Logarithms.

Add {	the Logarithm of . . . . .	9=0.954242
	the Logarithm. of . . . . .	7=0.845098

The Sum is the Log. of the Prod. =63=1.799340

3. Secondly, by the Logarithmic Scale.

(*Note.* If the smallest Divisions in the Line AB represent *Numbers*, the Logarithms begin from C e in the Line A e ; if the middle Divisions in A B be *Numbers*, the Logarithms begin from G ; but if the largest Divisions in AB be taken for *Numbers*, then the Logarithms begin from H, in the said Line A e. And since the *smallest Divisions* are too small, and the *largest Divisions* too large for Examples, we must consequently chuse the *mean Divisions* in A B to represent the *Numbers* 1, 2, 3, 4, &c. or 10, 20, 30, 40, &c. or 100, 200, 300, &c. and so the Logarithms begin from G. Therefore) Set one Foot of the Compasses in G, and extend the other to the Logarithm of the Multiplier 7=ag, which you'll find to be G a=845, and since G b=954 is the Logarithm of the Multiplicand 9=bh ; therefore with the same Extent G a in the Compasses, set one Foot in b, the other will fall on d ; therefore G d is the Logarithm of the Product dm=63 in A B, the Number sought.

4. Thirdly, by the Gunter with Compasses.

Set one Foot of the Compasses in the Beginning of the Line at 1, and extend the other to 7 ; with that Extent in the Compasses set one Foot in 9, the other will fall on 63, the Product required.

*Note.*

The Log. of  $1. 10'. 6''. 4''' = 1.8774 = 0.273562$   
 Divide by . . . . 3

The Quotient is the Log. of  $1.23553 = 0.091854$   
 But  $1.23553 = 1^f. 2'. 9''. 11'''$ . the *Side* of the *Cube*  
 required.

These few Examples abundantly shew with how much more *Ease*, *Brevity* and *Expedition* the Operations of *Duodecimals* are performed by *Logarithms*, than by the *ordinary Method*.



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First, by Logarithms.

Add {	the Logarithm of . . . . .	9=0.954242
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---

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## 3. Secondly, by the Logarithmic Scale.

(*Note.* If the smallest Divisions in the Line AB represent *Numbers*, the Logarithms begin from C e in the Line A e ; if the middle Divisions in A B be *Numbers*, the Logarithms begin from G ; but if the largest Divisions in A B be taken for *Numbers*, then the Logarithms begin from H, in the said Line A e. And since the *smallest Divisions* are too small, and the *largest Divisions* too large for Examples, we must consequently chuse the *mean Divisions* in A B to represent the Numbers 1, 2, 3, 4, &c. or 10, 20, 30, 40, &c. or 100, 200, 300, &c. and so the Logarithms begin from G. Therefore) Set one Foot of the Compasses in G, and extend the other to the Logarithm of the Multiplier 7=ag, which you'll find to be Ga=845, and since G b=954 is the Logarithm of the Multiplicand 9=bh ; therefore with the same Extent G a in the Compasses, set one Foot in b, the other will fall on d ; therefore G d is the Logarithm of the Product dm=63 in A B, the Number sought.

## 4. Thirdly, by the Gunter with Compasses.

Set one Foot of the Compasses in the Beginning of the Line at 1, and extend the other to 7 ; with that Extent in the Compasses set one Foot in 9, the other will fall on 63, the Product required.

*Note.*

*Note.* When the Numbers are *small* the *larger Divisions* may be used, as in the present Case; but if the Numbers be *large*, the *lesser Divisions* must be used.

5. Fourthly, by the *Sliding-Gunter*.

In this Case, there is one Line of Numbers on the *Rule*, and another on the *Slider*, both mark'd with N, at the End. And it is easy to conceive that by sliding one of these by the other, the same Effects are produced as before with the *Compasses*; that is, any Part of the Line on the *Rule* is transfer'd to, or compar'd with any other Part of the said Line by means of the sliding Line.

Therefore set 1 on the *Slider* to 7 in the Line on the *Rule*; then against 9 on the *Slider*, you find 63 on the *Rule*, and that is the *Product* sought.

6. Examp. II. By the *Gunter*. What is the *Product* of 27 by 18?

Here the *lesser Divisions* must be used, and the greater ones reckoned 10, 20, 30, &c. on the *first Radius*; and consequently on the *second Radius* they will be 100, 200, 300, &c. if the *double Radius* be used. For then it will be  $10 : 180 :: 27 : \text{the Product}$  sought. But since  $10 : 180 :: 1 : 18$ ; therefore if you make  $1 : 18 :: 27 : \text{the Product}$ ; the *single Radius* will give the Answer in the same manner; only remembering that the *fourth Number* sought will be of the same Denomination with the *second*, which in this Case is *Hundreds*.

Therefore set one Foot of the *Compasses* in the Beginning of the Line, and extend the other to 18, the same Extent will reach from 27 to 486, the *Product* sought.

And by the *Sliding-Rule*, thus; set 1 on the *Slider* to 18 on the *Rule*, and then against 27 on the *Slider*

you

you find 486 on the Rule, which is the same Product as before.

7. Examp. III. What is the Product of 257 by 34?

Take in your Compasses the Distance from 1 to 34 on the Line of Numbers, the same Extent will reach from 257 to 8738, the Product required.

By the Sliding-Rule, thus; Set 1 on the Slider to 34 on the Rule, and against 257 on the Slider, you see 8738 on the Rule, which is the Product as before.

8. Examp. IV. What is the Product of 215 by 108?

With the Compasses, take the Distance from 1 to 108 on the *Gunter*, the same Extent of the Compasses will reach from 215 to 23220, the Product required.

By the Sliding-Rule, thus; Set 1 on the Slider to 108 on the Rule, and against 215 on the Slider you find 23220 on the Rule, the Product sought.

9. When the Product becomes so large, it must be a very large Line of Numbers indeed to shew it near the Truth; the Use of these Lines being principally where the Numbers are small; or where great Exactness is not required. They who understand the foregoing Doctrine of Logarithms can never be at any loss to know how many Places of Figures are contained in the Number sought, in this, or any of the following Rules.

# 10. DIVISION.

Examp. I. What is the Quotient of 63 divided by 9?

First, by Logarithms.

Y

From



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From the Logarithm of . . .  $63 = 1.799340$   
 Subduct the Logarithm of . . .  $9 = 0.954242$

The Diff. is the Log. of the Quot.  $= 7 = 0.845098$

### 11. Secondly, by the Logarithmic Scale.

From  $Gd = 1799$  the Logarithm of  $dm = 63$ , take  $Gb = 954$  the Logarithm of  $bh = 9$ ; and there will remain  $Ga = 845$ , the Logarithm of  $a = 7$ , the Quotient sought.

### 12. Thirdly, by Gunter's Line and Compasses.

Set one Foot of the Compasses in 1, and extend the other to 9, and then with that Extent of the Compasses set one Foot in 63, the other will fall (towards the beginning of the Line) on 7, the Quotient sought.

### 13. Fourthly, by the Sliding-Rule.

Because  $9 : 63 :: 1 : \text{the Quotient}$ ; therefore set 9 on the Slider to 63 on the Rule, and then against 1 on the Slider is 7 on the Rule, which is the Quotient sought.

### 14. Examp. II. What is 486 divided by 18?

By the Gunter and Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 486 (downward) to 27, the Quotient required.

By the Sliding-Rule.

Set 18 on the Slider to 486 on the Rule, then against 1 on the Slider you find 27 on the Rule, the Quotient sought.

All other Operations of Division being performed in the very same manner, 'tis needless to add any more Examples of this kind.

15. INVOLUTION.

Examp. I. What is the Square of 9?

First, by Logarithms.

The Logarithm of . . .	9=0.954242
Multiply by the Index . . .	2

The Prod. is the Log. of the Square = 81 = 1,908484

16. Secondly, by the Logarithmic Scale.

Let the Logarithms begin from G in the Line Ae, as before; then with the Compasses take the Distance  $gb=954$  the Logarithm of  $bh=9$ ; and with one Foot remaining in b, turn the Compasses, the other Foot will fall on n; then shall  $Gn=(2Gb=)1908$  the Logarithm of  $no=81$ , which therefore is the *square Number* sought.

17. Thirdly, by the *Gunter* and Compasses.

Set one Foot in 1, and extend the other to 9, where keep it fix'd, and turn the Compasses, the other Foot will fall on 81, the *square* sought.

18. Fourthly, by the Sliding-Rule.

Because  $1:9::9:$  the Square required, therefore set 1 on the Slider to 9 on the Rule, then against 9 on the Slider is 81 on the Rule; which is the *square Number* sought.

19. Examp. II. What is the Cube of 9?

By the *Gunter* and Compasses.

Extend the Compasses from 1 to 9, that Extent will reach from 9 to 81, and again from 81 to 729, the Cube Number required.

By the Sliding-Rule.

Set 1 on the Slider to 9 on the Rule, then against 9 on the Slider is 81 on the Rule, and against 81 on the Slider (remaining unmov'd) is 729 on the Rule, the Cube Number required.

20. Examp. III. What is the Square and Cube of the Number 37?

By the Gunter and Compasses.

Extend the Compasses from 1 to 37, that Extent will reach from 37 to 1369 the *Square*; and the same Extent will reach from 1369 to 50653 the *Cube*; both as required.

By the Sliding-Rule.

Set 1 on the Slider to 37 on the Rule, then against 37 on the Slider is 1369 on the Rule, which is *Square*; and against 1369 on the Slider (remaining unmov'd) is 50653 on the Rule, which is *Cube* of 37; both as before.

## 21. EXTRACTION of ROOTS.

Examp. I. What is the *Square Root* of 81?

First, by Logarithms.

The Logarithm of . . . .	81 = 1.908484
which divide by the Index . . . .	2

The Quotient is the Log. } . . . . = 9 = 0.954242  
of the *Square Root*

22. Secondly, by the Logarithmic Scale.

Bisect Gn=1908 the Logarithm of 10=81, in b; then shall Gb=954 be the Logarithm of the *Square Root*, viz. bh=9, the Number sought.



23. Thirdly, by the *Gunter* and *Compasses*.

Take with the *Compasses* the Distance between 1 and 81, and bisect it; then take one Half in the *Compasses*, and it will reach from 1 to 9, the *square Root* sought.

24. Fourthly, by the *Sliding-Rule*.

Move the *Slider* forwards and backwards till you make the same Number on the *Rule* answer 1 on the *Slider*, as on the *Slider* answers 81 on the *Rule*; which Number will be the *square Root* sought, and in the present Case will be found 9.

25. Examp. II. What is the *Cube Root* of 50653?

Divide the Distance between 1 and 50653 into 3 equal Parts; the first Division will fall on 37, the *Cube Root* required.

*Note*, In the double Line of Numbers, if the grand Divisions be esteem'd *Units* in the first Radius, those in the second Radius will be *Tens*; if those in the first be *Tens*, viz. 10, 20, 30, &c. those of the second will be *Thousands*, as 1000, 2000, 3000, &c. with regard to *square Numbers*: and consequently in *Extraction*; if the Number whose *square Root* is sought be less than 100, yet greater than 10, the Number it self will be found on the *second Radius*; and its Root a Number of *Units* on the *first Radius*. But if the *Square* be less than 10, both it self and Root will be found in the *first Radius*. Again, if the *square Number* be between 1000 and 10000, the Number it self will be found on the *second Radius*, and its Root a Number of *Tens* on the first Radius. But if it be between 100 and 1000, both the Number and its Root of *Tens* will be found on the *first Radius*. After the same manner you may reason concerning the *Cube Number* and its *Root*.

26. Since the Logarithm of the *Square* is *double* the Logarithm of the *Root*; and the Logarithm of the *Cube* *triple* the Logarithm of the *Root*: therefore if a Line of Numbers of a *single Radius*, be equal to another of a *double Radius*, and these two appositely laid together, beginning from the same Point; then against any Number on the *single Radius*, you see its correspondent *Square* on the *double Radius*; and such Lines you have on some *Sliding-Rules*.

Also if a Line of *single Radius* were made equal to another of a *triple Radius*, and these exactly and properly placed together, then the Numbers on the *latter* would be the Cubes of those on the former; and so the *Square* and *Cube Roots* of any Number; and *vice versâ*, would be discoverable by *Inspection*.

27. Moreover by means of a *single* and *double Line* of Numbers made to slide by each other, 'tis very easy to find a *mean Proportional* between any two given Numbers; as suppose 13 and 23. Thus; set 13 on the *double Line* to 13 on the *single one*, then against 23 on the *double Line* is 17.35 on the *single one*, which is the Mean required between 13 and 23. Or if you set 23 to 23, then against 13 on the *double* you find 17.35 on the *single Line*, the *Mean* required as before.

28. In like manner, by means of a *single* and a *triple Line* of Numbers, *two mean Proportionals* may be easily found between any two Numbers, as 2 and 54; thus; set 2 on one Line to 2 on the other, then against 54 on the *triple Line* is 6 on the *single one*, which is the *first Mean*; then set 2 on the *triple Line* to 6 on the *single one*, and against 54 is 18, the *second Mean*, on the *single Line*; so the four Numbers are  $2 : 6 :: 18 : 54$ . And thus you may find *two Means* between any other two Numbers, which in many Cases is a most useful Problem.

29. In the foregoing Operations I have made no mention of *Decimals*, because they are to be respected

as

as Whole Numbers in the Management of them by *Instrumental Operations*, in the same manner as they were by *Numerical Logarithms*; the Number of *decimal Places* in any *Product*, *Quotient*, *Power*, *Root*, &c. being always determined here, as in all other Methods of working them, by the Rules proper to *Decimal Arithmetic*.

30. Thus it appears what *Similarity*, *Coherence*, and mutual Relation there is between the foregoing Methods of solving *Arithmetical Questions* by *Logarithms*, both *Numerical* and *Instrumental*; and that they are all one in Nature, and differ only in the *Modus operandi*, or Manner and Form of working. By this Chapter, I presume, 'twill be easy for the Learner to observe how any common Question in *Arithmetic*, or the Mensuration of *Artificers Work*, as *Joinery*, *Masonry*, *Carpentry*, *Painting*, *Timber-Measure*, *Gauging*, &c. may be most readily performed by the *Line of Numbers*, with *Compasses*, or by the *Sliding-Rule*, which is much the best Way.






## C H A P. X.

*Sheweth the ANALOGIES or PROPORTIONS for the SOLUTION of all the CASES of Plain and Spherical TRIANGLES, both Right and Oblique angled.*

1. IF any desire to be thoroughly instructed in the *Theory of Plain and Spherical Trigonometry*, I must refer them to my *Young Trigonometer's Guide*; since all I intend here is only to shew the *great and most excellent Use of Logarithms* in the practical Resolution of *Plain and Spherical Triangles*. A Synopsi


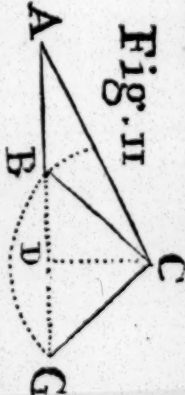


nopsis of all the Cases of a Right-angled plain Triangle here follows.

Cases	Given	Sought	Fig. I 	Fig. II 	Fig. III 
I	B AB	AC BC	$sc: AB:: sb: AC.$ $sb: AC:: R: BC.$	$R: AB:: tb: AC.$ $R: AB:: sb: BC.$	$tc: AB:: R: AC.$ $R: AC:: sec: BC.$
II	B AC	AB BC	$sb: AC:: sc: AB.$ $sb: AC:: R: BC.$	$tb: AC:: R: AB.$ $R: AB:: sb: BC.$	$R: AC:: tc: AB.$ $A: AC:: sec: BC.$
III	B BC	AB AC	$R: BC:: sc: AB.$ $R: BC:: sb: AC.$	$sb: BC:: R: AB.$ $R: AB:: tb: AC.$	$sec: BC:: tc: AB.$ $tc: AB:: R: AC.$
IV	AB AC	B BC	$\sqrt{ABq + ACq} = BC.$	$AB: R:: AC: tb.$ $R: AB:: sb: BC.$	$AC: R:: AB: tc.$ $R: AC:: sec: BC.$
V	AB BC	AC C	$BC: R:: AB: sc.$ $R: BC:: sb: AC.$	$AB: R:: BC: sb.$ $R: AB:: tb: AC.$	$\sqrt{BCq - ABq} = AC.$
VI	AC BC	B AB	$BC: R:: AC: sb.$ $R: BC:: sc: AB.$	$\sqrt{BCq - ACq} = AB.$	$AC: R:: CB: sec.$ $R: AC:: tc: AB.$

shown whence the Angle C may be easily found.

2. A Synopsis of the Analogies for the Solution of all the Cases of Oblique-angled plain Triangles.

Cases	Given	Sought	Fig. I	Fig. II
I	AB C, A, B.	AC. BC.	$\begin{aligned} & \text{SC} : \text{AB} :: \text{SB} : \text{AC} \\ & \text{SC} : \text{AB} :: \text{SA} : \text{BC}. \end{aligned}$ 	$\begin{aligned} & \text{SC} : \text{AB} :: \text{SB} : \text{AC} \\ & \text{SC} : \text{AB} :: \text{SA} : \text{BC}. \end{aligned}$ 
II	AC BC A	AB C, B	$\begin{aligned} & \text{BC} : \text{SA} :: \text{AC} : \text{SB}. \\ & \text{SB} : \text{AC} :: \text{SC} : \text{AB}. \end{aligned}$	
III	AC BC C	A, B, AB	$\text{AC} + \text{CB} : \text{AC} - \text{CB} :: t \frac{A+B}{2} : t \frac{A-B}{2}.$ <p>Whence the Angles are known.</p>	
IV	AB AC CB	A, B, C.	$\text{AB} : \text{AC} + \text{CB} :: \text{AC} - \text{CB} : \text{AD} - \text{DB}.$ <p>In the first Triangle, 'tis <math>\text{AD} - \text{DB}</math> } AG in the second Triangle, <math>\text{AD} + \text{DB}</math> }</p> <p>Then <math>\text{AB} - \text{AG} = \text{GB} = 2\text{DB}</math>. Thus each Tri- angle is reduced to two Right ones, viz. ADC, and BDC; in either of which two Sides are known whence the Angle C may be easily found.</p>	

3. A Synopsis of the Canons, and the Analogies (form'd from them for the Solution of all the Cases of Right-angled Spherical Triangles.

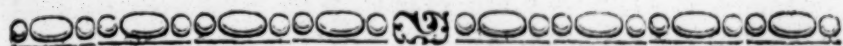
Cases	Given	Sought	Canons.	Analogies.	The Triangle.
I	AB B	AC BC C	sBE : sBA :: tFE : tAC. sDE : sDF :: tAE : tCF. .....	R : sBA :: tB : tAC. R : csB :: ctAB : ctBC. R : csAB :: sB : csC.	
II	AC B	AB BC C	tEF : tAC :: sBE : sAB. sEF : sAC :: sBF : sBC. sCD : sCH :: sDF : sHI.	tB : tAC :: R : sAB. sB : sAC :: R : sBC. csAC : R :: csB : sC.	
III	BC B	AB AC C	sDF : sDE :: tCF : tAE. sBF : sBC :: sEF : sAC. sCF : sCI :: tDF : tHI.	R : csB :: tBC : tAB. R : sBC :: sB : sAC. csBC : R :: ctB : tC.	
IV	AB AC	BC B C	sAD : sDC :: sAE : sCF. sAB : sBE :: tAC : tEF. .....	R : csCA :: csAB : csBC. sAB : R :: tAC : tB. sAC : R :: tAB : tC.	
V	BC AC	AB B C	sDC : sDA :: sCF : sAE. sBC : sBF :: sAC : sEF. tIF : tDH :: sGI : sGH.	csAC : R :: csBC : csAB. sBC : R :: sAC : sB. tBC : R :: tAC : csC.	
VI	B C	AB AC BC	..... sHI : sDF :: sCH : sCD. tHI : tDF :: sCI : sCF.	sB : csC :: R : csAB. sC : csB :: R : csAC. tC : ctB :: R : csBC.	





5. Being thus furnished with Proportions, we shall soon see with what incomparable Pleasure and Ease the several Cases of Triangles before going are resolved by the Canon of Logarithmical *Sines*, *Tangents*, and *Secants* ; and also by the *Line of Numbers*, both with the *Compasses* and by the *Sliding-Rule* ; I say, we shall see in the two next Chapters, with how much more Ease and Pleasure they are resolved by *these two Methods*, above what is attainable by any other Way yet invented for this Purpose,





## C H A P. XI.

*The SOLUTION of PLAIN TRIANGLES by the  
CANON of LOGARITHMICAL SINES and  
TANGENTS ; by GUNTER'S SCALE and  
COMPASSES ; and by the SLIDING RULE.*

∴ I Have already described and taught the Use of the Logarithmic Canon, so far as to find the Logarithms of any *Number, Sine, Tangent, or Secant* proposed. I shall therefore here only observe, that what I call *Gunter's Scale* is such a *Plane Scale* as hath upon it *Gunter's Line of Numbers*, and of *Artificial Sines and Tangents* ; whose Nature, Construction, and Design, have been before discoursed of in the Theory. The *Sliding-Rule* has the same Lines, which are contrived to slide by one another as you please ; and to avoid Repetitions, I shall call the Line of Numbers on the Rule it self, A ; and that on the Slider, B ; also I shall call the Lines of Sines and Tangents on the Rule Sr, Tr ; and those on the Slider Ss, and Ts. You must know also that the End of each Line marked 10, 90, 45, is here called Radius. Having premised these things, we proceed immediately to the Solution of

### 2. *Right-angled Plain Triangles.*

Case I. In the Right-angled Triangle ABC, there is given the Base AB=285, and the Angle at Base B=32° 48' ; to find the Perpendicular AC, and the Hypothenuse BC. The Hypoth. Radius, Fig. I.

The Analogy for AC is,  $sC : AB :: sB : AC$ .

In



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In Words ;

As the Sine of the Angle  $C=57^{\circ} 12' = 9.924572$

is to the given Side or Base  $AB=285 = 2.454845$  } add  
So is the Sine of the Ang.  $B=32^{\circ} 48' = 9.733765$

12.188610

To the Perpendicular  $AC=183,67 = 2.264038$

The Analogy for BC is,  $sB : AC :: R : BC.$

In Words ;

As the Sine of . . .  $B=32^{\circ} 48' = 9.733765$

is to the Perpendicular  $AC=183,67 = 2.264038$

So is Radius . . . .  $90^{\circ} 00' = 10.000000$

to the Hypothenufe . . .  $BC=339,06 = 2.530273$

3. *By Scale and Compasses. To find AC.*

Set one Foot of the Compasses to  $32^{\circ} 48' = B$ , in Line of Sines, and extend the other upwards to  $57^{\circ} 12' = C$ ; the same Extent will reach from 285 = AB downwards in the Line of Numbers to 183,67 = AC, the Perpendicular sought.

To find BC.

Extend the Compasses from  $32^{\circ} 48' = B$  to Radius  $90^{\circ}$  in the Line of Sines; the same Extent will reach (in the Line of Numbers) from 183,67 = AC, to 339,06 = BC, the Hypothenufe sought.

4. *By the Sliding-Rule. To find AC.*

Make the Line of Numbers slide by the Line of Numbers, then will the Lines of Sines slide by each other, and also the Line of Tangents by the Line of Tangents. Having thus prepared the Rule; set

57°

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57° 12' on Ss to 32° 48' on SR; then against 285 on B is 183,67 on A, the Perpendicular sought.

To find BC.

Set 32° 48' on Ss to Radius 90° on SR, then against 183,67 on B is 339,06 on A, which is the Hypothenufe required.

5. Case II. Given the Angles  $B=32^{\circ} 48'$ , and  $C=57^{\circ} 12'$ , and the Side  $AC=183,67$ ; to find AB, and BC.

The Base made Radius. Fig. II.

As the Tangent of the Ang.  $B=32^{\circ} 48'=9.809193$   
is to the Side . . . .  $AC=183,67=2.264038$   
So is Radius . . . .  $90^{\circ} 00'=10.000000$

to the Side or Base . . .  $AB=285=2.454845$

To find BC.

As Radius . . . .  $90^{\circ} 00'=10.$   
is to the Side . . . .  $AB=285=2.454845$   
So is the Secant of the Angle  $B=32^{\circ} 48'=10.075428$

to the Side or Hypoth.  $BC=339,06=2.530273$

*Note*, I have wrought this last in *Secants* for Variety sake, and that the Reader may see the Conclusions are the same every Way. But this Case is much better resolved by making BC Radius, as in Fig. I.

6. *By the Plane Scale and Compasses.*

As there is no *Line of Artificial Secants* on the *Scale* or *Sliding-Rule* (as being useless) so this Case will be best performed Instrumentally by the Analogies of Fig. I. where BC is made Radius.

Therefore

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Therefore, extend the Compasses from  $32^{\circ} 48'$  to  $57^{\circ} 12'$  in the Line of Sines, the same Extent will reach from 183,76 to 285 in the Line of Numbers; thus  $285=AB$ , the Side required.

Or thus, By the first Analogy of this Case of Fig. II. extend the Compasses from  $32^{\circ} 48'$  to Radius 45 in the Line of Tangents, the same will reach from 183,67 to  $285=AB$  (as before) in the Line of Numbers.

To find BC.

Extend the Compasses from  $32^{\circ} 48'$  to Radius 90 in the Line of Sines, the same will reach from 183,67 to  $339,06=BC$ ; in the Line of Numbers.

7. *By the Sailing-Rule.*

To find AB.

Set  $32^{\circ} 48'$  on Ts to Radius  $45^{\circ}$  on TR; then against 183,67 on B is  $285=AB$ , on A.

To find BC.

Set  $32^{\circ} 48'$  on Ss to Radius  $90^{\circ}$  on SR; then against 183,67 on B is  $339,06=BC$ , on A.

8. Case III. Given the Angles  $B=32^{\circ} 48'$  and  $C=57^{\circ} 12'$ , and the Hypothenufe  $BC=339,06$ ; to find the Sides AB and AC.

The Hypothenufe made Radius. Fig. I.

As Radius . . . .  $90^{\circ} 00' = 10$ .  
is to the Hypothenufe  $BC=339,06=2.530273$   
So is the Sine of the Angle  $C=57^{\circ} 12'=9.924572$

To the Side or Base . . . .  $AB=285=2.454845$

And so is the Sine of the Ang.  $B=32^{\circ} 48'=9.733765$

to the Side . . . .  $AC=183,67=2.264038$

Or



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Or thus; AB made Radius, Fig. II. To find AC.

As Radius . . . .	90° 00' = 10.
is to the Side . . . .	AB = 285 = 2.454845
So is the Tangent of . . . .	B = 32° 48' = 9.809193
to the Side . . . .	AC = 183,67 = 2.264038

9. *By the Scale and Compasses.*

Extend the Compasses from Radius 90 to 57° 12' in the Line of Sines, the same Extent will reach from 339,06 to 285 = AB, in the Line of Numbers.

And then again, extend them from 90 to 32° 48' in the Line of Sines, the same will reach from 339,06 to 183,67 = AC, in the Line of Numbers.

10. *By the Sliding-Rule.*

This Case may be solved by once setting the Rule, thus; Make the Line of Sines to slide by the Line of Numbers: Then set Radius 90 on Ss to 339,06 on A; thus against 57° 12' on Ss you see 285 = AB, and against 32° 48' is 183,67 = AC, on A. Such is the great Conveniency of this small Instrument.

11. Case IV. Given the two Sides, AB = 285 and AC = 183,67; to find the Side BC, and the Angles B and C.

The Base made Radius, Fig. II. To find B.

As the Side . . . .	AB = 285 = 2.454845
is to Radius . . . . .	10.
So is the Side . . . . .	AC = 183,67 = 2.264038

To the Tangent of the Angle B = 32° 48' = 9.809193

Or thus, making AC Radius, Fig. III. To find C.

A 2	As
-----	----

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As the Side . . . . .  $AC=183,67=2.264038$   
 is to Radius . . . . .  $10.$

So is the Side . . . . .  $AB=285=2.454845$

To the Tang. of the Angle  $C=57^{\circ}48'=10.190707$

12. To find the Side BC.

This may be done directly with the Secants, or by finding the Angles first, with the Sines; as is manifest from the Synopsis. But since if the required Side BC be made Radius, it can't be found by the common Analogies of *Trigonometry*; I have given an Equation formed on the Principles of *Geometry* for that purpose, *viz.*  $\sqrt{ABq+ACq}=BC$ ; which I shall resolve by Logarithms, as follows.

The Log. of the Side . . .  $AB=285=2.454845$   
 the Double thereof }  
 is the Log. of . . .  $ABq=81225=4.909690$

Also the Log. of the Side  $AC=183,67=2.264038$   
 The Double of which }  
 is the Log. of  $ACq=33734,66=4.528076$   
 Add the Square . . . . .  $ABq=81225$

The Log. of }  
 the Sum }  $ABq+ACq=114959.66=5.060546$

Half which is the Log. of  
 $\sqrt{ABq+ACq}=BC=339,06=2.530273$   
 the Side required.

13. *By Scale and Compasses.* To find the Ang. B.

Extend the Compasses from 285 to 183,67 in the Line of Numbers, the same will reach from Radius  $45^{\circ}$  to  $32^{\circ}48'$ , in the Line of Tangents, the Angle B required.

Set 3  
 By 90° on

*By the Sliding-Rule.*

Set 285 on A to 183,67 on B, then against Radius 45° on TR is 32° 48' = the Angle B, on Ts.

Having thus found the Angles, the Side BC is found as in the foregoing Cases.

14. Case V. Given the Hypothenufe  $BC=339,06$  and the Side  $AB=285$ ; to find the Angles B and C, and the Side AC.

To find the Angle C. Fig. I. BC Radius.

As the Side . . . . .  $BC=339,06=2.530273$   
is to Radius . . . . . 10.  
So is the Side . . . . .  $AB=285=2.454845$

To the Sine of the Angle  $C=57^{\circ} 12'=9.924572$

To find the Side AC.

As Radius . . . . . 10.  
is to the Side . . . . .  $BC=339,06=2.530273$   
So is the Sine of the Angle  $B=32^{\circ} 48'=9.733765$   
to the Side . . . . .  $AC=183,67=2.264038$

Note, The Side AC may be found Geometrically, as taught in Art. 12. the Equation being  $\sqrt{BCq-ABq}=AC$ .

15. *By Scale and Compaffes.* To find the Angle C.

Extend the Compaffes from 339,06 to 285 in the Line of Numbers, the same will reach from Radius 90° to 57° 12' = C, in the Line of Sines.

*By the Sliding-Rule.*

Set 339,06 on A to 285 on B, then against Radius 90° on SR is 57° 12' = C, on Ss.

A a 2

The



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The Angles being thus found, the Side AC may be found by *Scale* or *Sliding-Rule*, as before.

16. Case VI. Given the Side AC=183,67 and BC=339,06; to find the Angles B, C, and the Side AB.

As this Case is, in the Nature of it, the same as the last, so the Solution is in all respects the same, and needs not be repeated.

17.      *Of Oblique-angled Plain Triangles.*

Case I. There is given the Angles C=82° 30', B=60° 00'; and the Side AB=365; to find the other two Sides AC and BC.

*Note,* The first Triangle in the Synopsis of Oblique Plain Triangles is that which I have regard to here, and is acute-angled; the Difference between this and the obtuse-angled one, Fig. II. will be taken notice of as I go along.

To find the Side AC.      Fig. I.      Co. Ar.

As the Sine of the Angle C=82° 30'=0.003732  
is to the Side . . . . . AB=365=2.562293

So is the Sine of the Angle B=60° 00'=9.937531

to the Side sought . . . . AC=318,82=2.503556

To find the Side BC.      Co. Ar.

As the Sine of the Angle C=82° 30'=0.003732  
is to the Side . . . . . AB=365=2.562293

So is the Sine of the Angle A=37° 30'=9.784447

to the Side required . . . . BC=224,11=2.350472

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18. *By the Scale and Compasses.* To find AC.

Extend the Compasses from  $82^{\circ} 30'$  to  $60^{\circ}$  in the Line of Sines, and the same Extent will reach from 365 to 318,82 in the Line of Numbers; therefore  $318,82 = AC$ .

To find BC.

Extend the Compasses from  $82^{\circ} 30'$  to  $37^{\circ} 30'$  in the Line of Sines, the same will reach from 365 to 224,11 = BC, in the Line of Numbers.

19. *By the Sliding-Rule.*

To find AC.

Set  $82^{\circ} 30'$  on SR to  $60^{\circ} 00'$  on Ss; then against 365 on A, you have  $318,82 = AC$ , on B.

To find BC.

Set  $82^{\circ} 30'$  on SR to  $37^{\circ} 30'$  on Ss; then against 365 on A, you have  $224,11 = BC$ , on B.

But since all the Angles are known, both the unknown Sides are found at once setting the Rule thus;

Let the *Line of Numbers* slide by the *Line of Sines*; and set  $82^{\circ} 30'$  on SR to 365 on B, then against  $60^{\circ} 00'$  is  $318,82 = AC$ ; and against  $37^{\circ} 30'$  is  $224,11 = BC$ , on the Line of Numbers.

20. Case II. Given two Sides,  $AC = 318,82$  and  $BC = 224,11$ ; and the opposite Angle  $A = 37^{\circ} 30'$ ; to find the Angle B, and the other Side AB.

To find the Angle B.

Co. Ar.

As the Side . . . . .  $BC = 224,11 = 7.649528$   
is to the Sine of the Angle  $A = 37^{\circ} 30' = 9.784447$   
So is the Side . . . . .  $AC = 318,82 = 2.503556$

to the Sine of the Angle . .  $B = 60^{\circ} 00' = 9.937531$

The

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The Angles being thus known, the Side AB is found as were the Sides AC and BC in the foregoing Case.

21. *By the Scale and Compasses.*

To find the Angle B.

Extend the Compasses from 224,11 to 318,82 in the Line of Numbers, the same Extent will reach from  $37^{\circ} 30'$  to  $60^{\circ} 00' = B$ , on the Line of Sines.

*By the Sliding-Rule.*

Set 224,11 on A to 318,82 on B, then against  $37^{\circ} 30'$  on SR is  $60^{\circ} 00' = B$ , the Angle sought, on Ss.

*Note,* In this Case of the Obtuse-angled Triangle, Fig. II. 'tis obvious the Angle here found is the outward Angle CBG, whose Complement therefore to 180 Degrees is equal to the internal obtuse Angle  $ABC = 120^{\circ} 00'$ .

22. Case III. Given two Sides  $AC = 318,82$  and  $BC = 224,11$ ; and the included Angle  $C = 82^{\circ} 30'$ ; to find the Angles A, B, and the Side AB.

To find the Angles A and B.

The Sum of the given Sides is  $AC + BC = 542,93$ ; their Difference is  $AC - BC = 94,71$ ; the Sum of the unknown Angles  $A + B = 97^{\circ} 30'$ ; therefore the half Sum is  $\frac{A+B}{2} = 48^{\circ} 45'$ ; whence by the Analogy in the Synopsis, say;

As



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Co. Ar.

As the Sum of } ...  $AC + BC = 542,93 = 7.265256$   
the Sides }  
is to their Difference  $AC - BC = 94,71 = 1.976396$   
So is the Tangent of }  
the half Sum of the }  $\frac{A+B}{2} = 48^{\circ} 45' = 10.057012$   
unknown Angles }

to the Tangent of half }  $\frac{A-B}{2} = 11^{\circ} 15' = 9.298664$   
their Difference }

Then to the half Sum of the Angles  $48^{\circ} 45'$   
add the half Difference . . .  $11^{\circ} 15'$

The Sum is the greater Angle  $B = 60^{\circ} 00'$

But the Diff. is the lesser Ang.  $A = 37^{\circ} 30'$

23. *By Scale and Compasses.*

To find the Half Difference of the Angles  $\frac{A-B}{2}$ .

Having prepared the Work as above, proceed thus ;

Extend the Compasses from the Sum of the Sides 542,93 to their Difference 94,71 on the Line of Numbers ; with this Extent set one Foot of the Compasses in Radius  $45^{\circ}$ , and pitch the other downwards in the Line of Tangents, where fix it while you bring the other Foot from  $45^{\circ}$  to  $48^{\circ} 45'$  ; then with this Extent apply one Foot in  $45^{\circ}$ , the other will reach to  $11^{\circ} 15'$  = the half Difference of the Angles A and B ; which therefore may be found as before.

Or if you have a Line of Tangents continued beyond  $45^{\circ}$ , then the Extent from 94,71 to 542,93 in the Line of Numbers will reach from  $48^{\circ} 45'$  to  $11^{\circ} 15'$  in the Tangent Line.

24.

*By the Sliding-Rule.*

Set 542.93 on A to 94.71 on B; then against  $45^{\circ}$  on TR observe the Degree and Minute on Ts, and bring that Point to  $48^{\circ} 45'$  on TR; then against  $45^{\circ}$  on TR you have  $11^{\circ} 15'$  on Ts, which is the half Difference as before, of the two enquired Angles A and B.

Or thus, if the Line of Tangents be continued on the Slider beyond  $45^{\circ}$ ; set 542.93 on A to  $48^{\circ} 45'$  on Ts, then against 94.71 on A is  $11^{\circ} 15'$  on Ts.

Having therefore the *Half Sum*, and *Half Difference* of those Angles, they are found as in Art. 22, and then the Side AB will be found to be 365 as in Case I. hereof.

25. Case IV. Given all three Sides  $AB=365$ ,  $AC=318.82$ , and  $CB=224.11$ , to find the Angles.

In order to this 'twill be necessary to reduce the oblique Triangle into two Right-angled ones ADC, and BDC, thus; Find the Sum of any two Sides  $AC+CB=542.93$ ; and their Difference  $AC-CB=94.71$ ; esteeming the other Side AB the Base, the Difference of whose Segments  $AD-DB=AG$ , is first of all to be found by this Proportion, *viz.*

Co. Ar.

As the Base . . . . .	$AB=365=.7437707$
is to the Sum of } the two Sides	$AC+BC=542.93=2.734744$
So the Diff. of the } two Sides	$AC-BC=94.71=1.976396$

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to the Diff. of } the Segments	$AD-DB=AG=141=2.148847$
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Therefore

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Therefore  $AB - AG = 224 = GB = 2BD$  ; therefore  $BD = 112$ , and  $AD = 253$  ; and so the whole Triangle  $ACB$  is reduced to two Right-angled ones  $ADC$  and  $BDC$ , in each of which there is two Sides given  $AD$  and  $AC$ ,  $DB$  and  $BC$  ; by which means the Angles  $ACD$  and  $DCB$  may be found, which together are equal to the Angle  $ACB$  ; and this being known, the other two are found with Ease by Case I. And thus the whole oblique Triangle is resolved.

*Note*, When the Perpendicular falls without, as in the obtuse-angled Triangle  $ABC$ , Fig. II. then it will be the Sum of the Segments  $AD + DB = AG$  ; and the Difference of the Angles  $ACD - BCD = ACB$ , the Angle required.

26. And since the several Problems of *Navigation*, whether in the *Plain*, *Mercator's*, *Middle Latitude*, *Oblique*, or *Traverse Sailing* ; as also of measuring *Heights* and *Distances*, *accessible* and *inaccessible* ; or *Fortification*, *Gunnery*, and divers Parts of *Astronomy*, &c. are all resolved by the *Doctrine of Plain Trigonometry*, as in the Method before-going ; it must be very easy for any who understands the *Solution* of Plain Triangles, to apply it to any practical Cases that may occur in any of the aforesaid Arts, without any farther Instructions or Examples. Yet those who would see the *Theory* of *Plain Trigonometry*, and its *Application*, in the largest Extent, may find it in the first Vol. of my *Young Trigonometer's Guide*.



B b

CHAP.





## C H A P. XII.

*The SOLUTION of SPHERICAL TRIANGLES by  
LOGARITHMS, by GUNTER'S SCALE, and  
by the SLIDING RULE.*

1. **A**S in the foregoing Chapter I have exemplified the Resolution of *Plain Triangles* both by the *Canon of Logarithms*, and *Logarithmical Instruments*, so I shall pursue the same Method here with respect to *Spherical Triangles* of both kinds; in each of which there are *six different Cases*, which in all their *Varieties* are resolvable according to the *Analogies* assigned in the *Synopsis*; on which, and on the Figure of the Triangle there, the Reader is desired to have his eye, thro' the whole Course of Examples. To begin then with

2.      *Right-angled Spherical Triangles.*

Case I. Given the Base  $AB=38^{\circ} 15'$ , and Angle at Base  $B=39^{\circ} 56'$  to find the Perpendicular  $AC$ , the Hypothenuse  $BC$ , and the Angle  $C$ .

1. To find the Perpendicular  $AC$ .

As Radius . . . . .  $90^{\circ} 00' = 10.000000$   
is to the Sine of the Side  $AB=38^{\circ} 15' = 9.791757$   
So is the Tang. of the Ang.  $B=39^{\circ} 56' = 9.922787$

to the Tang. of the Perp.  $AC=27^{\circ} 23\frac{1}{4}' = 9.714544$

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### 2. To find the Hypothenufe BC.

As Radius . . . . .  $90^{\circ} 00' = 10.000000$   
 to the Co-Sine of the Angle  $B = 39^{\circ} 56' = 9.884677$   
 So is the Co-T. of the Side  $AB = 38^{\circ} 15' = 10.103288$   
 to the Co-T. of the Hypoth.  $BC = 45^{\circ} 48' = 9.988065$

### 3. To find the Angle C.

As Radius . . . . .  $90^{\circ} 00' = 10.000000$   
 to the Co-Sine of the Side  $AB = 38^{\circ} 15' = 9.895045$   
 So is the Sine of the Angle  $B = 39^{\circ} 56' = 9.807465$   
 to the Co-Sine of the Angle  $C = 59^{\circ} 44' = 9.702510$

### 3. *By Scale and Compasses.*

#### 1. To find AC.

Extend the Compasses from  $90^{\circ}$  to  $39^{\circ} 56'$  in the Line of Sines, that will reach from  $38^{\circ} 15'$  in the Line of Tangents to  $27^{\circ} 23\frac{1}{4}' = AC$ , the Side required.

#### 2. To find the Hypoth. BC.

Extend the Compasses from Radius  $90^{\circ}$  to the Co-Sine of  $B$ ,  $50^{\circ} 04'$  in the Line of Sines; then apply that Extent from  $45^{\circ}$  in the Line of Tangents downwards, where fix that lower Foot, and bring the other to  $51^{\circ} 45'$  the Co-Tangent of  $AB$ , this last Extent will reach from  $45^{\circ}$  to  $44^{\circ} 12'$  the Co-Tangent of  $BC = 45^{\circ} 48'$ , the Side required.

#### 3. To find the Angle C.

Extend the Compasses from  $90^{\circ}$  to  $51^{\circ} 45'$  the Co-Sine of  $AB$ , the same will reach from the Sine  $39^{\circ} 56'$  to  $30^{\circ} 16'$  the Co-Sine of  $C = 59^{\circ} 44'$ .

4. *By the Sliding-Rule.*

## 1. To find AC.

Set  $90^{\circ}$  on SR to  $38^{\circ} 15'$  on Ss; then against  $39^{\circ} 56'$  on TR is  $27^{\circ} 23\frac{1}{4}' = AC$ , on Ts.

## 2. To find BC.

Set 90 on SR to  $50^{\circ} 04'$  the Co-Sine of B, on Ss; and mark the Degree and Minute on Ts against  $45^{\circ}$  on TR, bring that Point to  $51^{\circ} 45'$  (the Co-Tangent of AB) on TR, on which against  $45^{\circ}$ , you have  $44^{\circ} 12'$  on Ts, the Co-Tangent of  $BC = 45^{\circ} 48'$ , as required.

## 3. To find the Angle C.

Set  $90^{\circ}$  on SR to the Co-Sine of AB,  $51^{\circ} 45'$  on Ss; then against  $39^{\circ} 56'$  on SR is  $30^{\circ} 16'$  on Ss, the Co-Sine of  $C = 59^{\circ} 44'$ , as required.

5. Case II. Given the Perpendicular  $AC = 27^{\circ} 23'$ , and the opposite Angle  $B = 39^{\circ} 56'$ ; to find the Side AB, the Hypothenufe BC, and Angle C.

## To find AB.

As the Tangent of . . . . .  $B = 39^{\circ} 56' = 9.922787$   
 is to the Tangent of . . .  $AC = 27^{\circ} 23\frac{1}{4}' = 9.714544$   
 So is Radius . . . . .  $90^{\circ} 00' = 10.$

to the Sine of the }  
 Side required } . . . . .  $AB = 38^{\circ} 15' = 9.791757$

The Operations of this Case being only the Converse of the foregoing, needs no further Examples, in Numbers.



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6. *By Scale and Compasses.*

1. To find AB.

Extend the Compasses from  $39^{\circ} 56'$  to  $27^{\circ} 23\frac{1}{4}'$  on the Tangents, the same will reach from  $90^{\circ}$  on the Sines, to  $38^{\circ} 15' = AB$ .

2. To find BC.

Extend the Compasses from  $39^{\circ} 56'$  to  $27^{\circ} 23\frac{1}{4}'$  on the Sines, the same will reach from  $90^{\circ}$  to (the Sine of  $BC =$ )  $45^{\circ} 48'$ , as required.

3. To find the Angle C.

Extend the Compasses from (the Co-Sine of AC)  $62^{\circ} 36\frac{1}{4}'$  to  $90^{\circ}$ , the same will reach from (the Co-Sine of B)  $50^{\circ} 04'$  to  $59^{\circ} 44'$  the Sine of the Angle C required.

7. *By the Sliding-Rule.*

1. To find AB.

Set  $39^{\circ} 56'$  on  $TR$  to  $27^{\circ} 23\frac{1}{4}'$  on  $Ts$ , then against  $90^{\circ}$  on  $SR$  is  $38^{\circ} 15' = AB$ , on  $Ss$ .

2. To find BC.

Set  $39^{\circ} 56'$  on  $SR$  to  $27^{\circ} 23\frac{1}{4}'$  on  $Ss$ ; then against  $90^{\circ}$  on  $SR$  is  $45^{\circ} 48' = BC$ , on  $Ss$ .

3. To find the Angle C.

Set (the Co-Sine of AC)  $62^{\circ} 36\frac{1}{4}'$  on  $SR$  to  $90^{\circ}$  on  $Ss$ ; then against (the Co-Sine of B)  $50^{\circ} 04'$  on  $SR$  is  $59^{\circ} 44' = C$ , on  $Ss$ .

8. Case III. Given the Hypothenufe  $BC = 45^{\circ} 48'$ , and an acute Angle  $B = 39^{\circ} 56'$ , to find the Legs AB, AC, and the Angle C.

1.

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1. To find AB.

As Radius . . . . .  $90^{\circ} 00' = 10.000000$   
 to the Co-Sine of . . .  $B = 39^{\circ} 56' = 9.884677$   
 So is the Tangent of . .  $BC = 45^{\circ} 48' = 10.012129$

to the Tang. of the Side }  $AB = 38^{\circ} 15' = 9.896806$   
 fought

2. To find AC.

As Radius . . . . .  $90^{\circ} 00' = 10.$   
 is to the Sine of . . .  $BC = 45^{\circ} 48' = 9.855465$   
 So is the Sine of . . .  $B = 39^{\circ} 56' = 9.807465$

to the Sine of the Side }  $AC = 27^{\circ} 23\frac{1}{4}' = 9.662930$   
 fought

3. To find the Angle C.

As the Co-Sine of . . .  $BC = 45^{\circ} 48' = 9.843336$   
 is to Radius . . . . .  $90^{\circ} 00' = 10.$   
 So is the Co-Tangent of  $B = 39^{\circ} 56' = 10.077213$

To the Tangent of . . .  $C = 59^{\circ} 44' = 10.233877$

9. *By Scale and Compasses.*

1. To find AB.

Extend the Compasses from Radius  $90^{\circ}$  to (the Co-sine of B)  $50^{\circ} 04'$  in the Sines, then set one Foot in  $45^{\circ}$  in the Tangents, and pitch the other downward; where fix it, while you bring the former from  $45^{\circ}$  to  $45^{\circ} 48'$ ; then will this last Extent reach from  $45^{\circ}$  to the Tangent of  $38^{\circ} 15' = AB$ , the Side required.

2. To find AC.

Extend the Compasses from  $90^{\circ}$  to  $45^{\circ} 48'$  in the Sines,

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Sines, the same will reach from  $39^{\circ} 56'$  to  $27^{\circ} 23\frac{1}{4}'$  = AC, in the same Line.

### 3. To find the Angle C.

Extend the Compasses from (the Co-Sine of BC)  $44^{\circ} 16'$  to Radius  $90^{\circ}$ ; the same applied to (the Co-Tangent of B)  $50^{\circ} 04'$ , will reach to the Tangent  $59^{\circ} 44'$  = C, the Angle required.

10.

*By the Sliding-Rule.*

### 1. To find AB.

Set Radius  $90^{\circ}$  on SR to  $50^{\circ} 04'$  (the Co-Sine of B) on Ss, and against  $45^{\circ}$  on TR mark the Point on Ts, bring that Point to  $45^{\circ} 48'$  on TR; and now against  $45^{\circ}$  on TR you have  $38^{\circ} 15'$  = AB, on Ts.

### 2. To find AC.

Set Radius  $90^{\circ}$  on S to  $45^{\circ} 48'$  on SR, then against  $39^{\circ} 56'$  on Ss, you'll see  $27^{\circ} 23\frac{1}{4}'$  on SR, the Side AC required.

### 3. To find the Angle C.

Set (the Co-Sine of BC)  $44^{\circ} 16'$  on Ss to Radius  $90^{\circ}$  on SR; then against (the Co-Tangent of B)  $50^{\circ} 04'$  on TR, is  $59^{\circ} 44'$  = C, on Ts.

11. Case IV. Given the Legs, AB =  $38^{\circ} 15'$ , and AC =  $27^{\circ} 23\frac{1}{4}'$ ; to find the rest.

To find the Hypothenufe BC.

As Radius . . . . .  $90^{\circ} 00' = 10$ .  
 is to the Co-Sine of . . AC =  $27^{\circ} 23\frac{1}{4}' = 9.948388$   
 So is the Co-Sine of . . AB =  $38^{\circ} 15' = 9.895045$   
 to the Co-Sine of . . . . BC =  $45^{\circ} 48' = 9.843433$

As



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As there is nothing new in finding the Analogies B and C, I shall pass them by.

12. *By Scale and Compasses.*

1. To find BC.

Extend the Compasses from  $90^\circ$  to (the Co-Sine of AC)  $62^\circ 37'$ ; the same applied to (the Co-Sine of AB)  $51^\circ 45'$  will extend to (the Co-Sine of BC)  $44^\circ 16'$ ; therefore  $BC = 45^\circ 48'$ .

2. To find the Angle B.

Extend the Compasses from  $38^\circ 15'$  to  $90^\circ$  in the Sines, the same will reach from  $27^\circ 23\frac{1}{4}'$  to  $39^\circ 56' = B$ , in the Line of Tangents.

3. To find the Angle C.

Extend the Compasses from  $27^\circ 23\frac{1}{4}'$  to  $90^\circ$  in the Sines; the same will reach from  $45^\circ$  to a Point in the Tangent-Line, where fix the Compasses while you bring the Leg from  $45^\circ$  to  $38^\circ 15'$ , then will this last Extent reach from  $45^\circ$  to  $59^\circ 44' = C$ , in the Tangents.

13. *By the Sliding-Rule.*

1. To find BC.

Set  $90^\circ$  on Ss to (the Co-Sine of AC)  $62^\circ 36\frac{1}{4}'$  on SR, then against (the Co-Sine of AB)  $51^\circ 45'$  on Ss is (the Co-Sine of BC)  $44^\circ 16'$  on SR; wherefore  $BC = 45^\circ 48'$ , as required.

2. To find the Angle B.

Set  $38^\circ 15'$  on Ss to  $90^\circ$  on SR; then against  $27^\circ 23\frac{1}{4}'$  on Ts is  $39^\circ 56'$  on TR, the Angle B required.

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### 3. To find the Angle C.

Set  $27^{\circ} 23'$  on Ss to  $90^{\circ}$  on SR ; and mark the Point on Ts against  $45^{\circ}$  on TR ; bring that Point to  $38^{\circ} 15'$  on TR ; then against  $45^{\circ}$  on TR you see  $59^{\circ} 44' = C$ , on Ts.

14. Case V. Given the Hypotenuse  $BC = 45^{\circ} 48'$ , and one Side  $AC = 27^{\circ} 23\frac{1}{4}'$  ; to find the rest,

### To find the Angle C.

The Tangent of . . . .  $BC = 45^{\circ} 48' = 10.012129$   
is to Radius . . . . .  $90^{\circ} 00' = 10.$

As the Tangent of  $AC = 27^{\circ} 23\frac{1}{4}' = 9.714544$

is to the Co-Sine of the Angle  $C = 59^{\circ} 44' = 9.702415$

15. *By the Scale and Compasses.*

Extend the Compasses from  $45^{\circ}$  to  $27^{\circ} 23\frac{1}{4}'$  in the Tangents, then set one Foot in  $45^{\circ} 48'$ , and pitch the other downward, where fix it while you move the other again from  $45^{\circ} 48'$  to  $45^{\circ}$ , then shall this last extent reach from Radius  $90^{\circ}$  to (the Co-Sine of C)  $30^{\circ} 16'$  in the Line of Sines ; whence the Angle  $C = 59^{\circ} 44'$ , as required.

16. *By the Sliding-Rule.*

Set  $45^{\circ}$  on TR to  $27^{\circ} 23\frac{1}{4}'$  on Ts, and mark the Point in Ts against  $45^{\circ} 48'$  on TR, then bring that Point to  $45^{\circ}$  on TR ; lastly, against  $90^{\circ}$  on SR, you have  $30^{\circ} 16'$  on Ss, which is the Co-Sine of  $C = 59^{\circ} 44'$ , as required.

*Note,* The Proportions for AB and the Angle B, contain nothing new or difficult either in Numbers or by Instrument, therefore shall give no Examples in them.

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17. Case VI. Given the Angles  $B=39^{\circ} 56'$ , and  $C=59^{\circ} 44'$ , to find the Sides AB, AC, BC.

1. To find AB.

As the Sine of . . . . .	$B=39^{\circ} 56'=9.807465$
is to the Co-Sine of . . .	$C=59^{\circ} 44'=9.702452$
So is Radius . . . . .	$90^{\circ} 00'=10.$

To the Co-Sine of the Side  $AB=38^{\circ} 15'=9.894987$

2. To find AC.

The Analogy is the same as for AB, by which you will find  $AC=27^{\circ} 23\frac{1}{4}'$  nearly.

3. To find BC.

As the Tangent of . . .	$C=59^{\circ} 44'=10.233905$
is to the Co-Tangent of	$B=39^{\circ} 56'=10.077213$
So is Radius . . . . .	$90^{\circ} 00'=10.$

to the Co-Sine of the Side  $BC=45^{\circ} 48'=9.843308$

18. *By Scale and Compasses.*

1. To find AB.

Extend the Compasses from  $39^{\circ} 56'$  to (the Co-Sine of C)  $30^{\circ} 16'$ , the same will reach from Radius  $90^{\circ}$  to (the Co-Sine of AB)  $51^{\circ} 45'$ , in the Line of Sines; therefore  $AB=38^{\circ} 15'$ . In the same manner you find  $AC=27^{\circ} 23\frac{1}{4}'$ .

2. To find BC.

Extend the Compasses from  $59^{\circ} 44'$  to (the Co-Tangent of B)  $50^{\circ} 04'$  in the Tangents, the same Extent will reach from Radius  $90^{\circ}$  to (the Co-Sine of BC)  $44^{\circ} 12'$  in the Sines; therefore  $BC=45^{\circ} 48'$ .



## Ch. XII. Solution of Spherical TRIANGLES. 195

19.

*By the Sliding-Rule.*

### 1. To find AB.

Set  $39^{\circ} 56'$  on Ss to (the Co-Sine of C)  $30^{\circ} 16'$  on SR; then against  $90^{\circ}$  on Ss is (the Co-Sine of AB)  $51^{\circ} 45'$ ; consequently  $AB = 38^{\circ} 15'$ .

In like manner, you find  $AC = 27^{\circ} 23\frac{1}{2}'$ .

### 2. To find BC.

Set  $59^{\circ} 44'$  on Ts to (the Co-Tangent of B)  $50^{\circ} 04'$  on TR; then against  $90^{\circ}$  on SR is  $44^{\circ} 12'$  (the Co-Sine of BC) on Ss. Wherefore  $BC = 45^{\circ} 48'$ .

20.

*Of Oblique Triangles.*

Oblique Spherical Triangles admit also of six different Cases, as follow.

Case I. Given two Angles,  $B = 34^{\circ} 30'$ , and  $D = 48^{\circ} 00'$ , and an opposite Side  $BC = 38^{\circ} 45'$ ; to find the other two Sides DC and BD; and Angle C.

Let fall a Perpendicular CA from the unknown Angle C to its opposite Side BD. Then is the oblique Triangle reduced to two Right ones, BAC and DAC. Then say by the first Analogy; To find the Angle C.

As the Co-Sine of . . . .  $BC = 38^{\circ} 45' = 9.892030$   
is to Radius . . . . .  $90^{\circ} 00' = 10.$

So is the Co-Tangent of  $B = 34^{\circ} 30' = 10.162866$

to the Tangent of . .  $BCA = 61^{\circ} 48'\frac{1}{2} = 10.270836$

Again, by the second Analogy, say;

C c 2

As

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Co. Ar.

As the Co-Sine of . . .  $B=34^{\circ} 30' = 0.084006$   
 is to the Sine of . . .  $BCA=61^{\circ} 48\frac{1}{2}' = 9.945159$   
 So is the Co-Sine of . . .  $D=48^{\circ} 00' = 9.825511$

to the Sine of the Angle  $DCA=45^{\circ} 41\frac{1}{2}' = 9.854676$

Now since the Perpendicular CA falls within the Triangle, 'tis plain the Sum of the two Angles now found makes the Angle sought, viz.  $BCA + DCA = BCD = 45^{\circ} 41\frac{1}{2}' + 61^{\circ} 48\frac{1}{2}' = 107^{\circ} 30'$ .

To find the Side BD.

21. By the first Analogy, say;

As Radius . . . . .  $90^{\circ} 00' = 10.$   
 to the Co-Sine of . . . .  $B=34^{\circ} 30' = 9.915994$   
 So is the Tangent of . . .  $BC=38^{\circ} 45' = 9.904497$

to the Tangent of the Side  $BA=33^{\circ} 29' = 9.820485$

Again, by the second Analogy, say;

Co. Ar.

As the Tangent of . . .  $D=48^{\circ} 00' = 9.954437$   
 is to the Tangent of . . .  $B=34^{\circ} 30' = 9.837134$   
 So is the Sine of . . . .  $AB=33^{\circ} 29' = 9.741698$   
 to the Sine of . . . .  $AD=19^{\circ} 57' = 9.533269$

Now 'tis evident, the Sum of the two Arches  $AB + AD = BD = 33^{\circ} 29' + 19^{\circ} 57' = 53^{\circ} 26'$ , the Side required.

22. To find the Side CD. Co. Ar.

As the Sine of the Angle  $D=48^{\circ} 00' = 0.128927$   
 is to the Sine of the Side  $BC=38^{\circ} 45' = 9.796571$   
 So is the Sine of the Angle  $B=34^{\circ} 30' = 9.753128$

to the Sine of the Side sought  $DC=28^{\circ} 30' = 9.678626$

Thus

## Ch.XII. *Solution of Spherical TRIANGLES.* 197

Thus the whole Triangle is resolved according to the Data of this Case.

23. Case II. Given two Angles  $B=34^{\circ} 30'$ , and  $C=107^{\circ} 30'$ ; and a Side included  $BC=38^{\circ} 45'$ ; to find the rest.

You let fall the Perpendicular  $CA$ , and find the Angle  $BCA=61^{\circ} 48\frac{1}{2}'$ , as in Art. 20. Then from the given Angle  $C=107^{\circ} 30'$ , take the Angle  $BCA=61^{\circ} 48\frac{1}{2}'$ , and there will remain the Angle  $DCA=45^{\circ} 41\frac{1}{2}'$ , by which you will find the Angle  $D=48^{\circ} 00'$ , according to the second Analogy of this Case. These Things being known, we may proceed

To find  $DC$ .

Co. Ar.

Thus, as the Co-Sine of  $DCA=45^{\circ} 41\frac{1}{2}'=0.155821$   
is to the Co-Sine of  $BCA=61^{\circ} 48\frac{1}{2}'=9.674329$   
So is the Tangent of . . .  $BC=38^{\circ} 45'=9.904491$

To the T. of the Side sought  $DC=28^{\circ} 30'=9.734641$

24. To find  $BD$ .

From the given Angle  $B$  let fall the Perpendicular  $BN$  to the unknown Side  $DC$  produced; then by the first Analogy, find the Angle  $CBN$ , thus;

As Radius . . . . .  $90^{\circ} 00'=10$ .  
to the Co-Sine of . . . .  $BC=38^{\circ} 45'=9.892030$   
So is the Tangent of  $CBN=72^{\circ} 30'=10.501278$

to the Co Tangent of  $CBN=22^{\circ} 00'\frac{1}{4}=10.393308$

Then by the second Analogy, say;

As the Co-Sine of . .  $DBN=56^{\circ} 30'\frac{1}{2}=0.258208$   
is to the Co-Sine of  $NBC=22^{\circ} 00'\frac{1}{4}=9.967151$   
So is the Tangent of . . .  $BC=38^{\circ} 45'=9.904491$

to the T. of the Side sought  $DC=53^{\circ} 26'=10.129850$



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25. Case III. Given two Sides  $BC=38^{\circ} 45'$ , and  $CD=28^{\circ} 30'$ , and an Angle opposite to one of them,  $B=34^{\circ} 30'$ ; to find the rest.

This Case being but the Converse of Case I. I shall not repeat the Examples.

Case IV. Given two Sides  $BC=38^{\circ} 45'$ , and  $BD=53^{\circ} 26'$ , and the Angle included  $B=34^{\circ} 30'$ ; to find the rest.

This Case contains nothing difficult if what goes before be well understood; the Analogies being plain and easy for the Operations, I shall leave them to the Learner's Exercise; and pass on to

26. Case V. Given all the three Sides,  $BC=38^{\circ} 45'$ ,  $DC=28^{\circ} 30'$ , and  $BD=53^{\circ} 26'$ ; to find the Angles.

To find the Angle C.

The Difference of the Legs containing the Angle sought, is  $BC-CD=AM=10^{\circ} 15'$ ; then  $\frac{BD+AM}{2}=31^{\circ} 50'\frac{1}{2}$ , and  $\frac{BD-AM}{2}=21^{\circ} 35'\frac{1}{2}$ ; wherefore, according to the Analogy for this Case in the Synopsis, proceed thus;

The Sine of the Side . . .  $BC=38^{\circ} 45'=9.796571$   
Add the Sine of the Side  $DC=28^{\circ} 30'=9.678663$

The Sum is the Log. of  $sBC \times sDC=19.475234$

Again, the Sine of  $\frac{BD+AM}{2}=31^{\circ} 50'\frac{1}{2}=9.722283$

Add the Sine of  $\frac{BD-AM}{2}=21^{\circ} 35'\frac{1}{2}=9.565836$

The Sum is  $s \frac{BD+AM}{2} \times s \frac{BD-AM}{2}=19.288119$

Then

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Then say ;

As . . . . .  $sBC \times sDC = 19.475234$   
is to the Square of Radius . . .  $Rq = 20.$

So is . . . .  $s \frac{BD+AM}{2} \times s \frac{BD-AM}{2} = 19.288119$

to the square Sine of  $\frac{1}{2}$  the } . .  $Sq_{\frac{1}{2}}C = 19.812885$   
Angle sought

The half whereof is . . . .  $s_{\frac{1}{2}}C = 53^{\circ} 44' = 9.906442$

Wherefore the Angle sought is  $C = 107^{\circ} 28'$ ; or  
more compendiously thus, by the Arithmetical Com-  
plement of the Sides BC, and DC.

27. The Sine of  $BC = 38^{\circ} 45' = 0.203429$  Co. Ar.

The Sine of  $DC = 28^{\circ} 30' = 0.321337$  Co. Ar.

The Sq. of Radius  $Rq = 20.000000$

The Sine of  $\frac{BD+AM}{2} = 31^{\circ} 50'\frac{1}{2} = 9.722283$

The Sine of  $\frac{BD-AM}{2} = 21^{\circ} 35'\frac{1}{2} = 9.565836$

The Sum of all . . . .  $sq_{\frac{1}{2}}C = 19.812885$

Therefore . . . .  $s_{\frac{1}{2}}C = 53^{\circ} 44' = 9.906442$

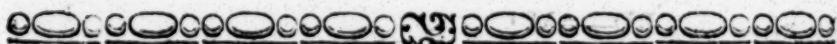
Consequently . . .  $C = 107^{\circ} 28'$ ; the same as before.

Having found one Angle, the others are easily found  
by the former Cases.

Note, By Case I. the Angle C was found to be  
 $107^{\circ} 30'$ , which is but  $2'$  different from what it is  
found by this Case; whence the Reader may observe  
the wonderful Certainty and Agreement of the most  
different Methods of Calculation, and from the great-  
est Diversity of Data.

28. The sixth Case is but the same with this, if the three Angles given be changed into Sides, taking for the greatest Angle in that Triangle and greatest Side in this, their Supplements to 180 Degrees.

29. Having thus shewn the best Methods, both for *Exactness* and *Ease*, of resolving the several Cases of *Right* and *Oblique-angled Triangles*, I shall leave the Application thereof to the various Problems of *Astronomy*, *Geography*, *Dialling*, *Orthodromics*, &c. for the Learner's own Recreation; as being in it self very easy, if what is here taught be understood; and also because I have both demonstrated the *Theory*, and very closely applied the *Doctrine* of *Spherical Trigonometry*, in the 2d Vol. of my *Young Trigonometer's Guide*, printed for Mr. J. Noon, at the *White Hart*, in *Cheapside*; and which I recommend to all unacquainted with, and who would have a good Notion of the noble and most useful Art of *Trigonometry*.



### C H A P. XIII.

MERCATOR'S SAILING performed by the CANON of LOGARITHMIC TANGENTS, without the MERIDIONAL PARTS.

1. **T**HE Property of *Mercator's Chart* is its having the Degrees of Latitude increased in the same Proportion as a Degree of Longitude decreases from the Equator to the Pole. Which Proportion is that of *Radius* to the *Co-Sine* of *Latitude*; or, of the *Secant* of the *Latitude* to the *Radius*; which is thus demonstrated.

2. Let AEB (Fig. VIII.) be a Sector in the Plane of the Equator, made by the Intersections of the Planes of two Meridians therewith, viz. AE, and B.

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Ch. XIII. of LOGARITHMIC TANGENTS. 201

BE, whose Inclination, or Angle BEA, that is, the Arch of the Equator AB is = 1 Degree. Also let EC be the Radius of any Parallel of Latitude; then shall DC be an Arch in that Parallel similar to (AB) one Degree of Longitude in the Equator, or it is that Degree diminished. Now (from the Elements) the Arch AB is to the same diminish'd in the given Parallel DC, as the Radius of the Equator AE to the Radius of that Parallel CE; but since AE=EC (AC being the given Latitude) and because of similar Triangles EcC and ESA, therefore Ec (EA) : EC :: ES : EA :: AB : DC. But EC=E c, the Co-Sine of the Latitude AC, and ES the Secant thereof; therefore, &c. which was to be demonstrated.

3. And because in *Mercator's Projection* or *Chart*, the Meridians and Parallels are all represented by *parallel Right-Lines*, the Arch CD in every Parallel is ever equal to that in the Equator AB; therefore that the true Proportion between Longitude and Latitude might be preserved on this Chart, as on the Globe it self, 'twas necessary the several Degrees of Latitude should be such as belong to those Circles whose Radii are severally equal to the Secants of those Latitudes.

4. Let A = the Arch of one Degree in any Latitude; a, b, c, &c. = the several enlarg'd Degrees on the Meridian; R = Radius; and S, s, s, &c. the Secants of those Latitudes. Then it will

$$\left. \begin{array}{l} A1st : a :: R : S. \\ A2d : b :: R : s. \\ A3d : c :: R : s, \text{ \&c.} \end{array} \right\} \begin{array}{l} \text{Therefore we shall have} \\ 3A : a+b+c :: 3R : S+s+s. \end{array}$$

That is, the Sum of the Secants of 1, 2, 3 Degrees is always equal to the Distance of the Parallel of those three enlarg'd Degrees from the Equator. And consequently by a continual Addition of the Secants of  $\frac{1}{2}^{\circ}$ ,  $1^{\circ}$ ,  $2^{\circ}$ ,  $3^{\circ}$ , &c. or their Doubles  $1^{\circ}$ ,  $3^{\circ}$ ,  $5^{\circ}$ ,  $7^{\circ}$ , &c. the Line or Table of *Meridional Parts* may be made; and by some has been made in this manner

D d

for

for every Minute of the Quadrant; but a truer way is that delivered in Chap. X. of the Theory, which see.

5. If then you draw any Line AB (Fig. X.) to represent an Arch of the Equator, as suppose 60 Degrees; and on every tenth Degree be erected Perpendiculars, these shall be the *Meridians*; on which if the Degrees of Latitude, enlarg'd in the Manner and Proportion above describ'd, be set off; and those Divisions join'd by Right-Lines, representing the Parallels of Latitude, *Mercator's Chart* will be constructed for the given Longitude and Latitude; and a *Meridian* thus graduated, is what is called the *Nautical* or *Meridian Line*. This Chart, as here drawn, includes  $60^{\circ}$  Longitude, and  $80^{\circ}$  Latitude, that the Reader may have a perspicuous Idea thereof.

6. Now suppose F be the Latitude whence you sail, and C the Latitude you arrive to, on the Rhumb FC; 'tis plain there will be formed a Right-angled plain Triangle FCD, wherein FD is the *enlarg'd Difference of Latitude*, and CD the *true Difference of Longitude*, and the Angle CFD the *Course* or *Rhumb*; and therefore any two of these being given, the rest are found by the Analogies of plain Triangles before-going.

7. Since every Degree is equal to 60 Minutes or Nautical Miles, let the *proper Difference of Latitude* be reduc'd to these Parts, and set (off the same Scale as AB was laid down by) from F to E, and draw EK parallel to DC; so shall there be form'd another Right-angled plain Triangle FEK; in which FE is the *proper Difference of Latitude*; EK the *Departure*; KF the *Distance* sail'd on the Rhumb or Course EFK. Any two of which Parts being given, the others are to be found as aforesaid.

8. And these two Triangles FCD, FKE, comprehending all the Particulars of *Mercator's Sailing*; 'tis obvious enough how they are all resolv'd, either by

### Ch.XIII. of LOGARITHMIC TANGENTS. 203

by *Projection* or *Trigonometrical Calculation*; but my Purpose is here to shew, that without those *common Methods*, they are all to be resolv'd by the Canon of Logarithm-Tangents only; in order to which, the following Articles must be premis'd and duly observ'd, *viz.*

9. First, That it has been shewn, the *Nautical* or *Meridian Line*, or Scale of *Mercator's Chart*, is no other than a Scale of Logarithm-Tangents of the Half-Complements of the Latitude. Secondly, that such Logarithm-Tangents of Mr. *Brigg's* Form, (or those in common Use) are a Scale of the Differences of Longitude upon the Rhumb, which makes an Angle of  $51^{\circ} 38' 9''$  with the Meridian. Thirdly, the Differences of Longitude on differing Rhumbs, are as the Tangents of the Angles of those Rhumbs with the Meridians; as is evident from the Triangle FCD (Fig. X.) And fourthly, that the Logarithm-Tangent of the Angle  $51^{\circ} 38' 9''$ , *viz.* 10.1015104 is a constant Factor in these Computations. On these Premises 'tis easy to operate the Propositions of *Mercator's Sailing*, as follows.

10. In Fig. 11. let F represent the *Lizard's Point*, whose Latitude North  $DF=49^{\circ} 55'$ ; also let K represent *Barbadoes* in Latitude North  $13^{\circ} 10'=CK$ ; then is  $FE=$  the Difference of Latitude;  $CD$  the Difference of Longitude; and  $EFK$  the Angle of the Rhumb,  $FK$  with the Meridian. Lastly, let  $FO$  be the Rhumb making an Angle of  $52^{\circ} 38' 9''$  with the Meridian; or that on which the Difference of Longitude  $QD$  is the Difference of the Logarithms of the Tangents of half the Complements of the Latitude  $PF$ ,  $PK$ , or  $PO$ : Then,

11. Case I. Given the Latitude of the *Lizard*,  $49^{\circ} 55'$  N. and that of *Barbadoes*  $13^{\circ} 10'$  N. and their Difference of Longitude  $53^{\circ} 00'=$

D a 2

3180



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3180 Nautic Miles : I demand the Course and Distance sail'd ?

$$\left\{ \begin{array}{l} \text{The Lat.} \\ \text{CK} = 13^{\circ} 10' \\ \text{DF} = 49^{\circ} 55' \end{array} \right\} \left\{ \begin{array}{l} \text{Comp.} \\ \text{PK} = 76^{\circ} 50' \\ \text{PF} = 40^{\circ} 05' \end{array} \right\} \left\{ \begin{array}{l} \text{the Halfs.} \\ 38^{\circ} 25' \\ 20^{\circ} 2\frac{1}{2}' \end{array} \right\}$$

Then the Log. Tangent of  $\left\{ \begin{array}{l} 38^{\circ} 25' = 9.899308 \\ 20^{\circ} 2\frac{1}{2}' = 9.562048 \end{array} \right\}$

The Difference of which is . . . QD = 3372,60

(Note, The four first Figures are *Integers*, according to the Theory)

Then say, for the Course KFE ;

As the Diff. of Log. Tang. QD = 3372,6 = 3.527965

to the given Diff. of Long. CD = 3180 = 3.502427

So is the const. Tang. of OFE = 51° 38' 9" = 10.101510

3.603937

to the Tangent of the } KFE = 49° 59' 10" = 10.075972  
Course sought

Secondly, For the Distance FK.

The Difference of Latitude is 49° 55' — 13° 10' = 36° 45' = 2205 Miles ;

Say, as Radius . . . . . 10.

to the Diff. of Lat. . . . EF = 2205 = 3.343409

So is the Secant } KFE = 49° 59' 10" = 10.191807  
of the Course

to the Distance sail'd FK = 3429,38 = 3.535216

By this Proposition you estimate the Course a Ship must steer, and the Distance of her Port.

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12. Case II. Given the Latitude of the *Lizard*  $49^{\circ} 55'$  N. and of *Barbadoes*  $13^{\circ} 10'$  N. and the Course  $49^{\circ} 59' \frac{1}{2}$ , to find the Difference of Longitude, and Distance sailed.

Things being prepared as before, say ;

As the Log. Tang. of  $OFE=51^{\circ} 38' 9''=10.101510$

to the Tang. of }  $KFE=49^{\circ} 59' 10''=10.075972$   
the Course

So is the Diff. of the Log. }  $QD=3372,6=3.527965$   
Tang. of the  $\frac{1}{2}$  Comp. }  
of the Latitudes

3.603937

to the Diff. of Long. required  $CD=3180=3.502427$

Which converted into Degrees makes  $53^{\circ} 00'$ . And so much is the Difference of Motion or Time between these two Places. The Distance on the Rhumb will be found  $3429,38=FK$ , as before.

13. Case III. Given the Latitude of the *Lizard*  $49^{\circ} 55'$  N. the Distance sailed  $3429,38$  Miles on a Course  $49^{\circ} 59' 10''$  Southwesterly, 'tis required to find the Latitude and Longitude of the Place to which the Ship is arrived.

As Radius . . . . . 10.

to the Distance sailed  $FK=3429,38=3.535216$

So is the Co-Sine of }  $KFE=49^{\circ} 59' 10''=9.808193$   
the Course

to the Diff. of Latitude  $FE=2205=3.343409$

Then from the given Lat. of the *Lizard*  $49^{\circ} 55'$

Subd. the Diff. of the Lat.  $2205=36^{\circ} 45'$

There remains the Lat. sought . . .  $=13^{\circ} 10'$

And

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And having thus obtained the Latitudes, the Difference of Longitude will be found, as *per* Case II. to be 3180 Miles or 53 Degrees; and so the Operation needs not to be repeated.

14. If it so happen that the Ship passes the *Equator*, and consequently has one Latitude *North*, the other *South*; then observe two things; first, that both the Complements of the Latitudes are to be estimated from the same Pole of the World. And therefore, secondly; suppose you sail from a *Northern* to a Southern Latitude, you must add  $90^{\circ}$  to the *former*, and subtract the latter from  $90^{\circ}$ ; then subtract this Sum and Remainder from  $180^{\circ}$ , and take the Difference of the Logarithm-Tangents of half the Remainders, as before.

15. Case IV. Suppose I sail from Latitude  $48^{\circ} 30' N.$  to Latitude  $23^{\circ} 45' S.$  on a Course  $43^{\circ} 50'$  Southwesterly; required the *Difference, Longitude, and Distance* sailed?

Then  $48^{\circ} 30' + 90 = 128^{\circ} 30'$ ; and  $90 - 23^{\circ} 45' = 66^{\circ} 15'$ . And

$$\begin{array}{l} 180 - 66^{\circ} 15' = 113^{\circ} 45' \\ 180 - 128^{\circ} 30' = 51^{\circ} 30' \end{array} \left. \vphantom{\begin{array}{l} 180 - 66^{\circ} 15' \\ 180 - 128^{\circ} 30' \end{array}} \right\} \frac{1}{2} \left\{ \begin{array}{l} 56^{\circ} 52\frac{1}{2}' = 10.185410 \\ 25^{\circ} 45' = 9.683356 \end{array} \right.$$

The Diff. of those Tangents is . . . 5020,54

Therefore say, for the Difference of Longitude;

As the Log. Tangent of  $51^{\circ} 38' 9'' = 10.101510$

is to the Diff. of the Log. Tang. 5020,54 = 3.700750

So is the Tang. of the Course  $43^{\circ} 50' = 9.982309$

3.683059

to the Diff. of Long. required  $3815,5 = 3.581549$

That is, in Degrees,  $= 63^{\circ} 35\frac{1}{2}'$ .

To



# Ch. XIII. of LOGARITHMIC TANGENTS. 207

To find the Distance fail'd ; say,

As Radius . . . . . 10.  
to the Sum of the Lat.  $72^{\circ} 15' = 4335 = 3.636989$   
So is the Secant of the Course  $43^{\circ} 50' = 10.141849$   
to the Dist. fail'd in Miles,  $6009,5 = 3.778838$

16. Thus you have all the *Practical Cases* of *Mercator's Sailing* performed by the *Canon of Logarithm-Tangents* only, without the *Meridional Parts* or *Chart*, as in the common Way. And since this is the most exact, and natural Method of Navigation (next to the *Globular Chart* it self) and wholly resolvable by *Logarithms*, it adds not a little to the (before invaluable) Estimation of those excellent Numbers ; and renders their Use to Navigators more necessary than before.



## C H A P. XIV.

Of the MENSURATION of SUPERFICIES and SOLID BODIES by LOGARITHMS.

1. **A**mongst the Variety of Methods for measuring the *Surfaces* and *Solidity* of *Bodies*, I intend here to shew the Excellency of that by *Logarithms* ; which may justly be allow'd the *Preference* to all others in point of *Ease* and *Brevity*, Advantages none of the least in common Estimation. And since these Operations consist altogether in *Multiplication* and *Division*, I need not here repeat them by the *Instruments*, as having already largely shewn the Manner thereof in a *Chapter* particularly on that Head.

2.

To

## 208 MENSURATION of Superficies and

### 2. To measure a SQUARE, Fig. XII.

The Logarithm of the Side multiplied by 2, gives the Logarithm of the *Area* or *superficial Content*.

Examp. Let the Side of the Square be  $AB=31,57$ .

Then the Logarithm of  $AB=31,57=1.499275$   
which multiply by . . . . . 2

The Product is the }  $ABCD=996,66=2.998550$   
Area of the Square }

### 3. To measure a PARALLELOGRAM, Fig. XIII.

The Sum of the Logarithms of the Length and Breadth is the Logarithm of the Area.

Examp. The Length  $AB=41,5$  and Breadth  $BC=31,57$ .

To the Log. of the Length  $AB=41,5=1.618048$   
add the Log. of the Breadth  $BC=31,57=1.499275$

The Sum is the Log. }  $ABCD=1312,155=3.117323$   
of the Parallelog. }

### 4. To measure a RHOMBUS ABCD, Fig. XIV.

The Sum of the Logarithms of a Side and the Perpendicular Height, is the Logarithm of the Area.

Examp. Let the Side  $AB=15,5$ ; and the Perpendicular  $BE=13,42$ .

Then to the Log. of the Side  $AB=15,5=1.190332$   
add the Log. of the Perpend.  $BE=13,42=1.127752$

The Log. of the Area of }  $\dots = 208,01 = 2.318084$   
the Rhombus }

5. *To measure a* RHOMBOIDES, ABCD, *Fig. XV.*

The Sum of the Logarithm of the longest Side, and perpendicular Height is the Logarithm of the Area.

*Exam.* Let the longest Side  $AB=19,5$ , and perpendicular Height  $BE=6,07$

Then to the Log. of the Side  $AB=19,5=1.290035$   
Add the Log. of the Perpend.  $BE=6,07=0.783189$

The Log. of the Area of the }  
Rhomboides . . . . . }  $=118,365=2.073224$

6. *To measure a* TRIANGLE ABC. *Fig. XVI.*

The Sum of the Logarithm of the Base, and of half the perpendicular Height (or *vice versa*) is the Logarithm of the Area.

*Exam.* Let the Base  $BC=65,25$ ; and the perpendicular Height  $AG=21,5$ ; then  $\frac{1}{2} AG=10.75$ .  
Therefore,

To the Log. of the Base . . .  $BC=65,25=1.814580$   
Add the Log. of half the Perp.  $\frac{1}{2} AG=10,75=1.031408$

The Log. of the Area . . . .  $=701,4375=2.845988$

7. *To measure a* CIRCLE ABCD. *Fig. XVII.*

In order to this, let  $\begin{cases} D=\text{the Diameter.} \\ P=\text{the Periphery.} \\ A=\text{the Area.} \end{cases}$

Then the Rules or Theorems for finding those several Parts are as follows ;

E e

Theo.



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Theo. I.  $3.1416D=P$ . Th. II.  $0.7854DD=A$ .  
 Theo. III.  $0.3183P=D$ . Th. IV.  $0.07957PP=A$ .  
 Theo. V.  $\sqrt{1.2732A=D}$ . Th. VI.  $\sqrt{12.5664A=P}$ .

8. Therefore the Diameter AB being given, suppose = 20,15, to find the Periphery P? *per* Theor. I.

Thus, to the Log. of the }  $AB=20.15=1.304275$   
 Diameter . . . . . }  
 Add the Log. of the con- }  $3.1416=0.497151$   
 stant Number . . . . . }

The Log. of the Periphery  $P=63,303, \&c.=1.801426$

9. Given the Periphery of a Circle  $P=63,303, \&c.$  to find the Diameter D? *per* Theor. III.

To the Log. of the Pe- }  $P=63.303, \&c.=1.801426$   
 riphery . . . . . }  
 Add the Log. of the }  $0.3183, \&c.=9.502837$   
 constant Number . . }

The Log. of the Diameter D or  $AB=20.15=1.304263$

10. Having the Diameter given, suppose = 20,15, to find the Area A of the Circle? *per* Theor. II.

The Log. of the Diameter ..  $AB=20.15=1.304275$   
 Multiply by . . . . . <sup>2</sup>

The Product is the Log. of . . . . .  $DD=2.608550$   
 To which add the Log. }  $0.7854=9.895091$   
 of the constant Numb. }

The Log. of the A- }  $ACBD=318,89, \&c.=2.503641$   
 rea of the Circle }

11. Or thus supposing the Periphery given, = 63,303,  
*Ec. per Theor. IV.*

To twice the Log. of . . . . .  $P=63.303=\left\{\begin{array}{l} 1.801426 \\ 1.801426 \end{array}\right.$

Add the Log. of the constant }  
 Number . . . . . }  $0.07957=8.900749$

The Sum is the Log. of }  
 the Area . . . . . }  $A=318,89, Ec.=2.503601$

12. To measure the SECTOR of a CIRCLE ACB,  
*Fig. XVIII.*

The Sum of the Logarithms of the *Radius*, and  
 the *Arch* (or of the Arch and half the *Radius*) is  
 the Logarithm of the *Area*.

*Examp.* Suppose the Radius  $AC=12.36$ , and the  
 Arch  $AB\ 10,12$ ; then  $\frac{1}{2}AB=5,11$ .

Then to the Log. of Radius  $AC=12,36=1.092018$

Add the Log. of . . . . .  $\frac{1}{2}AB=5,11=0.707570$

The Log. of the Area }  
 of the Sector . . . . . }  $ACB=63,2596=1.799588$

13. To measure the SEGMENT of a Circle, as  
 AFBG, *Fig. XIX.*

The best way is to find the Centre C; as by this

Theorem, *viz.*  $\overline{FB}^2 - FG = N$ , then  $\frac{N+FG}{2} = CG$ ;

whence C is given; and finding the Area of the  
 whole Sector ACBG, (as *per* last Article.) and the  
 Area of the Triangle ABC, (*per* Art. 6.) if the latter  
 Area be subducted from the former, it will leave  
 the Area of the Segment required, AFBG.

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### 14. To measure a SPHERICAL TRIANGLE, ABC. *Fig. XX.*

From the Sum of the three Angles A, B, C, take 180 Degrees; then from the Logarithm of the Remainder subduct the Logarithm of the constant Number 720; to that Remainder add the Logarithm of the Superficies of the whole Sphere; the Sum shall be the Logarithm of the Area required.

*Exam.* Let the  $\left\{ \begin{array}{l} A=37^{\circ} \ 30' \\ B=92 \ 30 \\ C=60 \ 45 \end{array} \right.$   
Angles be

$$\begin{array}{rcl} \text{Their Sum is} & \dots & 190^{\circ} \ 45' \\ \text{Subtract} & \dots & 180 \ 00 \\ \text{There remains} & \dots & 10^{\circ} \ 45' = 10,75 = 1.031403 \\ \text{Subduct the Logarithm of} & \dots & 720 = 2.857332 \end{array}$$

$$\hline 8.174076$$

Suppose the Superficies of the }  
Sphere  $\dots \dots \dots$  }  $= 1257,3 = 3.099439$

The Sum is the Log. of the }  
Area of the Triangle.. }  $ABC = 23,623 = 1.273515$

### 15. To measure an ELLIPSIS, as ACBD, *Fig. XXI.*

The Sum of the Logarithms of the Transverse Diameter AB, the Conjugate Diameter CD, and the constant Number 0.7854; is the Logarithm of the Area.

*Exam.* Let  $AB=61,6$ , and  $CD=44,4$ .

Then add the Logarithms of  $\left\{ \begin{array}{l} AB=61,6=1.789581 \\ CD=44,4=1.647383 \\ N.o. 7854=9.895092 \end{array} \right.$

The Log. of the Area }  
of Ellipsis  $\dots \dots$  }  $= 2148,1004, \&c. = 3.33205$



16. To measure a PARABOLA, as ACD, Fig. 22.

From the Sum of the Logarithms of the Base AB, the perpendicular Height CD, and the Number 2, subtract the Logarithm of 3; the Remainder is the Logarithm of the Area.

*Exam.* Let AB=61,6, and CD=44,4; as in the Ellipse.

Then add the Logarithms of  $\left\{ \begin{array}{l} AB=61,6=1.789581 \\ CD=44,4=1.647383 \\ \text{and N. 2.}=0.301030 \end{array} \right.$

3.737994

Subtract the Logarithm of . . . . . 3=0.477121

3.260873

The Log. of the Area required=1823,36=3.260873

17. To measure any REGULAR POLYGON, Fig. XXIII.

In order to this the following Table will be very expedient.

	Sides.	Names.	Numbers.
A Table for the more ready find- ing the A- rea of any Regular Polygon.	5	Pentagon.	1.72048
	6	Hexagon.	2.59808
	7	Heptagon.	3.63896
	8	Octagon.	4.82843
	9	Enneagon.	6.18183
	10	Decagon.	7.69421
	11	Endecagon.	8.51425
	12	Dodecagon.	9.33012

Then the Sum of double the Logarithm of the Side of the Polygon, and of the Number in the Table proper to it, is the Logarithm of the Area.

*Exam.* Let the *Pentagon* ABCDE be proposed, and let its Side be AB=14,6.

Then

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Then twice the Logarithm of  $AB=14,6$   $\left\{ \begin{array}{l} 1.164353 \\ 1.164353 \end{array} \right.$   
 Add the Log. of the *tabular Numb.*  $1.72048=0.235649$   
 The Log. of the Area required  $=366,74=2.564355$

In the same manner you find the Area of any other Polygon mention'd in the Table.

## 18. Of the MENSURATION of SOLIDS.

To measure a CUBE ABCDFGE, *Fig. XXIV.*

Three times the Logarithm of the Side is the Logarithm of the Solidity.

*Exam.* Let the Side  $AB = 31.57 = 1.499275$   
 Multiply by ..... 3

The Log. of the Solidity ...  $31464,81=4.497825$

## 19. To measure a PARALLELOPIPEDON AD, *Fig. XXV.*

The Sum of the Logarithms of the *Breadth*, *Depth*, and *Length*, is the Logarithm of the Solidity.

*Exam.* Let the Width  $AB=21,56$ ; the Length  $AG=31,57$ ; and Depth  $GF=9,03$ .

Then add the Logarithms of  $\left\{ \begin{array}{l} AG=31,57=1.499275 \\ AB=21,56=1.333649 \\ GF=9,03=0.955688 \end{array} \right.$

The Log. of the Solidity }  $=6146,2623=3.788612$   
 required .....

## 20. To measure a PRISM ABCDEF, *Fig. XXVI.*

First find the *Area* of the *Base*, whether a *Triangle*, *Square*, &c. Then the Sum of the Logarithms of the

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the said *Area*, and Length of the *Prism*, is the Logarithm of its *Solidity*.

*Exam.* Suppose a *Prism* of a *triangular Base*, as in the Figure, then let its Area be  $ABC=701,4375$ , and its Length  $BD=70,15$ .

Then add the Logarithms of both . . .  $\left\{ \begin{array}{l} ABC=701,4375=2.845988 \\ BD=70,15=1.846028 \end{array} \right.$

The Log. of the Solidity  $=49205,84, \&c.=4.692016$

21. To measure a PYRAMID ABCDE. *Fig. XXVII.*

First find the *Area* of its *Base*, whether *triangular*, *quadrangular*, &c. then the Sum of the Logarithms of the Area of the Base, and  $\frac{1}{3}$  of the *perpendicular Height*, is the Logarithm of the *Solidity*.

*Exam.* Let the Prism have a *quadrangular Base*  $ABCD=342,25$ ; and the perpendicular Height  $GE=180$ ; then  $\frac{1}{3}GE=60$ .

Therefore add the Logarithms of . . . . .  $\left\{ \begin{array}{l} ABCD=342,25=2.534343 \\ \frac{1}{3}GE=60=1.778151 \end{array} \right.$

The Log. of the Solidity . . . . .  $=20535=4.312494$

22. To measure a CYLINDER ACBDEF, *Fig. XXVIII.*

The Sum of the Logarithms of the *Area* of its Base, and of the *Height*, is the Logarithm of the *Solidity*.

*Exam.* Let the Area of the Base be  $380,15=AGBH$ ; and the Height  $BD=50,05$ .

Then add the Logarithms of . . . . .  $\left\{ \begin{array}{l} AGBH=380,15=2.579955 \\ BD=50,05=1.699404 \end{array} \right.$

Log. of the Solidity . . .  $=19026,5075=4.279359$



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22. To measure a CONE AEBFD, Fig. XXIX.

The Sum of the Logarithms of the *Area* of the *Base*, and  $\frac{1}{3}$  of the perpendicular Height, is the Logarithm of the *Solidity*.

*Exam.* Let the Area of the Base be  $100.75 = AEBF$ ; and the perpendicular Height  $CD = 19.95$ ; and therefore  $\frac{1}{3}CD = 6.65$ .

Then add the Logarithms of  $\left. \begin{array}{l} AEBF = 100.75 = 2.003245 \\ \frac{1}{3}CD = 6.65 = 0.822822 \end{array} \right\}$

The Sum is the Log. of the Solidity  $\left. \begin{array}{l} \text{The Sum is the Log. of the} \\ \text{Solidity} \end{array} \right\} 669.9875 = 2.826067$

23. To measure the FRUSTUM of a Pyramid. Fig. XXX.

If it be a square  $\left\{ \begin{array}{l} D = \text{Side of the greater Base AB.} \\ d = \text{Side of the lesser Base EF.} \end{array} \right.$  then put  $\dots D - d = x$  and  $H = \text{the Height GO.}$

Then we have this Theorem  $Dd + \frac{1}{3}xx \times H = \text{the Solidity.}$

*Exam.* Suppose  $D = 50$ ,  $d = 21$ ; and  $H = 105.6$ ; then  $D - d = x = 29$ .

Then add the Logarithms of  $\left\{ \begin{array}{l} D = 50 \dots = 1.698970 \\ d = 21 \dots = 1.322219 \\ H = 105.6 = 2.023664 \end{array} \right.$

The Logarithm of  $\dots DdH = 110880 = 5.044853$

Again, add the Logarithms of  $\left\{ \begin{array}{l} \frac{1}{3}xx = 280, x = 2,447673 \\ H = 105.6 = 2.023664 \end{array} \right.$

The Logarithm of  $\dots \frac{1}{3}xxH = 29603.2 = 4.471337$   
To which add  $\dots DdH = 110880$

The Sum is  $\dots Dd + \frac{1}{3}xxH = 140483.2 = \text{the Solidity required.}$

24. But if the Bases of the Fruustum be *triangular*; the Theorem is  $\overline{Dd} + \frac{1}{3}xx \times 0,433H = \text{the Solidity}$ . Again, if the Base be any of the regular Polygons; then put  $N =$  the Number in Table at Art. 17. proper to the Polygon, and the Theorem will be  $\overline{Dd} + \frac{1}{3}xx \times NH = \text{the Solidity}$ .

25. To measure the Fruustum of a RIGHT CONE.  
*Fig. XXXI.*

Let  $D =$  the Diameter of the great Base  $AB$ , and  $d =$  the Diameter  $CD$  of the lesser Base;  $D - d = x$ , and  $H =$  the perpendicular Height, as before; then the Theorem for the Solidity will be  $\frac{\overline{Dd} + \frac{1}{3}xx \times H}{1,27323}$ , or thus  $\overline{Dd} + \frac{1}{3}xx \times 0,7854H = \text{Solidity}$ .

*Exam.* Let  $AB = 16$ ,  $CD = 12$ , and  $GO = 9$ : then  $D - d = x = 4$ ; and  $\frac{1}{3}xx = 5,3$ : also  $\overline{Dd} = 192$ ; and so  $\overline{Dd} + \frac{1}{3}xx = 197,3$ .

Then add the Logarithms of . . . . .  $\left\{ \begin{array}{l} \overline{Dd} + \frac{1}{3}xx = 197,3 = 2.295199 \\ 0.7854 = 9.895091 \\ H = 9 = 0.954242 \end{array} \right.$

The Log. of the Solidity . . =  $1394,8704 = 3.144532$

26. To measure a GLOBE or SPHERE  $ACBD$ ,  
*Fig. XXXII.*

The Sum of the triple Logarithm of the Diameter, and the Logarithm of the constant Number  $0.5236$  is the Logarithm of the Solidity of the Sphere.

*Exam.* Suppose the Diameter of a Sphere  $AB = 50,37$ .

F f

Then

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$$\text{Then add the Logarithms } \left\{ \begin{array}{l} AB=50,37=1.702172 \\ 1.702172 \\ 1.702172 \\ 0,5236=.9.718999 \end{array} \right.$$

$$\text{The Log. of the Solidity of the Sphere } \dots\dots\dots \} = 66913,8 = 4.825515$$

27. To measure the Superficies of a Sphere.

The Sum of Logarithm of double the Diameter, and the constant Number 3.1416, is the Logarithm of the Superficies of the Sphere.

*Exam.* Let the Diameter  $AB=50.37$ .

$$\text{Then add the Logarithms of } \left\{ \begin{array}{l} AB=50.37=1.702172 \\ 1.702172 \\ 3,1416=0.497151 \end{array} \right.$$

$$\text{The Log. of the Superficies } \dots\dots\dots 7970,7 = 3.901495$$

28. To measure the SEGMENT of a Sphere,  
*Fig. XXXIII.*

Let  $\left\{ \begin{array}{l} D = \text{The Axis or Diameter of the Sphere } CD. \\ C = \text{Half the Diam. of the Segment's Base } EB. \\ H = \text{The Height of the Segment } ED. \end{array} \right.$

Then we have the two following Theorems for finding the Solidity.

$$\text{viz. } \left\{ \begin{array}{l} \text{Theor. I. } \frac{3CCH + H^3}{\dots} \times 0.5236 = \text{the Solidity} \\ \text{Theor. II. } \frac{3DH^2 - 2H^3}{\dots} \times 0.5236 = \text{the Solidity} \end{array} \right.$$

29. To measure a SPHEROID ACBDA,  
*Fig. XXXIV.*

The Sum of the double Logarithm of the Lesser the Logarithm of the Greater Diameter, and the constant



*constant Number* 0.5236, is the Logarithm of the solid Content of the *Spheroid*.

*Exam.* Let the lesser Diameter  $CD=33$ ; and the greater Diameter  $AB=55$ .

Then the double Log. of ...  $CD=33 \left\{ \begin{array}{l} =1.518514 \\ =1.518514 \end{array} \right.$

The Logarithm of ...  $AB=55=1.740363$

The Logarithm of ...  $0.5236=9.719000$

The Log. of the Solidity ..  $=31361,022=4.496391$

30. To measure the SEGMENT of a SPHEROID.

As the Solidity of the Sphere AFBE is to the Solidity of its Segment AGK; so is the Solidity of the Spheroid ACBD to its like Segment AHI.

31. To measure a PARABOLIC CONOID ACBD, *Fig. XXXV.*

The Sum of the double Logarithm of the Diameter of the Base, the Logarithm of the Height, and *constant Number* 0.3927; is the Logarithm of the Solidity of the Conoid.

*Exam.* Let the Diameter of the Base  $AB=55$ ; and its Height  $CD=33$ .

Then the double Log. of ...  $AB=55 \left\{ \begin{array}{l} =1.740363 \\ =1.740363 \end{array} \right.$

The Logarithm of ...  $CD=33=1.518514$

The Logarithm of ...  $0.3927=9.594061$

The Logarithm of the Solidity  $39201,4=4.593301$

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32. To measure the FRUSTUM of a PARABOLIC CONOID, *Fig. XXXVI.*

To this end, let  $\begin{cases} D = \text{Diameter of the greater Base AB.} \\ d = \text{Diameter of the lesser Base CD.} \\ H = \text{the Height of the Frustum FE.} \end{cases}$

Then we have this Theorem . . . .  $\{ DD + dd \times 0.3927H = \text{the Solidity.} \}$

33. To measure a PARABOLIC SPINDLE, *Fig. XXXVII.*

The Sum of the double Logarithm of its Thickness, the Logarithm of its Length, and the Logarithm of the constant Number 0.41888, is the Logarithm of the Solidity of the Spindle.

*Exam.* Let the Diameter of its greatest Circle, or Thickness  $AB = 43.45$ ; and the Length  $CD = 50.075$ .

Then the double Log. of  $AB = 43.45 \begin{cases} = 1.637990 \\ = 1.637990 \end{cases}$

The Logarithm of . . . . .  $CD = 50.075 = 1.699621$

The Logarithm of . . . . .  $0.41888 = 9.622090$

The Log. of the Solidity . .  $= 39599.6 = 4.597691$

34. To measure any of the FIVE REGULAR BODIES, *Fig. XXXVIII.*

For this purpose the following Table is necessary.

Names.	Superficies.	Solidity.
Tetrahedron	1.73205	0.11785
Octahedron	3.46410	0.47140
Hexahedron	6.00000	1.00000
Icosahedron	8.66025	2.18169
Dodecahedron	20.64573	7.66312

*Ex.*

*Exam.* Suppose the Side  $AB=12$ , of the *Icosahedron*,  
Fig. XXXVIII.

The Sum of the Logarithm of the *tabular Number*, and double Logarithm of the Side is the Logarithm of the Superficies; and the Sum of the Logarithm of the *tabular Number*, and triple Logarithm of the Side, is the Logarithm of the Solidity.

Thus the double Log. of . . .  $AB=12$   $\left\{ \begin{array}{l} =1.079181 \\ =1.079181 \end{array} \right.$

The Log. of the *tab. Number* ..  $8,66025=0.937530$

The Log. of the *Superficies*  $=1247,0688=3.095892$

Again the Logarithm of . . .  $AB=12=1.079181$   
3

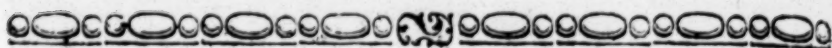
Add the Log. of the *tab. Numb.*  $2.18169=0.338793$   
3.237543

The Log. of the Solidity ..  $=3769,9458=3.576336$

35. These Propositions are sufficient for measuring any *Superficies* or *Solid Body* in common Use; and the great Ease and Conciseness of performing the same by Logarithms, is abundantly manifest from these Examples.







## C H A P. XV.

*The DOCTRINE of MENSURATION applied to GAUGING, MEASURING TIMBER, and SURVEYING; wherein the PRACTICAL USE of the PLAIN SCALE and SLIDING RULE, for these Purposes, is clearly explain'd.*

1. **I**N the preceeding Chapter you have the *practical Method of absolute or general Mensuration of the Content of Superficies and Solids*, laid down in divers Propositions; my Business is here to apply that general Doctrine to particular Uses as those of *Gauging, Timber-Measure, and Surveying*. For tho' I there shew'd how the *Dimensions of Bodies* might be express'd in *Numbers*; yet since these Numbers represent *divers kinds of Quantities*, as *Inches, Feet, Yards, Poles, Chains, &c.* at pleasure, so they are equally subservient to those several Arts above-mention'd, and must be reduced to those *Denominations or Measures* which are *peculiar* to each. I shall begin therefore with

2.

*Gauging.*

In this Art the *Dimensions* are taken in *Inches*, and *Decimal Parts* thereof (or must be reduced to such;) and by *Inches* is here to be understood *solid or cubic Inches*; and that as well in *superficial* as *solid Measure*. For though it be improper, *geometrically speaking*, to ascribe *Thicknejs* to a *Superficies*, the *Gaugers* always consider them as *one Inch deep*, and accordingly compute the *superficial Content* in *Gallons*, or *solid Measure*.

3. As *Inches* are the *lineal Dimensions* of *Gaugers*, so *Gallons* are the common *Quantities* of their *solid Measure*; of *liquid Substances* especially; and *Busbels* for *solid dry Measure*, as *Malt*, *Corn*, &c. Now the *standard Gallons* and *Busbel*, as used in *England*, are as follows: the *Ale or Beer Gallon* = 282 *solid Inches*; the *Wine Gallon* = 231; the *Corn Gallon* = 268,8; and the *Corn Busbel* = 2150,42 *Cubic Inches*.

4. From whence it follows, that, supposing the *Dimensions* of the *Superficies* in the foregoing Chapter taken in *Inches*, the several *Area's*, or *superficial Contents*, will by the *Gauger* be understood to be so many *solid or cubic Inches*; which therefore if he divide by 282, 231, 268,8, or 2150,42, the *Quotients* will be the several *Contents* in the respective *Gallons* or *Busbels*. For instance, in Fig. XII. the Side of the Square AB=31,57 *Inches*, and the Area was therefore found (Art. 2.) to be 996,66 *solid Inches*.

Conse- quently	{	282	)996,66(=3,53	Ale Gallons.
		231	)996,66(=4,31	Wine Gallons.
		268,8	)996,66(=3,707	Corn Gallons.
		2150,42	)996,66(=0,46	Corn Busbels.

5. But since *Multipliers* are the *Reciprocals* of *Divisors*, therefore *Unity* divided by these *Divisors* will produce so many *Multipliers*. Thus,

282	)1.00000(=0.003546	the Mult. for A. Gall.
231	)1.00000(=0.004329	the Mult. for W. Gall.
2688	)1.00000(=0.003722	the Mult. for C. Gall.
2150,42	)1.00000(=0.000465	the Mult. for C. Bush.

Consequently, if any Area be

Divided by ..	{	282	}	or multi- plied by	{	0 003546	}	the Area { A. Gall. will be { W. Gall. expressed { C. Gall. in . . . { C. Bush.
		231				0 004329		
		268.8				0 003722		
		2150,42				0 000465		

6. In order to work these Dimensions by the *artificial Line of Numbers*, either on the *Plain Scale*, or *Sliding-Rule*; put  $B$ =Breadth,  $L$ =Length,  $D$ =Depth or Thickness,  $G$ =Standard Gallon; and  $A$ =Area, and  $S$ =Solid Content in those Gallons; &c. Then for *Superficies*,  $\frac{LB}{G} = A$ ; and so  $LB = AG$ ; therefore  $G : B :: L : A$ . That is, the *Ratio*, or *Logarithm* of the *Ratio* of the *Standard Gallon* to the *Breadth*, is equal to that of the *Ratio* of the *Length* to the *Area* in Gallons. Wherefore supposing  $B=31,57$ ,  $L=41,5$  (as in the *Parallelogram* Fig. XIII. Art. 3.) if you set one Foot of the *Compasses* in 282, and extend the other to 31.57, that Extent will reach from the Length 41,5 to  $4\frac{6}{10}$  Gallons, Ale Measure. Or, on the *Sliding-Rule*, set 282 on the Rule to 31,57 on the Slider, then against 41,5 on the Rule is  $4\frac{6}{10}$  on the Slider, the *Area* in Ale Gallons, as before; and thus the *Area* is found in *Wine* or *Corn Gallons*.

7. Again, Since  $AD=S \times 1$ , therefore  $1 : A :: D : S$ . That is, the *Logarithm* of the *Ratio* of Unity 1 to the *Area* in Gallons  $A$ , is equal to that of the *Ratio* of the *Depth* (or *Height*)  $D$  to the *Solidity* (or *Capacity*)  $S$ , in Gallons of the same kind. Wherefore if  $B=21,56$ ,  $L=31,57$ ,  $D=9,03$ , as in *Parallelopipedon*, Fig. XXV. then  $G=282 : B=21,56 :: L=31,57 : A=2\frac{4}{10}$  the *Area* in Ale Gallons; consequently  $1 : A=2\frac{4}{10} :: D=9,03 : S=21\frac{7}{10}$ , the *Solid Content* in Ale Gallons. Where extend the *Compasses* from Unity to the *Area*  $2\frac{4}{10}$ , the same Extent will reach from the *Depth* 9,03 to the *Solid Content*  $21\frac{7}{10}$  Gallons. Or, on the *Sliding Rule*, set Unity 1 on the Rule to the *Area*  $2\frac{4}{10}$  on the Slider, then against the *Depth* 9,03 on the Rule you have  $21\frac{7}{10}$  the *Solid Content* in Ale Gallons on the Slider. Thus the Method of finding the *Content* or *Capacity* of square or rectilinear *Area's* or *Bodies* in Ale,

*Wine,*



*Wine, or Corn Gallons* is exceeding plain and easy by the *Single Gunter* and *Sliding-Rule*.

8. In case of *circular Area's*, since they are all in the Ratio of the Squares of their Diameters; and supposing the Diameter of a Circle 1 Inch, the Area will be 0,7854 Decimal Parts of a Cubic Inch; therefore having the Diameter of any Circle given in Inches, if the Square thereof be multiplied by 0.7854, the Product will be the Area of that Circle in Cubic Inches. Let  $D$  = the Diameter of a Circle,  $a = 0.7854$ ,  $G$  = Standard Gall. and  $A$  = Area of the Circle in such Gallons, as before. Then  $1 : a :: DD : DDa = \text{Area in Inches}$ ; therefore  $DDa = GA$ ; and so  $G : a :: DD : A = \text{Area in Gallons}$ . But  $a : G :: 1 : \frac{G}{a} = \text{the Square of the Diameter of that Circle, whose Area is } G$ . Wherefore since  $DD = \frac{G}{a} A$ ,

therefore if the Square of the Diameter  $DD$  be divided by  $\frac{G}{a} =$

$\left\{ \begin{array}{l} 0.7854 \end{array} \right\}$	$282.0000 (= 359.05$
$\left\{ \begin{array}{l} 0.7854 \end{array} \right\}$	$231.0000 (= 294.12$
$\left\{ \begin{array}{l} 0.7854 \end{array} \right\}$	$268.8000 (= 342.24$
$\left\{ \begin{array}{l} 0.7854 \end{array} \right\}$	$2150.4200 (= 2737.92$

or the several Divisors 359,05; 294,12, &c. then these Quotients will be the Area of the Circle in Ale, Wine, &c. Gallons.

9. Or thus, since  $DD \times \frac{a}{G} = A$ ; therefore if the Square of the Diameter  $DD$

be multiplied by  $\frac{a}{G} =$

$\left\{ \begin{array}{l} 282 \\ 231 \\ 2688 \\ 2150,42 \end{array} \right\}$	$\left\{ \begin{array}{l} 0.7854(0.002785 \\ 0.7854(0.003389 \\ 0.7854(0.002922 \\ 0.7854(0.0000036 \end{array} \right\}$
---	---

the several Products will be the Area in Ale, Wine, &c. Gallons, as before. Suppose the Diameter of a Circle  $D = 50$  Inches, then  $DD = 2500$ ; and put the constant Divisor  $\frac{G}{a} = dd = 359,05; 294,12, \&c.$

G g

Then

Then since  $1 \times DD = ddA$ , therefore  $dd : DD :: 1 : A = \text{the Area in Gallons}$ . Therefore with the Compasses set one Foot in 359,05 extend the other to 2500, the same Extent will reach from 1 to  $6 \frac{1}{2} =$  the Gallons in the *Area* of the proposed Circle.

10. Otherwise thus; put  $\frac{a}{G} = dd = 0.002785$ ; 0.002389, &c. (see Art. 9.) then because  $DDdd = A \times 1$ ; therefore we have  $1 : DD :: dd : A$ ; or  $1 : dd :: DD : A$ . If then you set 1 on the Rule to 0.00278 on the Slider, you will see against 2500 on Rule  $6 \frac{1}{2}$  on the Slider, the Gallons of Ale in the Area of that Circle; and thus you find the Gallons of Wine, Corn, &c. both by the *Sliding-Rule* and *Plain Scale*.

11. Again, putting  $H = \text{Height of a Cylinder}$ , and  $D = \text{the Diameter of its circular Base}$ ; also  $S = \text{the solid Content or Capacity in Gallons}$ ; then  $dd : DD :: (1 : A ::) H : S$ . Now let  $D = 50$  Inches, and  $H = 15$ , the Area of such a Cylinder will be thus found by the *Sliding-Rule*. Set 359,05 on the Slider, to 2500 on the Rule; then against 15 on the Slider, is 104 on the Rule; and so many Gallons of Ale would such an hollow Cylinder contain.

12. But since  $dd : 1 :: DD : A$ , (Art. 9.) therefore  $d : 1 :: D : \sqrt{A}$ ; and for the same Reason  $d : \sqrt{H} :: D : \sqrt{S}$ . Consequently, if a *single Line of Numbers* be made to slide by a *double one*, if you set  $d$  on the *single one* to 1 on the *double one*, then against  $D$  on the *former*, you have  $A$  on the *latter*. Also, if against  $d$  on the *single Line*, you set  $H$  on the *double one*; then against  $D$  on the *former*, is  $S$  on the *latter*. By this means therefore, having only the *Diameter* and *Height* of a *Cylinder*, the Area of the Base, and solid Content of the said Cylinder, is immediately known. The Reason of the Method

here

here delivered, appears from what has been already said in the Chapter of *Instrumental Operations*.

13. Now  $d =$

$$\left\{ \begin{array}{l} \sqrt{359,05} = 18,95 \\ \sqrt{294,12} = 17,15 \\ \sqrt{342,24} = 18,5 \\ \sqrt{2737,92} = 52,32 \end{array} \right\} \text{Which Numbers } \left\{ \begin{array}{l} \text{Ale Gall.} \\ \text{Wine Gall.} \\ \text{Gauge Points} \\ \text{for } \dots\dots\dots \end{array} \right\} \left\{ \begin{array}{l} \text{Corn Gall.} \\ \text{Corn Bush.} \end{array} \right.$$

And accordingly on the single Line of Numbers against those *Gauge Points* there is a small Stroke ('), by which are set the Capitals AG, WG, MB, to signify they are the *Gauge Points* for *Ale Gallons*, *Wine Gallons*, and *Malt Bushels*.

14. Therefore in the foremention'd Cylinder, whose Height H was = 15 Inches; and the Diameter of its Base D = 50; in order to find the Area of the Base, and Capacity of the Cylinder in Gallons, by the Sliding-Rule; set 1 on the Slider to the *Gauge Point* for Ale 18.95 on the *single Line*, then against the Diameter D = 50, on the single Line is  $6 \frac{1}{8}$  on the *double Line* on the Slider, the Gallons of Ale in the Area of the Base.

Secondly, Set the *Gauge Point* 18.95 to the Height 15 on the Slider, then against the Diameter 50 on the single Line is 104 = S on the double Line or Slider; therefore the Capacity of the Cylinder is 104 Gallons, *Ale-Measure*, as before.

15. Or thus, with the *single Line* and *Compasses* only. Set one Foot in the *Gauge Point* 18,95, and extend the other to D = 50, this Extent turn'd twice over from 1 will fall on  $6 \frac{1}{8} = A =$  the Area of the Base in Gallons of Ale. The solid Content, in this Case, will fall beyond the single Line, or else might be performed in the same manner.

16. With the double Lines of Numbers, on the Sliding Rule, thus. Set the Gauge-Point 18.95 on the Rule to the Diameter 50 on the Slider, and mark the Number on the Slider against 1 on the Rule;

G g 2

then



then against that Number on the Rule you have  $6 \frac{2}{3} = A$  the Area of the Base on the Slider. Also mark the Number on the Slider against the Height 15 on the Rule, then against that Number on the Rule is 104 on the Slider, the Capacity of the Cylinder as before; and thus both the Area and Solidity are found at once setting the Rule. The same is performed by the double Line and Compasses, thus; Set one Foot in the Gauge Point 18,95, and extend the other to the Diameter 50, the same Extent will reach at twice from 1 to  $6 \frac{2}{3}$  the Area; and from the Height 15 to 104 solid Capacity in Ale Gallons.

17. In this manner may Gauge-Points be found for right-lined Areas: for let any such Area given in *Cubic Inches* be called  $a$ ; then (by Art. 6.)  $1 \times a = GA$ , and so  $G : a :: 1 : A$ ; or thus  $G : 1 :: a : A$ ; therefore  $\sqrt{G} : 1 :: \sqrt{a} : \sqrt{A}$ .

$$\text{Therefore } \sqrt{G} = \begin{cases} \sqrt{282} & = 16.79 = \\ \sqrt{231} & = 15.19 = \\ \sqrt{2150.42} & = 46.36 = \end{cases} \text{ the Gauge Point for } \begin{cases} \text{Ale G.} \\ \text{W. G.} \\ \text{Bush.} \end{cases}$$

Call these Gauge Points  $d$ ; then  $d : 1 :: \sqrt{a} : \sqrt{A} =$  the Area in Gallons. Also  $d : \sqrt{H} :: \sqrt{a} : \sqrt{S} =$  the Solidity in Gallons. Whence you may observe, that *right-lined Areas*, as well as *circular ones*, may be found in Gallons, by the *single* and *double Line* of Numbers sliding by each other; but not the *Solidity* of such Solids, there being *three Terms* of the *four* variable, in the Analogy for that.

18. But notwithstanding this, there is a Method whereby the Solidity or Capacity of Solids or Vessels may be found (without knowing the Areas at all) by the *Breadth*, *Length*, and *Depth* only; which call  $B$ ,  $L$ ,  $D$ ; and  $S =$  Solidity, and  $G =$  the Gallon or Bushel, as before. Now since  $BDL = GS$ , therefore  $BD = \frac{G}{L} S = G \times \frac{1}{L} \times S$ ; and so  $G \times \frac{1}{L} : B$

∴ D : S. But because  $\frac{1}{L}$  is the Reciprocal of L, therefore the Line of Numbers on which you seek L must be *inverted*; and because the Logarithm of G is added, the Number G must always begin the *inverted Line*, or be placed equal to 1 on the *direct Line*. These things premised, 'tis plain that if B on the *Slider* be set to L on the *inverted Line* order'd as before, then against D on the *direct Line* you have S on the *Slider*.

19. For Example; Suppose a Cistern be 80 Inches long, 50 broad, and 40 deep. Quære the Content in Bushels?

Here  $G = 2150,42$ , and for the Purpose of *Malt-Gauging* there is on some Rules an *inverted Line of Numbers* fix'd on one Side the *Slider*; beginning at 2150,42 as before said; then on such a Rule set  $50 = B$  on the *Slider* to  $80 = L$  on the *inverted Line*, and against  $40 = D$  on the *direct Line* on the Rule, you have  $74 = S$  = the Number of Bushels the *Cistern* will contain. And thus you might proceed for Gallons, had you *inverted Lines* beginning at 282, 231, 268.8.

20. In *gauging Casks*, the principal Consideration is the *Curvature* of the *Staves*; as A B D. Fig. XXXIX. which *Gaugers* reduce to *four Degrees*, or *Varieties*, viz.

*Variety I.* Those *Casks* whose *Staves* are most *curved* or *bent*, are consider'd as the *middle Zone* or *Frustum* of a *Spheroid*, such as Fig. XXXIV.

*Variety II.* If the *Staves* are not quite so much *arching* or *bent*, the *Cask* is supposed to be the *middle Zone*, or *Frustum* of a *Parabolic Spindle*, as Fig. XXXVII.

*Variety III.* When the *Staves* of *Casks* are but very little *curved*, they are reputed to be in the Form of the *Frustums* of two equal *Parabolic Conoids*.

*noids*, join'd together at the widest Bases. See Fig. XXXVI.

*Variety IV.* When the Staves are strait from the Bung to the Head, as the prick'd Lines AB, BD, (or very nearly so) then 'tis plain such a Cask consists of the *Frustums* of two equal right *Cones*, set together at the greater Bases. Fig. XXXIX.

21. The Casks being reduced to these four Varieties, if you multiply the Difference between the Head and Bung Diameters

$$\text{By } \left\{ \begin{array}{l} 0.7 \\ 0.65 \\ 0.6 \\ 0.55 \end{array} \right\} \text{ for the } \left\{ \begin{array}{l} \text{I.} \\ \text{II.} \\ \text{III.} \\ \text{IV.} \end{array} \right\} \text{ Variety,}$$

and then add the Product to the Head Diameter, then that Sum shall be a *mean Diameter*, or that of a *Cylinder*; whose *Height* and *Capacity* is equal to that of the *Cask*, as near as possible.

22. For Example; Let there be a Cask ADEG, whose *Bung-Diameter* BF = 31.5 Inches, and *Head-Diameter* AG = 24.5; then their Difference is BF - AG = 7.

$$\text{Consequently } \left\{ \begin{array}{l} 7 \times 0.55 = 28.35 \\ 7 \times 0.6 = 28.7 \\ 7 \times 0.65 = 29.05 \\ 7 \times 0.7 = 29.4 \end{array} \right\} \begin{array}{l} \text{The mean Diameter} \\ \text{of the Cylinder} \\ \text{equal to the Cask.} \end{array}$$

$24.5 +$

Having found the Areas (by Art. 8, 9, 10, 12) to be 2,2385; 2,2941, &c. Ale Gallons; and supposing the Length of the Cask 42 Inches, = HL.

$$\text{Then the Contents } \left\{ \begin{array}{l} 2,2385 \times 42 = 94.03 \\ 2,2941 \times 42 = 96.35 \\ 2,3504 \times 42 = 98.71 \\ 2.4073 \times 42 = 101.10 \end{array} \right\} \begin{array}{l} \text{Ale} \\ \text{Gallons.} \end{array}$$

according to the  
several Varieties  
will be : . . . . .

And



And these being all the principal Articles, wherein the Use of *instrumental Logarithms*, or the *artificial Line of Numbers* is directly concerned, I shall say no more on the Head of *Gauging*; but proceed to the next Article of

*Timber Measure.*

23. Every Piece of *Timber* is a Solid, like to some one or other of those, whose *absolute Mensuration* was shewn in the last Chapter, viz. The *Frustum* of a *Cone*, a *Cylinder*, the *Frustum* of a *Pyramid*, a *Parallelopipedon*, *Prism*, &c.

24. In whatever Form the Piece happens to be, find the *Content* or *Solidity* in Inches, as there taught; which divide by 1728, (the *solid Inches* in one *Foot* of *Timber*) the *Quotient* is the *solid Content* in *Feet*. But that the Analogies for Operation by Instruments may be evident, I shall make use of the foregoing Method in *Gauging*, by putting  $B$  = the Breadth,  $D$  = Depth,  $L$  = Length,  $G = 12$ , and  $S$  = Solidity, all in Inches. Then since  $\frac{BDL}{G^3} = S$ ,

therefore  $BDL = G^3 S$ ; but  $\frac{L}{12} =$  the *Length* in *Feet*, which let be  $F$ , then  $BD \frac{L}{12} = GGS = BDF$ ; and consequently  $GG : BD :: F : S$ .

25. Now if the Piece of *Timber* be in form of a *square Prism*, then the Base  $BD$  is a *square Number*, which call  $gg$ ; whence  $GG : gg :: F : S$ ; and therefore  $G : g :: \sqrt{F} : \sqrt{S}$ . Wherefore, having a *single* and *double Line* of *Numbers*, by the *Sliding-Rule*, set  $G = 12$  on the *single Line*, to the Length in Feet  $F$  on the *double one*; then against on the *single Line* is  $S$  = Solidity on the *double Line* on the Slider. Example, Suppose a Piece of *Timber*, the Side of whose square Base is  $g = 15$  Inches,

Inches, the Length 18 Feet = F; Quere the solid Content S?

Set the constant Point  $G = 12$  on the *single Line* to the Length in Feet  $F = 18$ ; then against the Side of the given *square Base*  $g = 15$  Inches, on the former, is  $28 = S =$  the Number of the solid Feet in the Piece, on the latter. Or with the Compasses, extend from 12 to 15 on the *single Line*, the same will reach from 18 to  $28 = S$  in the double Line, the Answer as before.

26. But if you have no *single Line*, proceed with the double Line on the *Sliding-Rule* thus: Set the constant Number  $GG = 144$  on the Rule to the given Square  $gg = 225$  on the Slider; then against the Length in Feet  $F = 18$ , on the Rule is  $28 = S =$  the solid Feet on the Slider. Or, with the Compasses, extend from 144 to 225, the same Extent will reach from 18 to 28 the solid Feet, as before. Otherwise by the Analogy  $G : g :: \sqrt{F} : \sqrt{S}$ ; thus, set the constant Point  $G = 12$ , on the Rule to the given Side  $g = 15$ , on the Slider, and mark the Point on the Slider against 15 on the Rule, bring that Point to 12 on the Rule; then against  $F = 18$  on the Rule, is  $28 = S$  on the Slider, the *Solidity* as before. But much better with the Compasses, thus; Extend from 12 to 15, that Extent turn'd twice over will reach from 18 to  $28 = S$ , as before.

27. If the Piece be in the Form of a *Parallelepipedon*, that is, hath its Breadth and Depth unequal; then the common way is to add the *Depth* and *Breadth* together; and to take half that Sum for the Side of a mean Square, viz.  $\frac{B+D}{2} = g$ , and then they imagine  $gg = BD$ , and so measure the Piece as before. But this is a very erroneous way; and the more so, as the Difference between the Breadth and Depth is greater. For since, (in order to reduce this Piece to a *square Prism*)  $BD = gg$ ; 'tis evident

$g =$

$g = \sqrt{BD}$ ; that is, a *mean Proportional* between B and D, (for  $B : \sqrt{BD} :: \sqrt{BD} : D$ ) and not their *half*

Sum  $\frac{B+D}{2}$ ; as they ignorantly suppose.

28. For Example; Suppose a piece of *betw* *Timber* in form as aforesaid, whose Breadth is 22,5 Inches = B, the *Depth* or *Thicknes* 10 Inches = D, and Length 18 Feet = F; what is the solid Content?

29. Find a *mean Proportional* between  $B=22,5$  and  $D=10$ , (as heretofore taught) it shall be  $g = 15 = \sqrt{BD}$ ; and therefore  $G^2 = 144 : g^2 225 :: F=18$  .  $S = 28$ , as before. But according to the common

false way,  $g = \frac{B+D}{2} = 16,25$ , and so  $gg = 264,0625$ , which is greater than the *true Area* of the Base BD = 225 by 39,0625 *square Inches*. Also set  $G = 12$  on the *single Line* of Numbers to the Length  $F = 18$  Feet on the double, then against (the false)  $g = 16,25$  on the *former*, is  $32\frac{3}{4} = S$  on the latter. But  $32\frac{3}{4} - 28 = 4\frac{3}{4}$  Feet more than is really in the Piece.

29. Because  $BDF = G^2 S$ ; therefore  $BD = \frac{G^2}{F} S$ ; and so we have  $G^2 \times \frac{1}{F} : B :: D : S$ . If then you have an *inverted Line* of Numbers beginning from  $G^2 = 144$  placed on one side the Slider, with a *direct Line* on the other; then may the *solid Content* in Feet be found by the B, D, and F, as taught in Art. 18. hereof, thus; Set  $B = 22,5$  on the Slider, to  $F = 18$ , on the *inverted Line*; then against  $D = 10$ , on the *direct Line* is  $28 = S$ , on the Slider, the solid Feet as before.

30. If the Timber be in Form of the *Frustum* of a *square Pyramid*, as Fig. XXX. the Theorem for its Solidity in Inches is  $HDd + \frac{1}{3}Hxx$ , as per last Chap. Art. 23. Therefore  $HDd + \frac{1}{3}Hxx = G^3 S$  (where  $S =$  Solidity in *cubic Feet*) and since  $\frac{1}{2}H = F =$  the Length in Feet, therefore  $F Dd + \frac{1}{3}Fxx =$

$$\begin{array}{ccc} & H & h \\ & \times & \\ & F & \end{array} \quad \begin{array}{c} G \\ \\ G \end{array}$$



$G^2 S$ , consequently  $G^2 : F :: Dd + \frac{1}{3}xx : S$ . Example; Suppose a Piece of Timber 25 Inches square at the greatest end, 9 Inches square at the lesser End, and 20 Feet long, how many solid Feet is there in such a Tree?

Here  $F = 20$ ,  $Dd = 225$ , and  $\frac{1}{3}xx = 85,3$ , therefore  $Dd + \frac{1}{3}xx = 310,3$ ; wherefore  $G^2 = 144 : F = 20 :: 310,3 : 43,1 = S$ , the Number of solid Feet as required. Note, the common way, by supposing the Square of  $\frac{D+d}{2} = Dd + \frac{1}{3}xx$  is very false; and in this Instance would not give the Solidity above 40,1 Feet, which is three Feet less than the Truth.

31. If the Bases of the *Frustum* be *Parallelograms*, as is the Case of most Pieces of *bew'd Timber*; then they may be reduced to Frustums of *square Pyramids*, thus; Let  $A$  = the Area of the greater Base,  $a$  = Area of the lesser. Then  $\sqrt{A} = D$ , and  $\sqrt{a} = d$ ; and  $\sqrt{A \times a} = Dd$ ; also since  $D^2 + 2 Dd + dd = xx$ , therefore  $A - 2 \sqrt{A \times a} + a = xx$ ; and so  $\frac{3Dd + xx}{3} = \frac{A + \sqrt{A \times a} + a}{3}$ ; consequently,  $G^2 : F :: \frac{1}{3} A + \sqrt{A \times a} + a : S$  = the Number of solid Feet in such a Tree.

32. For Example; Suppose a Piece of squared Timber be 32 Inches broad, and 20 deep at the largest End; and 10 broad and 6 deep at the lesser End; the Length 18 Feet; Quære the solid Content?

Here  $A = 32 \times 20 = 640$ , and  $a = 10 \times 6 = 60$ ; and  $\sqrt{A \times a} = \sqrt{38400} = 195,959$ ; therefore  $\frac{1}{3} A + \sqrt{A \times a} + a = 298,653$ . Then  $G^2 = 144 : F = 18 :: 298,653 : S = 37,33$  the solid Feet required. Note, the Product of  $\frac{1}{3}$  of the Girt of each End taken for a *mean Area* is very false; and cannot be practised without considerable Error, tho' it be the common or customary way.

33. If the Timber be in form of a Cylinder, *Fig. XXVIII.* then putting  $a = 0.7854$ , and  $G = 12$  Inches, we have (as per Art. 8.) this Analogy ;  $1 : a :: DD : DDa$  = the Area of the Cylinder's Base in Inches (supposing  $D$  = Diameter in Inches ; ) let  $H$  = Height in Inches ; then  $HDDa$  = the Inches solid, whereof  $1728 = G^3$  = one *solid Foot*. Consequently, if  $S$  = solid Content in Feet,  $HDDa = G^3 S$  ; and  $\frac{H}{12} = F$  = the Length in Feet, there-

fore  $FDDa = G^3 S$  ; and  $FDD = \frac{G^2}{a} S$  ; and putting  $\frac{G^2}{a} = dd = 183,34$  ; then  $FDD = ddS$  ; wherefore  $dd : DD :: F : S$ , or  $d : D :: \sqrt{F} : \sqrt{S}$ .

34. Or with the Length in Inches, to find the Solidity in Feet, thus ; since  $HDDa = G^3 S$ , therefore putting  $\frac{G^3}{a} = dd$ , we shall have  $HDD = ddS$  ; consequently  $dd : DD :: H : S$  = Solidity in Feet ; or  $d : D :: \sqrt{H} : \sqrt{S}$ . Wherefore, if there be a *single Line* of Numbers on your Sliding Rule ; set the constant Number  $d = 46.9$  on the single Line to  $H$  = the Height in Inches, on the double Line on the Slider ; then against  $D$  = the Diameter of the Cylinder's Base in Inches on the *single Line*, is  $S$  = the Number of solid Feet on the Slider as required. And if the Length be given in Feet ; then the Solidity is found, as above, Art. 13.

35. If you have no *single Line* on your Rule, you must work with the Analogies  $dd : DD :: F : S$  ; or  $dd : DD :: H : S$ . In the first  $dd = 183,34$ , and  $dd = 2200,152$  ; therefore  $d = 13.54$  = Diameter of a Circle, when the Area is 144 ; and  $d = 46,9$  = the Diameter of a Circle, whose Area is 1728 = the Inches in a solid Foot. And these Numbers being constant in all Operations, and the Method of operating such kind of Analogies every way on the ar-

*tificial Lines of Logarithms* already sufficiently exemplified in the preceeding Articles; I shall not here again repeat it.

36. The common way of measuring *round Timber*, is by girting them about the *Middle* with a *String*, and taking  $\frac{1}{4}$  of the Girt for the *Side* of a *Square* equal to a *mean circular Area*, such as would reduce the proposed Piece to a *Cylinder*. But this is also very false and ungeometrical. For, since the Area of that Circle, whose Circumference is 1, is 0.07958; and the Area of that Square, whose Side is .25 ( $=\frac{1}{4}$  of the said Circumference) is 0.0625, and the Solidities being in proportion to these Numbers, *viz.* as 0.07958 : 0.0625; that is, as 23 to 18; 'tis evident, the Content by this false way is above  $\frac{1}{3}$  less than what it really is; which *Error*, if it be not considerable enough to be regarded and corrected, is great pity indeed.

37. Therefore to measure a Piece of *round tapering Timber* truly, it must be considered as the *Frustrum* of a *right Cone*; whose solid Content in Inches is found by the Theorem  $Dd + \frac{1}{3}xx \times 0.7854H$ , as per Art. 25. of the last Chapter. Now putting  $0.7854 = a$ , and  $G^2 = 1728$ , the cubic Inches in a solid foot; the Theorem will become  $Dd + \frac{1}{3}xx \times aH = G^2S$ ; and again since  $\frac{1}{2}H = F$ , the Length in Feet; therefore  $Dd + \frac{1}{3}xx \times aF = G^2S$ ; also put  $\frac{G^2}{a} = dd$ ; and then  $\overline{Dd + \frac{1}{3}xx} \times F = ddS$ ; wherefore we have  $dd : \overline{Dd + \frac{1}{3}xx} :: F : S =$  the *solid Content* in Feet.

38. For Example; Suppose a Piece of *round Timber* be 36 Inches Diameter at one End, and 9 Inches Diameter at the other; and 24 Feet long; quare the Solidity in Feet?

Here  $D = 36$ ,  $d = 9$ ,  $D - d = x = 27$ ,  $Dd = 324$ ,  $\frac{1}{3}xx = 243$ ,  $\overline{Dd + \frac{1}{3}xx} = 567$ , and  $F = 24$ .  
There-



Therefore the Analogy is as  $dd = 183,34 : 567 :: F = 24 : S = 74,22$  the solid Feet required ; as may be wrought by the *Lines of Numbers* in any of the before-mention'd ways.

### Of MEASURING LAND.

39. What I principally design here, is to shew how the true Area or Content of a Plot of Land is to be found by the *artificial Line of Numbers*, in *Acres* and *Decimal Parts*. The Dimensions of a Field are commonly taken in Rods or *Poles* ; each containing  $16\frac{1}{2}$  Feet. Of these Poles, 40 in Length and 4 in Breadth make an Acre ; or an Acre is = 160 square Poles. Some (and indeed most) use a Chain, called *Gunter's Chain*, in taking Dimensions ; which consisteth of 100 Links, and the whole in Length = 4 Poles or Rods. And so 10 of these *square Chains* make an *Acre*.

40. The Field being measur'd with the Pole, if it be in Form of a *Parallelogram*, put  $L$  = Length, and  $B$  = Breadth ; and then it will be  $\frac{LB}{160} = A$  = the Number of Acres ; therefore  $LB = 160 A$  ; and so we have  $160 : L :: B : A$  = the Acres. For Example, suppose a Field be 35 Pole broad, and 185 Pole in length ; how many Acres doth it contain ? Set 160 on the Rule to 185 on the Slider, then against 35 on the Rule is  $40\frac{1}{2}$  nearly, the Number of Acres required.

41. If the Field be in Form of a Triangle, and measur'd with a Rod or Pole ; then, having plotted it and measur'd the *Base* and perpendicular Height, which call  $B$  and  $H$  ; then  $B \times \frac{1}{2}H = 160A$  ; and so  $160 : B :: \frac{1}{2}H : A$  = the Acres as before.

42. If the Field be in Form of a *Trapezium*, as Fig. XL. then because, by drawing the Diagonal  $AB$ , it is reduced to two Triangles  $ACB$ ,  $ADB$ ,  
and

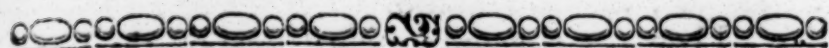
238 MENSURATION *applied, &c.*

and calling the two Perpendiculars  $Cc$  and  $Dd$ ,  $H$  and  $b$ ; and the common Base  $AB, B$ ; we have  $\frac{BH}{2} + \frac{Bb}{2} = \overline{H + b} \times \frac{1}{2}B = 160A$ . Therefore  $160 : \frac{1}{2}B :: H + b : A =$  the Acres contain'd in the Trapezium  $ACBD$ .

43. If the Field be of a *multangular Form*, it must, when plotted, be reduced to several *Triangles* and *Trapezia*; and then measur'd, as *per* Art. 41, 42. In case you take the Measures with a Chain of 4 Rods or 100 Links, then the Analogies for Operation will be the same as above, only instead of 160 you must use 10; thus  $10 : L :: B : A$ . Art. 40. and  $10 : B :: \frac{1}{2}H : A$ . Art. 41, &c. All which is so easy as to need no Example, nor any thing more to be said concerning it.



CHAP.



# CHAP. XVI.

## The Practical Use of the LOGISTICAL LOGARITHMS.

1. **T**HE Nature and Construction of Logistical Logarithms, having been sufficiently inculcated in Chap. IV. it remains now that I shew their Use in some Cases of practical Astronomy, for in other respects it is very little. But in finding the Places, Distances, &c. of the *heavenly Bodies*, the Calculation of Eclipses, &c. they are very necessary for finding the proportional Parts, as will, in some Degree, appear by the following Examples.

2. Example 1. Admit the *mean Anomaly* of the *Sun* be  $4^s 7^o 48' 14''$ ; what is the *true Equation*, and Logarithm of his Distance from the Earth?

Mean A-	$\{ 4^s 7^o \}$	Equa-	$\{ 1^o 33' 49'' \}$	Lo-	$\{ 4.995607 \}$
nomalies	$\{ 4 8 \}$	tions	$\{ 1 32 36 \}$	gar.	$\{ 4.995501 \}$
Differences	$\underline{1=60'}$		$\underline{1 13}$		$\underline{106}$

Then for the *Proportional Parts* of the Equation, say by the Logistical Logarithms,

If one Degree, or	.....	$60' 00'' =$	0
give the Difference	.....	$1 13 =$	16930
what gives the Anomaly	.....	$48 14 =$	948
Answer, the prop. Part	.....	$0 59 =$	17878

This subtracted, (because the Equation is decreasing) from the Equation  $1^o 32' 49''$  (answering the



and calling the two Perpendiculars  $Cc$  and  $Dd$ ,  $H$  and  $b$ ; and the common Base  $AB, B$ ; we have  $\frac{BH}{2} + \frac{Bb}{2} = \overline{H+b} \times \frac{1}{2}B = 160A$ . Therefore  $160 : \frac{1}{2}B :: H + b : A =$  the Acres contain'd in the Trapezium  $ACBD$ .

43. If the Field be of a *multangular Form*, it must, when plotted, be reduced to several *Triangles* and *Trapezia*; and then measur'd, as *per* Art. 41, 42. In case you take the Measures with a Chain of 4 Rods or 100 Links, then the Analogies for Operation will be the same as above, only instead of 160 you must use 10; thus  $10 : L :: B : A$ . Art. 40. and  $10 : B :: \frac{1}{2}H : A$ . Art. 41, &c. All which is so easy as to need no Example, nor any thing more to be said concerning it.



## C H A P. XVI.

*The Practical Use of the LOGISTICAL LOGARITHMS.*

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2. Example 1. Admit the *mean Anomaly* of the *Sun* be  $4^{\circ} 7' 48'' 14''$ ; what is the *true Equation*, and Logarithm of his Distance from the Earth?

Mean A-	$\{ 4^{\circ} 7' \}$	Equa-	$\{ 1^{\circ} 33' 49'' \}$	Lo-	$\{ 4.995607 \}$
nomalies	$\{ 48 \}$	tions	$\{ 13236 \}$	gar.	$\{ 4.995501 \}$
Differences	$1=60'$		$113$		$106$

Then for the *Proportional Parts* of the Equation, say by the Logistical Logarithms,

If one Degree, or . . . . .  $60' 00'' = 0$   
 give the Difference . . . . .  $113 = 16930$   
 what gives the Anomaly . . . . .  $4814 = 948$

Answer, the prop. Part . . . . .  $059 = 17878$

This subtracted, (because the Equation is decreasing) from the Equation  $1^{\circ} 32' 49''$  (answering the

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the Anomaly  $4^{\circ} 7'$  leaves the *true Equation* =  $1^{\circ} 32' 50''$ , as was requir'd.

3. Then for the proportional Part of the Logarithm say

One Degree, or .....  $60' 00'' = 0$   
 gives the Difference of Logarithms...  $106 = 15310$   
 what gives the Anomaly .....  $48' 14'' = 948$

Answer, the Proportional Part. ....  $85 = 16258$

Which subduct from the Logarithm 4,995607, answering to the Anom.  $4^{\circ} 7'$  there remains 4,995522, the *true Logarithm* of the *Sun's Distance* from the Earth, as required.

4. Example II. Suppose the Moon's *annual Argument* be  $29^{\circ} 51' 37''$ , what is the Equation of the *Apogee* and *Eccentricity* of her Orb?

Annual	{ $29^{\circ}$ }	Equa-	{ $8^{\circ} 53' 8''$ }	Eccen-	{ $60392$ }
Argum.	{ $30$ }	tion	{ $9 07 14$ }	tricity	{ $61043$ }
Differences .	$1 = 60'$		$14 6$		$653$

Then for the Equation of the Apogee, say

If one Degree, or .....  $60' 00'' = 0$   
 give the Difference .....  $14' 6'' = 6289$   
 what gives .....  $51' 37'' = 654$

Answer, the Prop. Part ....  $12' 8'' 6943$   
 To which add the Equation  $8^{\circ} 53' 8''$ , agreeing to the an. Arg.

$29^{\circ}$ ; the Sum is .....  $9^{\circ} 05' 16''$  the *true Equation* of the Apogee as required.

5. Then for the true Eccentricity, say



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If one Degree, or . . . . . 60' 00" = 0  
 give the Difference of Eccentricity . . 653 = 7414  
 what will the Part . . . . . 51' 37" = 654

Answer, the Proportional Part . . . . . 562 = 8068  
 To which add the Eccentricity . . . 60392

The Sum is the *true Eccentricity* . . 60954, as required.

6. In like manner the Equation of the Moon's Center, Latitude, Inclination of Limit, and Logarithmic Distance from the Earth may be found. Also the same things are in like manner found for any of the Planets, of which there needs no more Examples.

7. Example III. In an Eclipse of the Moon, admit her horary Motion be 30' 31", and the Sun's 2' 27"; then the horary Motion of the Moon from the Sun will be 28' 4": and suppose the Moon hath pass'd the Sun by the Distance 1° 19' 4" = 4744"; What is the Time requisite for that Motion? Say thus,

As the horary Motion of the Moon { 28' 4" = 3300  
 from the Sun . . . . . {  
 is to one Hour . . . . . 60 0 = 0  
 So is the Distance pass'd above 56' { 22' 56" = 4177  
 8", viz. . . . . {

To the Time above 2 Hours, viz. . . 49' 00" = 877

Therefore 2<sup>h</sup> 49' were pass'd since the *true Opposition*, or Moment of the Eclipse.

Note, because in this Case the Motion of the Moon from the Sun, performed in 1 Hour, is 28' 4"; therefore 56' 8" will be pass'd in 2 Hours: But the present Distance of the Moon from the Sun is

I i

79'

79' 4", which because it exceeds the Table of *Logistical Logarithms*, therefore subduct the Motion of two Hours, viz. 79' 4" — 56' 8" = 22' 56"; and this Excess of Motion will give the Excess in Time above 2 Hours; as in the Example. And thus you proceed when the Distance of the Moon from the Sun exceeds one Degree, or the Table.

8. Example IV. Suppose, in a *Lunar Eclipse*, the *Semidiameter* of the Moon be 15' 15", and the *Difference* between the *Moon's Latitude*, and *Sum* of the *Semidiameters* of the Moon and the Earth's Shadow be 9' 01"; Quære the Digits eclipsed?

*Note*, the *Semidiameter* of the *Moon* is always supposed to be 6 *Digits*, or equal to 6 Degrees. Therefore say,

As the Semidiameter of the Moon    15' 15" = 5949  
Is to six Digits . . . . . 6° 00' 00" = 10000  
So is the said Difference . . . . . 9' 1" = 8231

To the Digits eclips'd . . . . . 3° 32' 51" = 12282

9. Example V. In a *Lunar Eclipse*, let the *Scruples of Incidence* be 30' 17", and the *horary Motion* of the Moon from the Sun be 28' 47"; to find the Time of *half Duration*?

Here because 28' 47" : 60' :: 30' 17" : a fourth Number greater than 60', and so consequently beyond the Extent of the Table; therefore (as in Art. 7.) from 30' 17" subduct the Motion for an Hour, viz. 28' 47", and to the *Remainder* 1' 30" find the Time thus;

As the Motion . . . . . 28' 47" = 3197  
Is to an Hour . . . . . 60' 00" = 10000  
So the remaining Scruples . . . . . 1' 30" = 1600

To the Time . . . . . 3' 8" = 128

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The Time sought therefore  $1^h 3' 8''$ ; the double of which is  $2^h 6' 16''$ , the Time of the whole Duration of the Eclipse.

10. Example VI. If the diurnal Motion of the Sun be  $59' 8''$ , what is the Motion for 7 Hours  $15'$ ?

Say, as one Day . . . . .	$24^h 00' 00'' =$	0
Is to its Motion . . . . .	$59' 8'' =$	63
So is the Time . . . . .	$7^h 15' 0'' =$	5197
		<hr/>
To the Motion therein . . . . .	$17' 52'' =$	5260

11. Example VII. If the mean Motion of the Moon in one Hour be  $32' 56''$ , how far doth she move in  $17^h 45'$ ?

Say, as one Hour . . .	$1^h 00' 00'' =$	13802
Is to its Motion . . . . .	$32' 56'' =$	20392
So is . . . . .	$17^h 45' 00'' =$	1309
		<hr/>
		21701
		<hr/>

To the Motion required ..  $9^o 44' 00'' = 7899$   
 Thus the Motion in 24 Hours, or one natural Day, will be found  $13^o 10' 35''$ .

12. The *Logistical Logarithms* may also be used with *common Logarithms*. Thus suppose you would find the Logarithmic Sine of  $18^o 16' 47''$ , proceed thus ;

The Logarithmic Sine of . . .	$18^o 16' =$	9.496154
	$18 17' =$	9.496537
		<hr/>
The Differences . . . . .	$1' =$	60'' 383
		<hr/>
I i 2		Now



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Now the Proportion is,  $60'' : 383 :: 47'' : \text{the Proportional Part}$ . But since the *Logistical Logarithms* of  $47''$  and  $60''$  are reciprocally as those Numbers, and that of  $60$  is nothing; therefore (the Differences of the common Logarithms being proportional to the Numbers  $60''$  and  $47''$ ) say,

As the Logistical Logarithm of . . . .  $43'' = 1447$   
 is to the Logistical Logarithm of . . .  $60 = 0$   
 So is the common Logarithm of . . .  $383 = 2.5998$

To the common Logarithm of the }  $285 = 2.4551$   
 proportional Part . . . . . }  
 Which add to the common }  $9.496154$   
 Logarithm . . . . . }

The Sum is the Logarithm . . .  $9.496439$  of  $18^\circ 16' 47''$  as required. And thus proceed in any other Case of like nature.

13. The *Logistical Logarithms* may be used likewise with the *Logarithmic Sines* and *Tangents*, as in finding the Parallaxes of the Planets, &c. Thus suppose the Horizontal Parallax of the Moon  $55' 12''$ ; the Angle of her Orb with the Horizon  $22^\circ 4'$ ; and her Longitude in her Orb from the Horizon  $81^\circ 27'$ , to find her Parallax in Longitude.

From the Logistical Logarithm of the Hor. Paral. }  $00^\circ 55' 12'' = 362$

Subduct the { Sine of the }  $22 \quad 4 \quad 0 = 9.5748$   
 Sum of the { Angle . . . }  
 { Co-Sine of }  $81 \quad 27 \quad 0 = 9.1722$   
 { Longitude.. }

*viz.*  $18.7470$

There remains the Logif. Logarithm of the Par. in Long. . . }  $00^\circ 3' 5'' = 12892$

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14. Because when the Angle of the Moon's Orb with the Horizon is greatest  $67^{\circ} 14' 20''$ , and she in the Horizon, her greatest Parallax in Longitude will be then; and will be found as in the last Example, thus;

$$\begin{array}{l} \text{Logist. Log. of the greatest} \\ \text{Hor. Par. . . . .} \end{array} \left. \vphantom{\begin{array}{l} \text{Logist. Log. of the greatest} \\ \text{Hor. Par. . . . .} \end{array}} \right\} 00^{\circ} \ 1' \ 24'' = 17680$$

$$\begin{array}{l} \text{The Sine of . . . . .} \\ \text{Radius . . . . .} \end{array} \begin{array}{l} 67^{\circ} \ 14' \ 20'' \\ 90^{\circ} \ 0 \ 0 \end{array} = \begin{array}{l} 9.9648 \\ 10.0000 \end{array}$$

$$\begin{array}{l} \text{Logist. Log. of the gr. Par.} \\ \text{in Long. . . . .} \end{array} \left. \vphantom{\begin{array}{l} \text{Logist. Log. of the gr. Par.} \\ \text{in Long. . . . .} \end{array}} \right\} 00^{\circ} \ 56' \ 37'' = 18032$$

15. To find the Moon's *Parallax in Latitude*.

$$\begin{array}{l} \text{From the Logist. Log. of} \\ \text{Horizontal Parallax . . .} \end{array} \left. \vphantom{\begin{array}{l} \text{From the Logist. Log. of} \\ \text{Horizontal Parallax . . .} \end{array}} \right\} 00^{\circ} \ 55' \ 12'' = 362$$

$$\begin{array}{l} \text{Subduct the Co-Sine of the} \\ \text{Angle of her Orb with} \\ \text{the Horizon . . . . .} \end{array} \left. \vphantom{\begin{array}{l} \text{Subduct the Co-Sine of the} \\ \text{Angle of her Orb with} \\ \text{the Horizon . . . . .} \end{array}} \right\} 22^{\circ} \ 4' \ 0'' = 9.9669$$

$$\begin{array}{l} \text{there remains the Logistical} \\ \text{Logarithm of . . . . .} \end{array} \left. \vphantom{\begin{array}{l} \text{there remains the Logistical} \\ \text{Logarithm of . . . . .} \end{array}} \right\} 00^{\circ} \ 51' \ 9'' = 693$$

which is the *Parallax of Latitude* sought.

16. To find the greatest and least *Parallaxes in Latitude*.

$$\begin{array}{l} \text{From the Logistical Log. of} \\ \text{the gr. Hor. Par. . . . .} \end{array} \left. \vphantom{\begin{array}{l} \text{From the Logistical Log. of} \\ \text{the gr. Hor. Par. . . . .} \end{array}} \right\} 1^{\circ} \ 1' \ 24'' = 17680$$

$$\begin{array}{l} \text{Subtract the Co-Sine of the least} \\ \text{Ang. of her Orb with the Hor.} \end{array} \left. \vphantom{\begin{array}{l} \text{Subtract the Co-Sine of the least} \\ \text{Ang. of her Orb with the Hor.} \end{array}} \right\} 9^{\circ} \ 41' \ 40'' = 9.9937$$

$$\begin{array}{l} \text{Logistical Logarithm of the} \\ \text{greatest Parallax . . . . .} \end{array} \left. \vphantom{\begin{array}{l} \text{Logistical Logarithm of the} \\ \text{greatest Parallax . . . . .} \end{array}} \right\} 1^{\circ} \ 00' \ 31'' = 17783$$

Again,

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Again, from the L. Log. of }  $00^{\circ} 54' 59'' = 379$   
the least *Horiz. Parallax* . . }

Subtract the Co-Sine of the }  $67^{\circ} 14' 20'' = 9.5876$   
greatest Angle of Orb with }  
the Horizon . . . . . }

---

Logistical Logarithm of the }  $00^{\circ} 21' 16'' 4503$   
least *Parallax* . . . . . }

17. By these Instances the Reader will be appriz'd of the great Usefulness and Expediency of *Logistical Logarithms* in his *Astronomical Calculations*; and as the common Logarithms are laid on a Rule, so likewise are these; and such a *Sliding-Rule* of Logistical Logarithms may be very useful to those who desire to be more *expeditious* than *exact* in their Calculations.

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# LOGARITHMOLOGY.

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## PART II.

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CONTAINING

*A Threefold TABLE or CANON*

O F

# LOGARITHMS.

I. A Canon of *LOGARITHMS* for NATURAL  
NUMBERS from 1 to 10000.

II. A Canon of *LOGARITHMS* of SINES and  
TANGENTS to every Degree and Minute of the *Quadrant*.

III. A Canon or Table of *Logistical LOGARITHMS*.

All which are digested in a *New and most Compendious*  
Form, and very correct.

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L O N D O N:

Printed in the Year MDCCXXXIX.





*A TABLE of LOGARITHMS of  
NATURAL NUMBERS from 1 to 10000.*

N <sup>o</sup>	0	1	2	3	4
0		000000	301030	477121	602060
1	000000	041393	079181	113943	146128
2	301030	322219	342423	361728	380211
3	477121	491362	505150	518514	531479
4	602060	612784	623249	633468	643453
5	698970	707570	716003	724276	732394
6	778151	785330	792392	799340	806180
7	845098	851258	857332	863323	869232
8	903090	908485	913814	919078	924279
9	954242	959041	963788	968483	973128
10	000000	004321	008600	012837	017033
11	041393	045323	049218	053078	056905
12	079181	082785	086360	089905	093422
13	113943	117271	120574	123852	127105
14	146128	149219	152288	155336	158362
15	176091	178977	181844	184691	187521
16	204120	206826	209515	212188	214844
17	230449	232996	235528	238046	240549
18	255272	257679	260071	262451	264818
19	278754	281033	283301	285557	287802
20	301030	303196	305351	307496	309630
21	322219	324282	326336	328380	330414
22	342427	344392	346353	348305	350248
23	361728	363612	365488	367356	369216
24	380211	382017	383815	385606	387390
25	397940	399674	401400	403120	404834
26	414973	416640	418301	419956	421204
27	431364	432969	434569	436163	437751
28	447158	448706	450249	451786	453318
29	462398	463893	465383	466868	468347
30	477121	478566	480007	481443	482874
31	491362	492761	494155	495544	496930
32	505150	506505	507856	509202	510545
33	518514	519828	521138	522444	523746
34	531479	532754	534026	535294	536558
35	544068	545307	546543	547775	549003
36	556302	557507	558700	559907	561101
37	568202	569374	570543	571709	572872
38	579784	580925	582063	583199	584331
39	591065	592177	593286	594393	595496

\*4 The Logarithms of Nat. Numbers to 400.

	5	6	7	8	9
0	698970	778151	845098	903090	954242
1	176091	204120	230449	255272	278754
2	397940	414973	431364	447158	462398
3	544068	556302	568202	579784	591065
4	653212	662758	672098	681241	690196
5	740363	748188	755875	763428	770852
6	812913	819544	826075	832509	838849
7	875061	880814	886491	892095	897627
8	929419	934498	939519	944483	949390
9	977724	982271	986772	991226	995635
10	021189	025306	029384	033424	037426
11	060698	064458	068186	071882	075547
12	096910	100370	103834	107210	110590
13	130334	133539	136721	139879	143015
14	161368	164353	167317	170262	173186
15	190332	193125	195900	198657	201397
16	217484	220108	222716	225309	227887
17	243038	245513	247973	250420	252853
18	267172	269513	271842	274158	276462
19	290035	292256	294466	296665	298853
20	311754	313867	315970	318063	320146
21	332438	334454	336460	338456	340444
22	352182	354108	356026	357935	359835
23	371068	372912	374748	376577	378398
24	389166	390935	392697	394452	396199
25	406540	408240	409933	411620	413300
26	423246	424882	426511	428135	429752
27	439333	440909	442480	444045	445604
28	454845	456366	457882	459392	460898
29	469822	471292	472756	474216	475671
30	484300	485721	487138	488551	489958
31	498311	499687	501059	502427	503791
32	511883	513218	514548	515874	517196
33	525045	526339	527630	528917	530200
34	537819	539076	540329	541579	542825
35	550228	551450	552668	553885	555095
36	562293	563481	564666	565848	567026
37	574031	575188	576341	577492	578639
38	585461	586587	587711	588832	589950
39	596597	597695	598790	599883	600973



	0	1	2	3	4	5	6	7	8	9
40	60.2060	3144	4226	5305	6381	7455	8526	9594	0660	1723
41	61.2784	3842	4897	5950	7000	8048	9093	0136	1176	2214
42	62.3249	4282	5312	6341	7366	8389	9410	0428	1444	2457
43	63.3468	4477	5484	6488	7490	8489	9486	0481	1474	2464
44	64.3453	4439	5422	6424	7383	8350	9335	0307	1278	2246
45	65.3212	4176	5138	6098	7056	8011	8965	9916	0865	1813
46	66.2758	3701	4642	5581	6518	7453	8386	9317	0246	1173
47	67.2098	3021	3942	4861	5778	6694	7607	8518	9428	0335
48	68.1241	2145	3047	3947	4845	5742	6636	7529	8420	9309
49	69.0196	1081	9659	2847	3727	4605	5482	6356	7229	8101
50	8970	9838	0704	1568	2430	3291	4150	5008	5864	6718
51	70.7570	8421	9270	0117	0963	1807	2648	3491	4330	5167
52	71.6003	6838	7670	8502	9331	0159	0985	1811	2634	3456
53	72.4276	5094	5912	6727	7541	8354	9165	9974	0782	1589
54	73.2394	3197	3999	4800	5599	6396	7193	7987	8781	9572
55	74.0363	1152	1939	2725	3510	4293	5075	5855	6634	7412
56	8188	8963	9736	0508	1279	2048	2816	3583	4348	5112
57	75.5875	6636	7396	8155	8912	9668	0422	1176	1928	2679
58	76.3428	4176	4923	5669	6413	7156	7898	8638	9377	0115
59	77.0852	1587	2322	3055	3786	4517	5246	5974	6701	7427
60	8151	8874	9596	0317	1037	1755	2473	3189	3904	4617
61	78.5330	6041	6751	7460	8168	8875	9581	0285	0988	1691
62	79.2392	3092	3790	4488	5185	5880	6574	7267	7960	8651
63	9340	0029	0717	1404	2089	2774	3457	4139	4821	5501
64	80.6180	6858	7535	8211	8886	9560	0232	0904	1575	2245
65	81.2913	3581	4248	4913	5578	6241	6904	7565	8226	8885
66	9544	0201	0858	1513	2168	2822	3474	4126	4776	5426
67	82.6075	6075	5722	7369	8015	8660	9304	9947	0589	1230
68	83.2509	3147	3784	4421	5056	5691	6324	6957	7588	8219
69	8849	2479	0106	0733	1359	1985	2609	3233	3855	4477
70	84.5098	5718	6337	6955	7573	8189	8805	9419	0033	0646
71	85.1258	1870	2480	3089	3698	4306	4913	5519	6124	6729
72	7332	7935	8537	9138	9739	0338	8937	1534	2131	2727
73	86.3323	3917	4511	5104	5696	6287	6878	7467	8056	8644
74	9232	9818	0404	0989	1573	2156	2739	3321	3902	4482
75	87.5061	5640	6218	6795	7371	7947	8522	9096	9669	0242
76	88.0814	1385	1955	2524	3093	3661	4229	4795	5361	5926
77	6491	7054	7617	8179	8741	9302	9862	0421	0980	1537
78	89.2095	2651	3207	3767	4316	4870	5422	5975	6526	7077
79	7627	8176	8725	9273	9820	0367	0913	1458	2003	2547
80	90.3090	3632	4174	4715	5256	5796	6335	6873	7411	7948
81	8485	9021	9556	0090	0624	1158	1690	2222	2753	3284
82	91.3814	4343	4872	5400	5927	6454	6980	7505	8030	8554
83	9078	9601	0123	0645	1166	1686	2206	2725	3244	3762
84	92.4279	4796	5312	5828	6342	6856	7370	7883	8396	8908
85	9419	9930	0440	0949	1458	1966	2474	2981	3487	3993
86	93.4498	5003	5507	6011	6514	7016	7518	8019	8520	9020
87	9519	0018	0516	1014	1511	2008	2504	3000	3494	3989
88	94.4483	4976	5469	5961	6452	6943	7434	7924	8413	8902
89	9390	9878	0365	0851	1337	1823	2308	2792	3276	3760

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
90	95.4242	4725	5206	5688	6168	6649	7128	7607	8086	8564
91	9041	9518	9995	0471	0946	1421	1895	2369	2843	3315
92	96.3788	4260	4731	5202	5672	6142	6611	7080	7548	8016
93	8483	8950	9416	9882	0347	0812	1276	1740	2203	2665
94	97.3128	3590	4051	4512	4972	5432	5891	6350	6808	7266
95	7724	8180	8637	9093	9548	0003	0457	0912	1365	1819
96	98.2271	2723	3175	3626	4077	4527	4977	5426	5875	6324
97	6772	7219	7666	8113	8559	9005	9450	9895	0339	0783
98	99.1226	1669	2111	2553	2995	3436	3877	4317	4757	5196
99	5635	6074	6512	6946	7385	7823	8259	8695	9130	9565
100	00.0000	0434	0865	1301	1734	2165	2598	3029	3460	3891
101	4321	4751	5180	5609	6038	6466	6894	7321	7748	8174
102	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415
103	01.2837	3259	3680	4100	4520	4940	5360	5779	6197	6615
104	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775
105	02.1189	1603	2016	2428	2841	3252	3664	4075	4486	4896
106	5306	5715	6124	6533	6942	7350	7757	8164	8571	8978
107	9384	9799	0195	0600	1004	1408	1812	2216	2619	3021
108	03.3424	3826	4227	4628	5029	5430	5830	6229	6630	7028
109	7426	7825	8223	8620	9017	9414	9811	0207	0602	0998
110	04.1393	1787	2182	2575	2969	3362	3755	4148	4540	4931
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830
112	9218	9606	9993	0380	0766	1152	1538	1924	2309	2694
113	05.3078	3463	3846	4230	4613	4996	5378	5760	6142	6524
114	6905	7285	7666	8046	8426	8805	9185	9563	9942	0320
115	06.0698	1075	1452	1829	2206	2582	2958	3333	3709	4083
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7814
117	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514
118	07.1882	2250	2617	2985	3352	3718	4085	4451	4816	5181
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819
120	9181	9543	9904	0266	0626	0987	1347	1707	2067	2426
121	08.2785	3144	3503	3861	4219	4576	4934	5291	5647	6004
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552
123	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071
124	09.3422	3772	4122	4471	4820	5169	5518	5866	6215	6562
125	6910	7257	7604	7951	8297	8644	8990	9335	9681	0026
126	10.0371	0715	1059	1403	1747	2090	2434	2777	3119	3462
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6870
128	7210	7549	7888	8227	8565	8903	9241	9578	9916	0253
129	11.0590	0926	1262	1598	1934	2270	2605	2940	3275	3609
130	3943	4277	4611	4944	5278	5610	5943	6276	6608	6940
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245
132	12.0574	0903	1231	1560	1888	2216	2543	2871	3198	3525
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781
134	7105	7429	7752	8076	8399	8722	9045	9367	9690	0012
135	13.0334	0655	0977	1298	1619	1939	2260	2580	2900	3219
136	3539	3858	4177	4496	4814	5133	5451	5768	6086	6403
137	6721	7037	7354	7670	7987	8303	8618	8934	9249	9564
138	9879	0194	0508	0822	1136	1450	1763	2076	2384	2702
139	14.3015	3327	3639	3951	4263	4574	4885	5196	5507	5818

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
140	14.6128	6438	6748	7058	7367	7676	7985	8294	8603	8991
141	9219	9527	9835	0142	0449	0756	1063	1370	1676	1982
142	15.2288	2594	2900	3205	3510	3815	4119	4424	4728	5032
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061
144	8362	8664	8965	9266	9567	9868	0168	0468	0769	1068
145	16.1368	1667	1967	2266	2564	2863	3161	3460	3757	4055
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022
147	7317	7613	7908	8203	8497	8792	9085	9380	9674	9968
148	17.0261	355	0848	1141	1434	1726	2019	2311	2603	2895
149	3186	3478	3769	4060	4351	4641	4931	5222	5512	5802
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689
151	8977	9264	9552	9839	0126	0413	0699	0986	1272	1558
152	18.1844	2129	2415	2700	2985	3270	3554	3839	4123	4407
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	0051
155	19.0332	0612	0892	1171	1451	1730	2010	2289	2567	2846
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623
157	5900	6176	6452	6729	7005	7281	7556	7832	8107	8382
158	8657	8932	9206	9481	9755	0029	0303	0577	0850	1124
159	20.1397	1670	1943	2216	2488	2761	3033	3305	3577	3848
160	4120	4391	4662	4933	5204	5475	5745	6016	6286	6556
161	6826	7095	7365	7634	7903	8172	8441	8710	8978	9247
162	9515	9783	0051	0318	0585	0853	1120	1388	1654	1921
163	21.2188	2454	2720	2986	3252	3518	3783	4048	4314	4579
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221
165	7484	7747	8010	8273	8535	8798	9060	9322	9584	9846
166	22.0108	0370	0631	0892	1153	1414	1675	1936	2196	2456
167	2716	2976	3236	3494	3755	4015	4274	4533	4792	5051
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630
169	7881	8144	8400	8657	8913	9170	9426	9682	9938	0193
170	23.0449	0704	0960	1215	1470	1724	1979	2233	2488	2742
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795
173	8046	8297	8548	8799	9049	9299	9550	9799	0050	0300
174	24.0549	0799	1048	1297	1546	1795	2044	2293	2541	2790
175	3038	3286	3534	3782	4030	4277	4524	4772	5019	5266
176	5513	5759	6006	6252	6499	6745	6991	7236	7482	7728
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	0176
178	25.0420	0664	0908	1151	1395	1638	1881	2125	2367	2610
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031
180	5272	5514	5755	5996	6236	6477	6718	6958	7198	7439
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833
182	26.0071	0310	0548	0787	1025	1263	1501	1738	1976	2214
183	2451	2688	2922	3162	3399	3636	3873	4109	4345	4582
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279
186	9513	9746	9980	0213	0446	0679	0912	1144	1377	1609
187	27.1842	2074	2306	2538	2770	3001	3233	3464	3696	3927
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232
189	6462	6691	6921	7151	7380	7609	7838	8067	8296	8525



N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
190	72.8754	8982	9210	9439	9667	9895	0123	0351	0578	0806
191	28.1033	1261	1488	1715	1942	2169	2395	2622	2848	3075
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578
194	7802	8025	8249	8473	8696	8920	9143	9366	9589	9812
195	29.0035	0257	0480	0702	0925	1147	1369	1591	1813	2034
196	2256	2478	2699	2920	3141	3363	3583	3804	4025	4246
197	4466	4687	4907	5127	5347	5567	5787	6007	6225	6446
198	6665	6884	7104	7323	7542	7760	7979	8198	8416	8635
199	8853	9071	9289	9507	9725	9942	0160	0378	0595	0812
200	30.1030	1247	1464	1681	1898	2114	2331	2547	2764	2980
201	3196	3412	3628	3844	4059	4275	4490	4705	4921	5136
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417
204	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542
205	31.1754	1966	2177	2389	2600	2812	3023	3234	3445	3656
206	3867	4078	4289	4499	4710	4920	5130	5340	5550	5760
207	5970	6180	6390	6599	6809	7018	7227	7436	7645	7854
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938
209	32.0145	0354	0562	0769	0977	1184	1391	1598	1805	2012
210	2219	2425	2633	2839	3040	3252	3458	3664	3871	4077
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131
212	6336	6541	6745	6950	7154	7359	7563	7767	7972	8176
213	8380	8583	8787	8991	9194	9398	9601	9804	0008	0211
214	33.0414	0617	0819	1022	1225	1427	1630	1832	2034	2236
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253
216	4454	4655	4856	5056	5257	5458	5658	5859	6059	6260
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257
218	8456	8656	8855	9054	9253	9451	9650	9849	0047	0246
219	34.0444	0642	0840	1039	1237	1434	1632	1830	2028	2226
220	2423	2620	2817	3014	3212	3409	3605	3802	3999	4196
221	4372	4569	4765	4961	5158	5354	5550	5746	5941	6137
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110
223	8305	8500	8694	8889	9083	9277	9472	9666	9860	0054
224	35.0248	0442	0636	0829	1023	1216	1410	1603	1796	1989
225	2182	2375	2568	2761	2954	3146	3339	3532	3724	3917
226	4108	4301	4493	4684	4876	5068	5260	5451	5643	5835
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744
228	7935	8125	8316	8506	8696	8886	9076	9265	9456	9645
229	9835	0025	0215	0404	0593	0783	0972	1161	1350	1539
230	36.1718	1917	2105	2294	2482	2671	2859	3048	3236	3424
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5300
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169
233	7356	7542	7728	7915	8101	8287	8473	8659	8844	9030
234	9216	9401	9587	9772	9958	0143	0328	0513	0698	0883
235	37.1068	1253	1437	1622	1806	1991	2175	2360	2544	2729
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6395
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216
239	8398	8580	8761	8943	9124	9305	9487	9668	9849	0030

*The Logarithms of Natural Numbers to 2900.*

\*9

No.	0	1	2	3	4	5	6	7	8	9
40	38.0211	0392	0573	0754	0934	1115	1296	1476	1656	1837
41	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636
42	3815	3995	4174	4353	4533	4712	4891	5070	5249	5427
43	5606	5785	5964	6142	6321	6499	6677	6855	7034	7212
44	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989
45	9166	9343	9520	9697	9875	0051	0228	0405	0582	0758
46	39.0935	1112	1288	1464	1641	1817	1993	2169	2345	2521
47	2679	2873	3048	3224	3400	3575	3751	3926	4101	4276
48	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025
49	6199	6374	6548	6722	6896	7070	7245	7418	7592	7766
50	7940	8114	0287	8461	8634	8808	8981	9154	9327	9501
51	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228
52	40.1400	1573	1745	1917	2089	2261	2433	2605	2777	2949
53	3120	3292	3464	3635	3807	3978	4149	4320	4492	4663
54	4834	5005	5175	5346	5517	5688	5858	6029	6199	6370
55	6540	6710	6881	7051	7221	7391	7561	7731	7900	8070
56	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764
57	9933	0102	0271	0440	0608	0777	0946	1114	1283	1451
58	41.1620	1788	1956	2124	2292	2460	2628	2796	2964	3132
59	3300	3467	3635	3802	3970	4137	4305	4472	4639	4806
60	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474
61	6640	6807	6973	7139	7306	7472	7638	7804	7970	8135
62	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791
63	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439
64	42.1604	1768	1933	2097	2261	2426	2590	2754	2918	3082
65	3246	3410	3573	3737	3901	4064	4228	4392	4555	4718
66	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349
67	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973
68	8135	8297	8459	8621	8782	8944	9106	9268	9429	9591
69	9752	9914	0075	0236	0398	0559	0720	0881	1042	1203
70	43.1364	1525	1685	1846	2007	2167	2328	2488	2649	2809
71	2969	3129	3290	3450	3610	3770	3930	4090	4249	4409
72	4569	4728	4888	5048	5207	5366	5526	5685	5844	6003
73	6163	6322	6481	6640	6798	6957	7116	7275	7433	7592
74	7751	7909	8067	8226	8384	8542	8700	8859	9017	9175
75	9333	9491	9648	9806	9964	0122	0279	0437	0594	0752
76	44.0909	1066	1224	1381	1538	1695	1852	2009	2166	2323
77	2480	2636	2793	2950	3106	3263	3419	3576	3732	3888
78	4045	4201	4357	4513	4669	4825	4981	5137	5293	5448
79	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003
80	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552
81	8706	8861	9015	9170	9324	9478	9633	9787	9941	0095
82	45.0249	0403	0557	0711	0865	1018	1172	1326	1479	1633
83	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165
84	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692
85	4845	4997	5149	5302	5454	5606	5758	5910	6062	6214
86	6366	6518	6670	6821	6973	7125	7276	7428	7579	7730
87	7882	8033	8184	8336	8487	8638	8789	8940	9091	9241
88	9392	9543	9694	9845	9995	0146	0296	0447	0597	0747
89	46.0898	1048	1198	1348	1498	1649	1799	1948	2098	2248

No.	0	1	2	3	4	5	6	7	8	9
290	46.2398	2548	2697	2847	2997	3146	3296	3445	3594	3744
291	3893	4042	4191	4340	4489	4639	4787	4936	5085	5234
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200
294	8347	8495	8643	8790	8938	9085	9233	9381	9527	9675
295	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145
296	47.1292	1438	1585	1732	1878	2025	2171	2317	2464	2610
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4070
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976
300	7121	7266	7411	7555	7700	7844	7989	8133	8278	8422
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863
302	48.0007	0151	0294	0438	0582	0725	0869	1012	1156	1299
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157
305	4300	4442	4584	4727	4868	5011	5153	5295	5437	5579
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410
308	8551	8692	8833	8973	9114	9255	9396	9537	9677	9818
309	9958	0099	0239	0380	0520	0661	0801	0941	1081	1222
310	49.1362	1502	1642	1782	1922	2062	2201	2341	2481	2621
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406
313	5544	5683	5822	5960	6099	6237	6376	6514	6653	6791
314	6930	7068	7206	7344	7482	7621	7759	7897	8035	8173
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550
316	9687	9824	9962	0099	0236	0374	0511	0648	0785	0922
317	50.1059	1196	1333	1470	1607	1744	1880	2017	2154	2290
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3654
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014
320	5150	5286	5421	5557	5692	5828	5963	6099	6234	6370
321	6505	6640	6775	6911	7046	7181	7316	745	7586	7721
322	7856	7991	8125	8260	8395	8530	8664	8799	8933	9068
323	9202	9337	9471	9606	9740	9874	0008	0143	0277	0411
324	51.0545	0679	0813	0947	1081	1215	1348	1482	1616	1750
325	1883	2017	2150	2284	2417	2551	2684	2818	2951	3084
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415
327	4548	4680	4813	4946	5079	5211	5343	5476	5609	5741
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382
330	8514	8645	8777	8909	9040	9171	9303	9434	9565	9697
331	9828	9959	0090	0221	0352	0483	0614	0745	0876	1007
332	52.1138	1269	1400	1530	1661	1792	1922	2053	2183	2314
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915
335	5045	5174	5304	5434	5563	5692	5822	5951	6081	6210
336	6339	6468	6598	6727	6856	6985	7114	7243	7372	7501
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8789
338	8917	9045	9174	9302	9434	9559	9687	9815	9943	0071
339	53.0200	0328	0456	0584	0712	0840	0968	1095	1223	1351



*The Logarithms of Natural Numbers to 2900.*

\*11

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
340	53.1479	1607	1734	1862	1990	2117	2245	2372	2500	2627
341	2754	2882	3009	3136	3263	3391	3518	3645	3772	3899
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167
343	5294	5421	5547	5674	5800	5927	6053	6179	6306	6432
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951
346	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204
347	54.0329	0455	0580	0705	0830	0955	1080	1205	1330	1454
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183
351	5307	5431	5554	5678	5802	5925	6049	6172	6296	6419
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652
353	7775	7898	8021	8144	8266	8389	8512	8635	8758	8881
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106
355	55.0228	0351	0473	0595	0717	0840	0962	1084	1206	1328
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2546
357	2668	2790	2911	3033	3154	3276	3397	3519	3640	3762
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973
359	5094	5211	5336	5457	5578	5699	5820	5940	6061	6182
360	6302	6423	6544	6664	6785	6905	7026	7146	7266	7387
361	7507	7627	7748	7868	7988	8108	8228	8348	8469	8589
362	8709	8828	8948	9068	9188	9308	9428	9548	9667	9787
363	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982
364	56.1101	1221	1340	1459	1578	1697	1817	1936	2055	2174
365	2293	2412	2531	2650	2768	2887	3006	3125	3244	3362
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548
367	4666	4784	4903	5021	5139	5257	5375	5494	5612	5730
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084
370	8202	8319	8436	8554	8671	8778	8905	9023	9140	9257
371	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426
372	57.0543	0660	0776	0893	1010	1126	1243	1359	1476	1592
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915
375	4031	4147	4263	4378	4494	4610	4726	4841	4957	5072
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226
377	6341	6456	6572	6687	6802	6917	7032	7147	7262	7377
378	7492	7607	7721	7836	7951	8066	8181	8295	8410	8525
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669
380	9784	9898	0012	0126	0240	0355	0469	0583	0697	0811
381	58.0925	1039	1153	1267	1381	1494	1608	1722	1836	1950
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085
383	3199	3312	3425	3539	3652	3765	3879	3992	4105	4218
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348
385	5461	5573	5686	5799	5912	6024	6137	6250	6362	6475
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599
387	7711	7823	7925	8047	8160	8272	8384	8496	8608	8720
388	8831	8944	9055	9167	9279	9391	9503	9614	9726	9838
389	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953

\*B2

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
390	59.1065	1176	1287	1397	1510	1621	1732	1843	1955	2066
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282
393	4393	4503	4613	4724	4834	4945	5055	5165	5276	5386
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8682
397	8790	8900	9009	9119	9228	9337	9446	9556	9665	9774
398	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864
399	60.0973	1082	1190	1299	1408	1517	1625	1734	1843	1951
400	2060	2169	2277	2386	2494	2602	2711	2819	2928	3036
401	3144	3252	3361	3469	3577	3685	3794	3902	4010	4118
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197
403	5305	5413	5521	5628	5736	5843	5951	6059	6166	6274
404	6381	6489	6586	6704	6811	6918	7026	7133	7240	7348
405	7455	7562	7677	7777	7884	7991	8098	8205	8312	8419
406	8526	8633	8740	8847	8954	9060	9167	9274	9381	9488
407	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554
408	61.0600	0767	0873	0979	1086	1192	1298	1405	1511	1617
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678
410	2784	2890	2996	3101	3207	3313	3419	3525	3630	3736
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845
413	5950	6055	6160	6265	6370	6475	6580	6685	6790	6895
414	7000	7105	7210	7313	7420	7524	7629	7734	7839	7944
415	8048	8153	8257	8362	8466	8571	8675	8780	8884	8988
416	9093	9198	9302	9406	9511	9615	9719	9823	9928	0032
417	62.0136	0240	0344	0448	0552	0656	0760	0864	0968	1072
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2111
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146
420	3249	3353	3456	3559	3663	3766	3869	3972	4076	4179
421	4282	4385	4488	4591	4694	4798	4901	5004	5107	5210
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7264
424	7366	7468	7571	7673	7775	7878	7980	8082	8184	8287
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308
426	9410	9511	9613	9715	9817	9919	0021	0123	0224	0326
427	63.0428	0530	0631	0733	0834	0936	1038	1139	1241	1343
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356
429	5427	2558	2660	2761	2862	2963	3064	3165	3266	3367
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4377
431	4477	4578	4679	4779	4881	4981	5081	5182	5283	5384
432	5484	5584	5685	5785	5886	5986	6086	6187	6287	6388
433	6488	6588	6688	6789	6889	6989	7089	7189	7289	7390
434	7490	7590	7690	7790	7890	7990	8090	8190	8289	8390
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9388
436	9486	9586	9685	9785	9885	9984	0084	0183	0283	0384
437	64.0841	0581	0680	0779	0879	0978	1077	1176	1276	1376
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2367
439	2464	2563	2662	2761	2860	2959	3058	3156	3255	3355

*The Logarithms of Natural Numbers to 4900. \*13*

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
440	643.453	551	650	749	847	946	044	143	242	340
441	644.439	537	635	734	832	931	029	127	226	324
442	645.422	520	619	717	815	913	011	109	208	306
443	646.404	502	600	698	796	894	991	089	187	285
444	647.383	481	579	676	774	872	969	067	165	262
445	648.360	458	555	653	750	848	945	043	140	237
446	649.335	432	530	627	724	821	919	016	113	210
447	650.307	405	502	599	696	793	890	987	084	181
448	651.278	375	472	569	666	762	859	956	053	150
449	652.246	343	440	536	633	730	826	923	019	116
450	653.212	309	405	502	598	695	791	888	984	080
451	654.176	273	369	465	562	658	754	850	946	042
452	655.138	234	331	427	523	619	714	810	906	002
453	656.098	194	290	386	481	577	673	769	864	960
454	657.056	151	247	343	438	534	629	725	820	916
455	658.051	107	202	298	393	488	584	679	774	870
456	965	060	155	250	346	441	537	631	726	821
457	659.916	011	106	201	296	391	486	581	676	771
458	660.865	960	055	150	245	340	434	529	623	718
459	661.813	907	002	096	191	285	380	474	569	663
460	662.758	852	947	041	135	230	324	418	512	607
461	663.701	795	889	983	078	172	266	360	454	548
462	664.642	736	830	924	018	112	205	299	393	487
463	665.581	675	769	862	956	050	143	237	331	424
464	666.518	612	705	799	892	986	079	173	266	359
465	667.453	546	640	733	826	920	013	106	199	293
466	668.386	479	572	665	758	852	945	038	131	224
467	669.317	410	503	596	689	782	874	967	060	153
468	670.246	339	431	524	617	710	802	895	988	080
469	671.173	265	358	451	543	636	728	821	913	005
470	672.098	190	283	375	467	560	652	744	836	929
471	673.021	113	205	297	390	482	574	666	758	850
472	942	034	126	218	310	402	494	586	677	769
473	674.861	953	045	136	228	320	412	503	595	687
474	675.778	870	961	053	145	236	328	419	511	602
475	676.694	785	876	968	059	150	242	333	424	516
476	677.607	698	789	881	972	063	154	245	336	427
477	678.518	609	700	791	882	973	064	155	246	337
478	679.428	519	610	700	791	882	973	063	154	245
479	680.335	426	517	607	698	789	879	970	060	151
480	681.241	332	422	513	603	693	784	874	964	055
481	682.145	235	326	416	506	596	686	777	867	957
482	683.047	137	227	317	407	497	587	667	767	857
483	947	037	127	217	307	396	486	576	666	756
484	684.845	935	025	114	204	294	383	473	563	652
485	685.742	831	921	010	100	189	279	368	457	547
486	686.636	726	815	904	994	083	172	261	351	440
487	687.529	618	707	796	885	975	064	153	242	331
488	688.420	509	598	687	776	865	953	042	131	220
489	689.309	398	486	575	664	753	841	930	019	107



No	0	1	2	3	4	5	6	7	8	9
490	690.196	285	373	462	550	639	727	816	905	993
491	691.081	170	258	347	435	523	612	700	788	877
492	965	053	142	230	318	406	494	583	671	759
493	692.847	935	023	111	199	287	375	463	551	639
494	693.727	815	403	991	078	166	254	342	430	517
495	694.605	693	781	868	956	044	131	219	306	394
496	695.482	569	657	744	832	919	007	094	182	269
497	696.356	444	531	618	706	793	880	968	055	142
498	697.229	316	404	491	578	665	752	839	926	013
499	698.100	188	275	362	448	535	622	709	796	883
500	970	057	144	230	317	404	491	578	664	751
501	699.838	924	011	098	184	271	357	444	531	617
502	700.703	790	877	963	050	136	222	309	395	482
503	701.568	654	741	827	913	999	086	172	258	344
504	702.430	517	603	689	775	861	947	033	119	205
505	703.291	377	463	549	635	721	807	893	979	065
506	704.150	236	322	408	494	579	665	751	837	922
507	705.008	094	179	265	350	436	522	607	693	778
508	864	949	034	120	205	291	376	462	547	632
509	706.718	803	888	974	059	144	229	315	400	485
510	707.570	655	740	826	911	996	081	166	251	336
511	708.421	506	591	676	761	846	930	015	100	185
512	709.270	355	440	524	609	694	779	863	948	033
513	710.117	202	287	371	456	540	625	710	794	879
514	963	048	132	216	301	385	470	554	638	723
515	711.807	891	976	060	144	229	313	397	481	565
516	712.650	734	818	902	986	070	154	238	322	406
517	713.490	574	658	742	826	910	994	078	162	246
518	714.330	414	497	581	665	749	832	916	000	084
519	715.167	251	335	418	502	586	669	753	836	920
520	716.003	087	170	254	337	421	504	588	671	754
521	838	921	004	088	171	254	338	421	504	588
522	717.670	754	837	920	003	086	169	252	336	419
523	718.502	585	668	751	834	917	000	083	165	248
524	719.331	414	497	580	663	745	828	911	994	077
525	720.159	242	325	407	490	573	655	738	821	903
526	986	068	151	233	316	398	481	563	646	728
527	721.811	893	975	058	140	222	305	387	469	552
528	722.634	716	798	881	963	045	127	209	291	374
529	723.456	538	620	702	784	866	948	030	112	194
530	724.276	358	440	522	603	685	767	849	931	013
531	725.094	176	258	340	422	503	585	667	748	830
532	912	993	075	156	238	320	401	483	564	646
533	726.727	809	890	972	053	134	216	297	379	460
534	727.541	623	704	785	866	948	029	110	191	273
535	728.354	435	516	597	678	759	841	922	003	084
536	729.165	246	327	408	489	570	651	732	812	893
537	974	055	136	217	298	378	459	540	621	701
538	730.782	863	944	024	105	186	266	347	428	508
539	731.589	669	750	830	911	991	072	152	233	313

*The Logarithms of Natural Numbers to 5900. \*15*

N <sup>o</sup>	0	1	2	3	4	5	6	7	8	9
540	732.394	474	555	635	715	796	876	956	037	117
541	733.197	277	358	438	518	598	679	759	839	919
542	999	079	159	240	320	400	480	560	640	720
543	734.800	880	960	040	120	200	279	359	439	519
544	735.599	679	758	838	918	998	078	157	237	317
545	736.396	476	556	635	715	795	874	954	033	113
546	737.192	272	352	431	511	590	670	749	828	908
547	987	067	146	225	305	384	463	542	622	701
548	738.781	860	939	018	097	177	256	335	414	493
549	739.572	651	730	810	889	968	047	126	205	284
550	740.363	442	521	599	678	757	836	915	994	073
551	741.152	230	309	388	467	545	624	703	782	860
552	939	018	096	175	254	332	411	489	568	647
553	742.725	804	882	961	039	118	196	274	353	431
554	743.510	588	666	745	823	902	980	058	136	215
555	744.293	371	449	528	606	684	762	840	918	997
556	745.075	153	231	309	387	465	543	621	699	777
557	855	933	011	089	167	245	323	401	478	556
558	746.634	712	790	868	945	023	101	179	256	334
559	747.412	489	567	645	722	800	878	955	033	110
560	748.188	256	343	421	498	576	653	731	808	885
561	963	040	118	195	272	350	427	504	582	659
562	749.736	814	891	968	045	122	200	277	354	431
563	750.508	585	663	740	817	894	971	048	125	202
564	751.279	356	433	510	587	664	741	818	895	972
565	752.048	125	202	279	356	433	509	586	663	740
566	816	893	970	047	123	200	277	353	430	506
567	753.583	660	736	813	889	966	042	119	195	272
568	754.348	425	501	578	654	730	807	883	959	036
569	755.112	189	265	341	417	494	570	645	722	799
570	875	951	027	103	179	256	332	408	484	560
571	756.636	712	788	864	940	016	092	168	244	320
572	757.396	472	548	624	700	775	851	927	003	079
573	758.155	230	306	382	458	533	609	685	760	836
574	912	987	063	139	214	290	366	441	517	592
575	759.668	743	819	894	970	045	121	196	272	347
576	760.422	498	573	649	724	799	875	950	025	100
577	761.176	251	326	402	477	552	627	702	777	853
578	928	003	078	153	228	303	378	453	528	603
579	762.679	754	829	903	978	053	128	203	278	353
580	763.428	503	568	633	727	802	877	952	027	101
581	764.176	251	326	400	475	550	624	699	774	848
582	923	998	072	147	221	296	370	445	519	594
583	765.669	743	817	892	966	041	115	190	264	338
584	766.413	487	562	636	710	784	859	933	007	082
585	767.156	230	304	378	453	527	601	675	749	823
586	898	972	046	120	194	268	342	416	490	564
587	768.638	712	786	860	934	008	082	156	230	303
588	769.377	451	525	599	673	746	820	894	968	041
589	770.115	189	263	336	410	484	557	631	705	778

\*16 *The Logarithms of Natural Numbers to 6400.*

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
590	770.852	926	999	073	146	220	293	367	440	514
591	771.587	661	734	808	881	955	028	102	175	248
592	772.322	395	468	542	615	688	762	835	908	981
593	773.055	128	201	274	347	420	494	567	640	713
594	773.786	859	933	006	079	152	225	298	371	444
595	774.517	590	663	736	809	882	955	028	100	173
596	775.246	319	392	465	538	610	683	756	829	902
597	775.974	047	120	192	265	338	411	483	556	628
598	776.701	774	846	919	992	064	137	209	282	354
599	777.427	499	572	644	717	789	862	934	006	079
600	778.151	224	296	368	441	513	585	658	730	802
601	778.874	947	019	091	163	236	308	380	452	524
602	779.596	669	141	813	885	957	029	101	173	245
603	780.317	389	461	533	605	677	749	821	893	965
604	781.037	109	181	253	324	396	468	540	612	684
605	781.755	827	899	971	042	114	186	258	329	401
606	782.473	544	616	688	759	831	902	974	046	117
607	783.189	260	332	403	475	546	618	689	761	832
608	783.904	975	046	118	189	261	332	403	475	546
609	784.617	689	760	831	902	974	045	116	187	259
610	785.330	401	472	543	614	686	757	828	899	970
611	786.041	112	183	254	325	396	467	538	609	680
612	786.751	822	893	964	035	106	177	248	319	390
613	787.460	531	602	673	744	815	885	956	027	098
614	788.168	239	310	380	451	522	593	663	734	804
615	788.875	946	016	087	157	228	299	369	440	510
616	789.581	651	722	792	863	933	003	074	144	215
617	790.285	355	426	496	567	637	707	778	848	918
618	790.988	059	129	199	269	340	410	480	550	620
619	791.691	761	831	901	971	041	111	181	252	322
620	792.392	462	532	602	672	742	812	882	952	022
621	793.092	161	231	301	371	441	511	581	651	721
622	793.790	860	930	000	070	139	209	279	349	418
623	794.488	558	627	697	767	836	906	976	045	115
624	795.185	254	324	393	463	532	602	671	741	810
625	795.880	949	019	088	158	227	297	366	436	505
626	796.574	644	713	782	852	921	990	060	129	198
627	797.267	337	406	475	544	614	683	752	821	890
628	797.960	029	098	167	236	305	374	443	512	582
629	798.651	720	789	858	927	996	065	134	203	272
630	799.340	409	478	547	616	685	754	823	892	960
631	800.029	098	167	236	305	373	442	511	580	648
632	800.717	786	854	923	992	060	129	198	266	335
633	801.404	472	541	609	678	747	815	884	952	021
634	802.089	158	226	295	363	432	500	568	637	705
635	802.774	842	910	979	047	115	184	252	320	389
636	803.457	525	594	662	730	798	867	935	003	071
637	804.139	208	276	344	412	480	548	616	684	753
638	804.821	889	957	025	093	161	229	297	365	433
639	805.501	569	637	705	773	840	908	976	044	112



N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
640	806.180	248	316	383	451	519	587	655	722	790
641	858	926	993	061	129	197	264	332	400	467
642	807.535	603	670	738	805	873	941	008	076	143
643	808.211	278	346	413	481	548	616	683	751	818
644	886	953	021	088	155	223	290	358	425	492
645	809.560	627	694	762	829	896	963	031	098	165
646	810.232	300	367	434	501	568	636	703	770	837
647	904	971	038	106	173	240	307	374	440	508
648	811.575	642	709	776	843	910	977	044	111	178
649	812.245	312	378	445	512	579	646	713	780	846
650	913	980	047	114	180	247	314	381	447	514
651	813.581	648	714	781	848	914	981	048	114	181
652	814.248	314	381	447	514	580	647	714	780	847
653	913	980	046	113	179	246	312	378	445	511
654	815.578	644	710	777	843	910	976	042	109	175
655	816.241	308	374	440	506	573	639	705	771	838
656	904	970	036	102	169	235	301	367	433	499
657	817.565	632	697	764	830	896	962	028	094	160
658	818.226	292	358	424	490	556	622	688	754	819
659	885	951	017	083	149	215	281	346	412	478
660	819.544	610	675	741	807	873	939	004	070	136
661	820.201	267	333	398	464	530	595	661	727	792
662	858	924	989	055	120	186	251	317	382	448
663	821.513	579	644	710	775	841	906	972	037	103
664	822.168	233	299	364	430	495	560	626	691	756
665	822	887	952	017	083	148	213	279	344	409
666	823.474	539	605	670	735	800	865	930	996	061
667	824.126	191	256	321	386	451	516	581	646	711
668	776	841	906	971	036	101	166	231	296	361
669	825.426	491	556	621	686	751	815	880	945	010
670	826.075	140	204	269	334	399	463	528	593	658
671	722	787	852	917	981	046	111	175	240	305
672	827.369	434	498	563	628	692	757	821	886	950
673	828.015	080	144	209	273	338	402	466	531	595
674	660	724	789	853	918	982	046	111	175	239
675	829.304	368	432	497	561	625	690	754	818	882
676	947	011	075	139	204	268	332	396	460	524
677	830.589	653	717	781	845	909	973	037	102	166
678	831.230	294	358	422	486	550	614	678	742	806
679	870	934	998	062	125	189	253	317	381	445
680	832.509	573	637	700	764	828	892	956	019	083
681	833.147	211	275	338	402	466	530	593	657	721
682	784	848	912	975	039	103	166	230	293	357
683	834.421	484	548	611	675	738	802	866	929	993
684	835.056	120	183	246	310	373	437	500	564	627
685	691	754	817	881	944	007	071	134	197	261
686	836.324	387	451	514	577	640	704	767	830	893
687	957	020	083	146	209	273	336	399	462	525
688	837.588	652	715	778	841	904	967	030	093	156
689	838.219	282	345	408	471	534	597	660	723	786

\*18 *The Logarithms of Natural Numbers to 7400.*

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
690	835.849	912	975	038	101	164	227	289	352	415
691	839.478	541	604	667	729	792	855	918	981	043
692	840.106	169	232	294	357	420	482	545	608	671
693	733	796	859	921	984	046	109	172	234	297
694	841.359	422	485	547	610	672	735	797	860	922
695	934	047	110	172	235	297	360	422	484	547
696	842.609	672	734	796	859	921	983	046	108	170
697	843.233	295	357	420	482	544	606	669	731	793
698	855	918	980	042	104	166	229	291	353	415
699	844.477	539	601	663	726	788	850	912	974	036
700	845.098	160	222	284	346	408	470	532	594	656
701	718	780	842	904	966	028	090	151	213	275
702	846.337	399	461	523	584	646	708	770	832	893
703	955	017	079	141	202	264	326	388	449	511
704	847.573	634	696	758	819	881	943	004	067	127
705	848.189	251	312	374	435	497	559	620	682	743
706	805	866	928	989	051	112	174	236	296	358
707	849.419	481	542	604	665	726	788	849	911	972
708	850.033	095	156	217	279	340	401	462	523	585
709	646	707	769	830	891	952	014	075	136	197
710	851.258	319	381	442	503	564	625	686	747	808
711	870	931	992	053	114	175	236	297	358	419
712	852.480	541	602	663	724	785	846	907	968	029
713	853.089	150	211	272	333	394	455	516	576	637
714	698	759	820	881	942	002	063	124	184	245
715	854.306	367	427	488	549	610	670	731	792	852
716	913	974	034	095	156	216	277	337	398	459
717	855.519	580	640	701	761	822	882	943	003	064
718	856.124	185	245	306	366	427	487	548	608	668
719	729	789	850	910	970	031	091	151	212	272
720	857.332	393	453	513	574	634	694	754	815	875
721	935	995	056	116	176	236	296	357	417	477
722	858.537	597	657	718	778	838	898	958	018	079
723	859.138	198	258	318	378	438	499	559	619	679
724	739	798	858	918	978	038	098	158	218	278
725	860.338	398	458	518	578	637	697	757	817	877
726	937	996	056	116	176	236	295	355	415	475
727	861.534	594	654	714	773	833	893	952	012	072
728	862.131	191	251	310	370	430	489	549	608	668
729	727	787	846	906	966	025	085	144	204	263
730	863.323	382	442	501	561	620	680	739	798	858
731	917	977	036	096	155	214	274	333	392	452
732	864.511	570	630	689	748	808	867	926	985	045
733	865.104	163	222	282	341	400	459	518	578	637
734	696	755	814	873	933	992	051	110	169	228
735	866.287	346	405	465	524	583	642	701	760	819
736	878	937	996	055	114	173	232	291	350	409
737	867.467	526	585	644	703	762	821	880	939	997
738	868.056	115	174	233	292	350	409	468	527	586
739	644	703	762	821	879	938	997	056	114	173

N <sup>o</sup>	0	1	2	3	4	5	6	7	8	9
740	869.232	290	349	408	466	525	584	642	701	760
741	818	877	935	994	053	111	170	228	287	345
742	870.404	462	521	579	638	696	755	813	872	930
743	989	047	106	164	223	281	339	398	456	515
744	871.573	631	690	748	806	865	923	981	040	098
745	872.156	215	273	331	389	449	506	564	622	681
746	739	797	855	913	972	030	088	146	204	262
747	873.321	378	437	495	553	611	669	727	785	843
748	902	960	017	076	134	192	250	308	366	424
749	874.482	540	598	656	714	772	830	887	945	003
750	875.061	119	177	235	293	351	409	466	524	582
751	640	698	756	813	871	929	987	045	102	160
752	876.218	276	333	391	449	506	564	622	680	737
753	795	853	910	968	026	083	141	198	256	314
754	877.371	429	486	544	602	659	717	774	832	889
755	947	004	062	119	177	234	292	349	407	464
756	878.522	579	637	694	751	809	866	924	981	038
757	879.096	153	211	268	325	383	440	497	555	612
758	669	726	784	841	898	956	013	070	127	185
759	880.242	299	356	413	471	528	585	642	699	756
760	814	871	928	985	042	099	156	213	270	328
761	881.385	442	499	556	613	670	727	784	841	898
762	955	012	069	126	183	240	297	354	411	468
763	882.524	581	638	695	752	809	866	923	980	036
764	883.093	150	207	264	321	377	434	491	548	605
765	661	718	775	832	888	945	002	059	115	172
766	884.229	285	342	399	455	512	569	625	682	739
767	795	852	909	965	022	078	135	191	248	305
768	885.361	418	474	531	587	644	700	757	813	870
769	926	983	039	096	152	209	265	321	378	434
770	886.491	547	603	660	716	773	829	885	942	998
771	887.054	111	167	223	280	336	392	448	505	561
772	617	674	730	786	842	898	955	011	067	123
773	888.179	236	292	348	404	460	516	573	629	685
774	741	797	853	909	965	021	077	134	190	246
775	889.302	358	414	470	526	582	638	694	750	806
776	862	918	974	030	085	141	197	253	309	365
777	890.421	477	533	589	644	700	756	812	868	924
778	980	035	091	147	203	259	314	370	426	482
779	891.537	593	649	705	760	816	872	927	983	039
780	892.095	150	206	262	317	373	428	484	540	595
781	651	707	762	818	873	929	985	040	096	151
782	893.207	262	318	373	429	484	540	595	651	706
783	762	817	873	928	984	039	094	150	205	261
784	894.316	371	427	482	538	593	648	704	759	814
785	870	925	980	036	091	146	201	257	312	367
786	895.422	475	533	588	643	699	754	809	864	919
787	975	030	085	140	195	251	306	361	416	471
788	896.526	581	636	691	747	802	857	912	967	022
789	897.077	132	187	242	297	352	407	462	517	572



N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
790	897.627	682	737	792	847	902	957	012	067	122
791	898.176	231	286	341	396	451	506	561	615	670
792	725	780	835	890	944	999	054	109	164	218
793	899.273	328	383	437	492	547	602	656	711	766
794	820	875	930	985	039	094	149	203	258	312
795	900.367	422	476	531	585	640	695	749	804	858
796	913	958	022	077	131	186	240	295	349	404
797	901.458	513	567	622	676	731	785	840	894	948
798	902.003	057	112	166	220	275	329	384	438	492
799	547	601	655	710	764	818	873	927	981	035
800	903.090	144	198	253	307	361	416	470	524	578
801	632	687	741	795	849	903	958	012	066	120
802	904.174	228	283	337	391	445	499	553	607	661
803	715	770	824	878	932	986	040	094	148	202
804	905.256	310	364	418	472	526	580	634	688	742
805	796	850	904	958	012	065	119	173	227	281
806	906.335	389	443	497	550	604	658	712	766	820
807	873	927	981	035	089	142	196	250	304	358
808	907.411	465	519	573	626	680	734	787	841	895
809	948	002	056	109	163	217	270	324	378	431
810	908.485	539	592	646	699	753	807	860	914	967
811	909.021	074	128	181	235	288	342	395	449	502
812	556	609	663	716	770	823	877	930	984	037
813	910.090	144	197	251	304	358	411	464	518	571
814	624	678	731	784	838	891	944	998	051	104
815	911.158	211	264	317	371	424	477	530	584	637
816	690	743	797	850	903	956	009	063	116	169
817	912.222	275	328	381	435	488	541	594	647	700
818	753	806	859	913	966	019	072	125	178	231
819	913.254	337	390	443	496	549	602	655	708	761
820	814	867	920	973	026	079	131	184	237	290
821	914.343	396	449	502	555	608	660	713	766	819
822	872	925	977	030	083	136	189	241	294	347
823	915.400	453	505	558	611	664	716	769	822	874
824	927	980	033	085	138	191	243	296	349	401
825	916.454	507	559	612	664	717	770	822	875	927
826	950	033	085	138	190	243	295	348	400	453
827	917.505	558	610	663	715	768	820	873	925	978
828	918.030	083	135	188	240	292	345	397	450	502
829	554	607	660	712	764	816	869	921	973	026
830	919.078	130	183	235	287	340	392	444	496	549
831	601	653	705	758	810	862	914	967	019	071
832	920.123	175	228	280	332	384	436	489	541	593
833	645	697	749	801	853	906	958	010	062	114
834	921.166	218	270	322	374	426	478	530	582	634
835	686	738	790	842	894	946	998	050	102	154
836	922.206	258	310	362	414	466	518	570	622	674
837	725	777	829	881	933	985	037	088	140	192
838	923.244	296	348	399	451	503	555	607	658	710
839	762	814	865	917	969	021	072	124	176	228

*The Logarithms of Natural Numbers to 8900* \*21

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
840	924.279	331	383	434	486	538	589	641	693	744
841	796	848	899	951	002	054	106	157	209	260
842	925.312	364	415	467	518	570	621	673	724	776
843	827	879	931	982	034	085	137	188	239	291
844	926.342	394	445	497	548	600	651	702	754	805
845	857	908	959	011	062	114	165	216	268	319
846	927.370	422	473	524	575	627	678	730	781	832
847	883	935	986	037	088	140	191	242	293	345
848	928.396	447	498	549	601	652	703	754	805	856
849	908	959	010	061	112	163	214	269	317	368
850	929.419	470	521	572	623	674	725	776	827	878
851	930	981	032	083	134	185	236	287	338	389
852	930.440	491	541	592	643	694	745	796	847	898
853	949	000	051	102	153	203	254	305	356	407
854	931.458	509	560	610	661	712	763	814	864	915
855	966	017	068	118	169	220	271	321	372	423
856	932.474	524	575	626	677	727	778	829	879	930
857	981	031	082	133	183	234	285	335	386	437
858	933.487	538	588	639	690	740	791	841	892	943
859	993	044	094	145	195	246	296	347	397	448
860	934.498	549	599	650	700	751	801	852	902	953
861	935.003	054	104	154	205	255	306	356	406	457
862	507	558	608	658	709	759	809	860	910	960
863	936.011	061	111	162	212	262	313	363	413	463
864	514	564	614	664	715	765	815	865	916	966
865	937.016	066	116	167	217	267	317	367	418	468
866	518	568	618	668	718	769	819	869	919	969
867	938.019	069	119	169	219	269	319	370	420	470
868	520	570	620	670	720	770	820	870	920	970
869	939.020	070	120	170	220	270	319	369	419	469
870	519	569	619	669	719	769	819	868	918	968
871	940.018	068	118	168	218	267	317	367	417	467
872	516	566	616	666	716	765	815	865	915	964
873	941.014	064	114	163	213	263	313	362	412	462
874	511	561	611	660	710	760	809	859	909	958
875	942.008	058	107	157	206	256	306	355	405	454
876	504	554	603	653	702	752	801	851	900	950
877	943.000	049	099	148	198	247	297	346	396	445
878	494	544	593	643	692	742	791	841	890	939
879	989	038	088	137	186	236	285	335	384	433
880	944.483	532	581	631	680	729	779	828	877	927
881	976	025	074	124	173	222	272	321	370	419
882	945.469	518	567	616	665	715	764	813	862	911
883	961	010	059	108	157	207	256	305	354	403
884	946.452	501	550	600	649	698	747	796	845	894
885	943	992	041	090	139	189	238	287	336	385
886	947.434	483	532	581	630	679	728	777	826	875
887	924	973	021	070	119	168	217	266	315	364
888	948.413	462	511	560	608	657	706	755	804	853
889	902	951	000	048	097	146	195	244	292	341

\*22 *The Logarithms of Natural Numbers to 9400.*

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
890	949.390	439	488	536	585	634	683	731	780	829
891	878	926	975	024	073	121	170	219	267	316
892	950.365	413	462	511	560	608	657	705	754	803
893	851	900	949	997	046	095	143	192	240	289
894	951.337	386	435	483	532	580	629	677	725	774
895	283	872	920	969	017	065	114	163	211	259
896	952.308	356	405	453	502	550	599	647	695	744
897	792	841	889	938	986	034	083	131	180	228
898	953.276	325	373	421	470	518	566	615	663	711
899	760	808	856	905	953	001	049	098	146	194
900	954.242	291	339	387	435	484	532	580	628	677
901	725	773	821	869	918	966	014	062	110	158
902	955.206	255	303	351	399	447	495	543	592	640
903	688	736	784	832	880	928	976	024	072	120
904	956.168	216	264	312	361	409	457	505	553	601
905	649	697	744	792	840	888	936	984	032	080
906	957.128	176	224	272	320	368	416	464	511	559
907	607	655	703	751	799	847	894	942	990	038
908	958.086	134	181	229	277	325	373	420	468	516
909	564	612	659	707	755	803	850	898	946	994
910	959.041	084	137	184	232	280	328	375	423	471
911	518	566	614	661	709	757	804	852	900	947
912	995	042	090	138	185	233	280	328	376	423
913	960.471	518	566	613	661	709	756	804	851	899
914	946	994	041	089	136	184	231	279	326	374
915	961.421	469	516	563	611	658	706	753	801	848
916	896	943	990	038	085	132	180	227	275	322
917	962.369	417	464	511	559	606	653	701	748	795
918	843	890	937	985	032	079	126	174	221	268
919	963.315	363	410	457	504	552	599	646	693	741
920	788	835	882	929	977	024	071	118	165	212
921	964.260	307	354	401	448	495	542	590	637	684
922	731	778	825	872	919	966	013	060	108	155
923	965.202	249	296	343	390	437	484	531	578	625
924	672	719	766	813	860	907	954	001	048	095
925	966.142	189	236	283	329	376	423	470	517	564
926	611	658	705	752	798	845	892	939	986	033
927	967.080	127	173	220	267	314	361	408	454	501
928	548	595	642	688	735	782	829	875	922	969
929	968.016	062	109	156	203	249	296	343	389	436
930	483	530	576	623	670	716	763	810	856	903
931	950	996	043	090	136	183	229	276	323	369
932	969.416	462	509	556	602	649	695	742	788	835
933	882	929	975	021	068	114	161	207	254	300
934	970.347	393	440	486	533	579	626	672	719	765
935	812	858	904	951	997	044	090	137	183	229
936	971.276	322	369	415	461	508	554	600	647	693
937	740	786	832	879	925	971	018	064	110	156
938	972.203	249	295	342	388	434	480	527	573	619
939	665	712	758	804	851	897	943	989	035	082



*The Logarithms of Natural Numbers to 9900.* \*23

N <sup>o</sup>	0	1	2	3	4	5	6	7	8	9
940	973.128	174	220	265	313	359	405	451	497	543
941	590	636	682	728	774	820	866	913	959	005
942	974.051	097	143	189	235	281	327	373	420	466
943	512	558	604	650	696	742	788	834	880	926
944	972	018	064	110	156	202	248	294	340	386
945	975.432	478	524	570	616	661	707	753	799	845
946	891	937	983	029	075	121	166	212	258	304
947	976.350	396	442	487	533	579	625	671	717	762
948	808	854	900	946	991	037	083	129	175	220
949	977.266	312	358	403	449	495	541	586	632	678
950	724	769	815	861	906	952	997	043	089	135
951	978.180	226	272	317	363	409	454	500	546	591
952	637	683	728	774	819	865	911	956	002	047
953	979.093	138	184	230	275	321	366	412	457	503
954	548	594	639	685	730	776	821	867	912	958
955	980.003	049	094	140	185	231	276	322	367	412
956	458	503	549	594	640	685	730	776	821	867
957	912	957	003	048	093	139	184	229	275	320
958	981.365	411	456	501	547	592	637	683	728	773
959	819	864	909	954	000	045	090	135	181	226
960	982.271	316	362	407	452	497	543	588	633	678
961	723	769	814	859	904	949	994	040	085	130
962	983.175	220	265	310	356	401	446	491	536	581
963	626	671	716	762	807	852	897	942	987	032
964	984.077	122	167	212	257	302	347	392	437	482
965	527	572	617	662	707	752	797	842	887	932
966	977	022	067	112	157	202	247	292	337	382
967	985.426	471	516	561	606	651	696	741	786	830
968	875	920	965	010	055	100	144	189	234	279
969	986.324	369	413	458	503	548	593	637	682	727
970	772	816	861	906	951	995	040	085	130	174
971	987.219	264	309	353	398	443	487	532	577	622
972	666	711	756	800	845	890	934	979	024	068
973	988.113	157	202	247	291	336	381	425	470	514
974	559	603	648	693	737	782	826	871	915	960
975	989.005	049	094	138	183	227	272	316	361	405
976	450	494	539	583	628	672	717	761	806	850
977	895	939	983	028	072	117	161	206	250	294
978	990.339	383	428	472	516	561	605	650	694	738
979	783	827	871	916	960	004	049	093	137	182
980	991.226	270	315	359	403	448	492	536	580	625
981	669	713	757	802	846	890	934	979	023	067
982	992.111	156	200	244	288	333	377	421	465	509
983	553	598	642	686	730	774	818	863	907	951
984	995	039	083	127	172	216	260	304	348	392
985	993.436	480	524	568	612	657	701	745	789	833
986	877	921	965	009	053	097	141	185	229	273
987	994.317	361	405	449	493	537	581	625	669	713
988	757	801	845	889	933	977	021	064	108	152
989	995.196	240	284	328	372	416	460	504	547	591

\* 24 *The Logarithms of Natural Numbers to 10000.*

N <sup>o</sup> .	0	1	2	3	4	5	6	7	8	9
990	995.635	679	723	767	811	854	898	942	986	030
991	996.074	117	161	205	249	293	336	380	424	468
992	512	555	599	643	687	730	774	818	862	905
993	949	993	037	080	124	168	212	255	299	343
994	997.386	430	474	517	561	605	648	692	736	779
995	823	867	910	954	998	041	085	128	172	216
996	998.259	303	346	390	434	477	521	564	608	652
997	695	739	782	826	869	913	956	000	043	087
998	999.130	174	218	261	305	348	392	435	478	522
999	565	609	652	696	739	783	826	870	913	957

A  
T A B L E  
O F

*Artificial or Logarithmic*  
SINES and TANGENTS,  
*To every Degree and Minute of the*  
*Quadrant.*



# A T A B L E

O F

*Artificial or Logarithmic*

## S I N E S,

*To every Degree and Minute of the  
Quadrant.*

<i>A Table of Logarithmic Sines.</i>						
Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min.
0	0	8.241855	8.542819	8.718800	8.843584	0
1	6.463726	49032	46422	21204	5387	1
2	6.764756	56092	49995	23595	7183	2
3	6.940347	63042	53539	25972	8971	3
4	7.055786	69881	57054	28337	8.850751	4
5	7.162696	76614	60540	8.730688	2524	5
6	7.241877	83243	63999	3027	4290	6
7	7.308824	89773	67431	5353	6049	7
8	7.366816	96207	70836	7667	7801	8
9	7.417968	8.302546	74214	9969	9546	9
10	7.463725	08794	77566	8.742259	8.861283	10
11	7.505118	14954	80892	4536	3014	11
12	7.542906	21027	84193	6801	4738	12
13	7.577668	27016	87469	9055	6454	13
14	7.609353	32924	90721	8.751297	8165	14
15	7.639316	38753	93948	3528	9868	15
16	7.667844	44504	97152	5747	8.871565	16
17	7.694173	50180	8.600332	7955	3255	17
18	7.718997	55783	03489	8.760151	4938	18
19	7.742477	61315	06623	2337	6615	19
20	7.764754	66777	09734	4511	8285	20

# A T A B L E

O F

*Artificial or Logarithmic*

## TANGENTS,

*To every Degree and Minute of the  
Quadrant.*

<i>A Table of Logarithmic Tangents.</i>						
Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min
0	0	8.241921	8.543083	8.719396	8.844044	0
1	6.463726	49101	46691	21806	6455	1
2	6.764756	56165	50268	24203	8260	2
3	6.940847	63115	53317	26588	8.850057	3
4	7.065786	69956	57330	28959	1846	4
5	7.162696	76691	60828	8.731317	3626	5
6	7.241878	83323	64291	366	5403	6
7	7.308825	89856	67727	5996	7172	7
8	7.366817	96292	71137	8317	8932	8
9	7.417970	8.302633	74520	8.740025	8.860580	9
10	7.463727	08884	77877	2922	2433	10
11	7.505120	15046	81208	5207	4172	11
12	7.542909	21122	84314	7479	5205	12
13	7.577671	27014	87794	9740	7612	13
14	7.609857	33025	91051	8.751989	9351	14
15	7.639820	38856	94283	1227	8.871064	15
16	7.667349	44610	97492	6453	2770	16
17	7.694179	50289	8.600677	8063	4169	17
18	7.719003	55895	03839	8.760872	6152	18
19	7.742454	61430	06978	3055	7849	19
20	7.764761	66894	10094	5245	9529	20

Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min.
21	7.785943	8.372171	8.612823	8.766675	8.879949	21
22	7.806146	8.77499	8.5891	8.8827	8.881607	22
23	7.25451	8.82762	8.8937	8.770970	8.3258	23
24	7.43934	8.87962	8.621962	8.3101	8.4903	24
25	7.61662	8.93101	8.4965	8.5223	8.6542	25
26	7.78695	8.98179	8.7948	8.7333	8.8174	26
27	7.95085	8.403199	8.630911	8.9434	8.9801	27
28	7.910880	8.08161	8.3854	8.781524	8.891421	28
29	7.26119	8.13068	8.6776	8.3605	8.3035	29
30	7.40842	8.17919	8.9680	8.5675	8.4643	30
31	7.55082	8.22717	8.642563	8.7735	8.6245	31
32	7.68870	8.27462	8.5428	8.9787	8.7842	32
33	7.82233	8.32156	8.8274	8.791828	8.9432	33
34	7.95198	8.36800	8.651102	8.3859	8.901017	34
35	8.007787	8.41394	8.3911	8.5881	8.2595	35
36	8.20021	8.45941	8.6702	8.7894	8.4168	36
37	8.31919	8.50140	8.9475	8.9897	8.5736	37
38	8.43501	8.54893	8.662230	8.801891	8.7297	38
39	8.54781	8.59301	8.4958	8.3876	8.8853	39
40	8.65776	8.63665	8.7689	8.5852	8.910404	40
41	8.76500	8.67985	8.670393	8.7819	8.1949	41
42	8.86965	8.72263	8.3080	8.9777	8.3488	42
43	8.97183	8.76498	8.5751	8.811726	8.5022	43
44	8.107167	8.80693	8.8405	8.3667	8.6550	44
45	8.16926	8.84848	8.681043	8.5598	8.8073	45
46	8.26471	8.88563	8.3665	8.7522	8.9591	46
47	8.35810	8.93040	8.6272	8.9436	8.921103	47
48	8.44953	8.97078	8.8862	8.821342	8.2610	48
49	8.53907	8.501080	8.691438	8.3240	8.4112	49
50	8.62681	8.00045	8.3998	8.5130	8.5609	50
51	8.71280	8.08974	8.6543	8.7011	8.7100	51
52	8.79713	8.12867	8.9073	8.8884	8.8587	52
53	8.87985	8.16726	8.701589	8.830749	8.930068	53
54	8.96102	8.20551	8.4090	8.2607	8.1544	54
55	8.504070	8.24343	8.6577	8.4456	8.3015	55
56	8.11895	8.28102	8.9049	8.6297	8.4481	56
57	8.19581	8.31828	8.711507	8.8130	8.5942	57
58	8.27133	8.35523	8.3952	8.9956	8.7398	58
59	8.34557	8.39186	8.6383	8.841714	8.8850	59
60	8.41855	8.42819	8.8800	8.3584	8.940296	60



*A Table of Logarithmic Tangents.*

•29

Min.	• Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min
21	7.785951	8.372291	8.613189	8.767417	8.881202	21
22	7.806154	8.77622	8.6261	8.9578	8.2869	22
23	7.25460	8.82889	8.9313	8.771727	8.4530	23
24	7.43944	8.88092	8.622343	8.3866	8.6185	24
25	7.61674	8.93234	8.5352	8.5995	8.7833	25
26	7.78708	8.98115	8.8340	8.8114	8.9470	26
27	7.95099	8.403338	8.631308	8.780222	8.891112	27
28	7.910894	8.08304	8.4256	8.2320	8.2742	28
29	7.26134	8.13213	8.7185	8.4108	8.4366	29
30	7.40858	8.18068	8.640093	8.6486	8.5984	30
31	7.55100	8.22869	8.2982	8.8554	8.7596	31
32	7.68889	8.27618	8.5853	8.790613	8.9203	32
33	7.82253	8.32315	8.8704	8.2662	8.900803	33
34	7.95219	8.36962	8.651537	8.4701	8.2398	34
35	8.007809	8.41560	8.4352	8.6731	8.387	35
36	8.20044	8.46110	8.7149	8.8752	8.5570	36
37	8.31945	8.50613	8.9928	8.800763	8.7147	37
38	8.43527	8.55070	8.662689	8.2765	8.8719	38
39	8.54809	8.59481	8.5433	8.4758	8.910285	39
40	8.65806	8.63849	8.8160	8.6742	8.1846	40
41	8.76531	8.68172	8.670870	8.8717	8.3401	41
42	8.86997	8.72453	8.3563	8.0683	8.4951	42
43	8.97217	8.76693	8.6239	8.2641	8.6495	43
44	8.107202	8.80892	8.8900	8.4589	8.8034	44
45	8.16963	8.85050	8.681544	8.6529	8.9567	45
46	8.26510	8.89170	8.4172	8.8461	8.921095	46
47	8.35851	8.93250	8.6784	8.820384	8.2619	47
48	8.44996	8.97293	8.9381	8.2298	8.4136	48
49	8.53952	8.501298	8.691963	8.4205	8.5649	49
50	8.62727	8.05267	8.4529	8.6103	8.7156	50
51	8.71328	8.09200	8.7081	8.7592	8.8658	51
52	8.79762	8.13098	8.9617	8.9874	8.930155	52
53	8.88036	8.16961	8.702139	8.831748	8.1647	53
54	8.96156	8.20790	8.4646	8.3613	8.3134	54
55	8.204126	8.24586	8.7139	8.5471	8.4616	55
56	8.11953	8.28349	8.9618	8.7321	8.6093	56
57	8.19641	8.32080	8.712083	8.9163	8.7565	57
58	8.27195	8.35779	8.4534	8.840998	8.9032	58
59	8.34621	8.39447	8.6972	8.2824	8.940494	59
60	8.41921	8.43084	8.9396	8.4644	8.1952	60

\*30 Logarithmic Sines, Degrees 5, 6, 7, 8, 9, 10, 11, 12. Logarithms

D	0	1	2	3	4	5	6	7	8	9
5. 0	8.94.0296	1738	3174	4505	6034	7456	8874	0287	1696	3099
10	95.4499	5894	7284	8670	0052	1429	2801	4170	5534	6893
20	96.8249	9500	0947	2289	3528	4962	6293	7619	8941	0259
30	98.1573	2883	4189	5491	6789	8083	9374	0660	1943	3222
40	99.4497	5765	7036	8299	9560	0816	2069	3318	4563	5805
50	9.00.7040	8278	9510	0737	1926	3182	4400	5613	6824	8031
6. 0	01.9235	0435	1632	2825	4016	5203	6386	7557	8744	9918
10	03.1059	2256	3421	4582	5741	6895	8048	9197	0342	1485
20	04.2615	3762	4895	6026	7154	8279	9400	0519	1635	2748
30	05.3859	4950	6071	7172	8271	9357	0460	1551	2639	3729
40	06.4806	5885	6952	8036	9107	0176	1242	2305	3366	4424
50	07.5480	6533	7583	8631	9676	0719	1759	2797	3832	4864
7. 0	08.5894	6922	7947	8970	9920	1008	2024	3037	4047	5056
10	09.6061	7065	8066	9065	0061	1056	2047	3037	4025	5010
20	10.5992	6973	7951	8921	9901	0873	1842	2809	3774	4737
30	11.5698	6656	7612	8566	9519	0469	1417	2362	3306	4248
40	12.5181	6125	7060	7993	8925	9854	0781	1706	2630	3551
50	13.4170	5387	6303	7216	8127	9037	9944	0850	1754	2655
8. 0	14.3555	4453	5350	6243	7136	8026	8915	9801	0686	1569
10	15.2451	3330	4208	5083	5957	6830	7700	8569	9435	0300
20	16.1664	2025	2885	3743	4600	5454	6307	7159	8008	8855
30	16.9702	0546	1389	2230	3070	3908	4744	5578	6411	7242
40	17.8072	8900	9726	0551	1374	2196	3016	3834	4651	5466
50	18.6280	7092	7903	8712	9519	0325	1130	1933	2734	3534
9. 0	19.4332	5129	5925	6719	7511	8302	9091	9879	0666	1451
10	20.2234	3017	3797	4577	5354	6131	6906	7679	8452	9222
20	9992	0759	1526	2291	3055	3818	4579	5338	6097	6854
30	21.7609	8363	9116	9868	0518	1367	2115	2861	3606	4349
40	22.5092	5833	6572	7311	8048	8784	9518	0252	0984	1714
50	23.2444	3172	3899	4625	5349	6073	6795	7515	8235	8953
10. 0	9670	0386	1101	1814	2526	3237	3947	4656	5363	6069
10	24.6775	7478	8181	8883	9583	0282	0980	1677	2375	3071
20	25.3761	4453	5144	5834	6523	7211	7898	8583	9268	9951
30	26.0633	1314	1994	2673	3351	4027	4704	5377	6051	6723
40	7394	8065	8734	9402	0069	0735	1399	2063	2726	3388
50	27.4094	4708	5367	6024	6681	7337	7991	8644	9297	9948
11. 0	28.0599	1245	1897	2544	3190	3836	4480	5124	5766	6408
10	7018	7687	8325	8964	9600	0235	0870	1504	2137	2768
20	29.3399	4029	4658	5286	5913	6539	7164	7788	8411	9034
30	9655	0276	0895	1514	2132	2748	3364	3979	4593	5207
40	30.5819	6430	7041	7650	8259	8867	9474	0080	0685	1289
50	31.1893	2455	3097	3698	4297	4896	5495	6092	6688	7284
12. 0	7879	8473	9066	9658	0249	0840	1430	2019	2607	3194
10	32.3700	4366	4950	5534	6117	6700	7281	7862	8442	9021
20	9599	0176	0753	1328	1903	2478	3051	3624	4195	4765
30	33.5337	5906	6475	7043	7610	8176	8742	9306	9871	0434
40	34.0996	1556	2119	2679	3239	3797	4355	4912	5469	6024
50	6579	7134	7687	8240	8792	9343	9893	0443	0992	1542

Logarithmic Tangents Degrees, 5, 6, 7, 8, 9, 10, 11, 12, Index 8, 9. \*31

D	0	1	2	3	4	5	6	7	8	9
5. 0	8.94.1952	3404	4552	6295	7734	9108	0597	2021	3141	4856
10	95.6267	7673	9075	0473	1806	3254	4339	6019	7394	8766
20	97.0133	1456	2855	4209	5560	6906	8248	9586	0920	2251
30	98.3577	4893	6217	7532	8842	0149	1451	2750	4045	5337
40	99.6624	7905	9188	0465	1737	3007	4272	5524	6792	8047
50	9.00.9298	0546	1790	3031	4268	5502	6732	7959	9183	0403
5. 0	02.1620	2834	4044	5251	6455	7655	8852	0045	1237	2425
10	03.3606	4791	5969	7144	8316	9485	0651	1813	2973	4130
20	04.5284	6434	7582	8727	9869	1008	2144	3277	4407	5535
30	05.6659	7781	8900	0016	1130	2240	3348	4453	5556	6655
40	06.7752	8840	9938	1027	2113	3197	4278	5356	6432	7505
50	07.8576	9644	0710	1773	2833	3891	4947	6000	7050	8098
7. 0	08.9144	0187	1228	2266	3302	4335	5367	6395	7422	8446
10	09.9468	0487	1504	2515	3532	4542	5550	6556	7559	8560
20	10.9559	0556	1551	2543	3533	4521	5507	6491	7472	8452
30	11.9429	0404	1377	2348	3317	4284	5249	6211	7172	8130
40	12.9086	0041	0993	1944	2893	3839	4783	5726	6666	7605
50	13.8542	9476	0409	1340	2269	3196	4121	5044	5965	6885
8. 0	14.7802	8718	9632	0544	1454	2363	3269	4171	5077	5978
10	15.6877	7775	8671	9565	0457	1347	2236	3123	4008	4892
20	16.5774	6654	7532	8409	9284	0157	1029	1899	2767	3634
30	17.4499	5362	6224	7084	7942	8799	9655	0508	1360	2211
40	18.3059	3907	4752	5596	6439	7280	8119	8957	9794	0629
50	19.1462	2294	3124	3953	4780	5606	6430	7253	8074	8891
9. 0	9712	0529	1345	2159	2971	3782	4592	5400	6207	7012
10	20.7816	8619	9420	0220	1018	1815	2611	3405	4198	4989
20	21.5779	6568	7356	8141	8926	9710	0492	1272	2052	2830
30	22.3606	4382	5156	5929	6700	7471	8239	9007	9773	0539
40	23.1302	2065	2826	3586	4345	5103	5859	6614	7368	8120
50	8971	9622	0371	1118	1865	2610	3354	4097	4839	5579
10. 0	24.6319	7057	7794	8530	9264	9998	0730	1461	2191	2900
10	25.3648	4374	5100	5824	6547	7269	7990	8710	9428	0145
20	26.0862	1578	2292	3005	3717	4428	5138	5847	6555	7261
30	7967	8671	9375	0077	0779	1479	2178	2876	3573	4269
40	27.4964	5658	6351	7043	7734	8424	9113	9801	0489	1174
50	28.1808	2542	3225	3907	4588	5268	5947	6624	7301	7977
11. 0	8652	9326	9999	0671	1342	2013	2682	3350	4017	4684
10	29.5349	6013	6677	7339	8001	8662	9322	9980	0638	1295
20	30.1951	2607	3261	3914	4567	5218	5869	6519	7167	7815
30	8463	9109	9754	0398	1042	1685	2327	2967	3608	4247
40	31.4885	5523	6159	6795	7430	8064	8697	9329	9961	0592
50	32.1222	1851	2479	3106	3733	4358	4983	5607	6230	6853
12. 0	7474	8095	8715	9334	9953	0570	1187	1803	2418	3033
10	33.3646	4259	4871	5482	6093	6702	7311	7919	8527	9133
20	9739	0344	0948	1552	2155	2757	3358	3958	4558	5157
30	34.5755	6353	6949	7545	8141	8735	9329	9922	0514	1106
40	35.1697	2287	2876	3465	4053	4640	5227	5813	6398	6982
50	7566	8149	8731	9313	9893	0474	1053	1632	2210	2787



\*32 *Logarithm Sines, Deg. 13, 14, 15, 16, 17, 18, 19, 20. Index 9.*

D	0	1	2	3	4	5	6	7	8	9
13. 0	35.2038	2635	3181	3726	4271	4815	5358	5901	6443	6984
10	7524	8064	8603	9141	9678	0215	0751	1287	1822	2356
20	36.2889	3422	3954	4485	5016	5546	6075	6604	7131	7659
30	8185	8711	9236	9761	0285	0808	1330	1852	2373	2894
40	37.3414	3933	4452	4970	5487	6003	6519	7035	7549	8063
50	8576	9089	9601	0113	0624	1134	1643	2152	2660	3168
14. 0	38.3675	4181	4687	5192	5697	6201	6704	7207	7709	8210
10	8711	9211	9711	0210	0708	1206	1703	2199	2695	3190
20	39.3685	4179	4673	5166	5658	6150	6641	7131	7621	8111
30	8600	9088	9575	0062	0549	1035	1520	2005	2489	2972
40	40.3455	3938	4420	4901	5381	5862	6341	6820	7299	7777
50	8254	8731	9207	9682	0157	0632	1106	1579	2052	2524
15. 0	41.2996	3467	3938	4408	4878	5345	5815	6283	6751	7217
10	7684	8149	8615	9079	9544	0007	0470	0933	1395	1857
20	42.2318	2778	3238	3697	4156	4615	5073	5530	5987	6443
30	6809	7354	7809	8263	8717	9170	9623	0075	0526	0978
40	43.1429	1879	2328	2778	3226	3675	4122	4569	5016	5462
50	5908	6353	6798	7242	7686	8129	8572	9014	9456	9897
16. 0	44.0330	0778	1218	1658	2095	2535	2973	3410	3847	4284
10	4720	5155	5590	6025	6459	6893	7326	7759	8191	8623
20	9054	9485	9915	0345	0775	1204	1632	2060	2488	2915
30	45.3342	3768	4194	4619	5044	5469	5893	6316	6739	7162
40	7584	8066	8427	8848	9268	9688	0108	0526	0945	1364
50	46.1782	2199	2616	3032	3448	3864	4279	4694	5108	5522
17. 0	5935	6348	6761	7173	7585	7997	8407	8817	9227	9637
10	47.0046	0455	0863	1271	1678	2086	2492	2898	3304	3710
20	4115	4519	4923	5327	5730	6133	6536	6938	7340	7741
30	8142	8542	8942	9342	9741	0140	0538	0937	1334	1731
40	48.2128	2525	2921	3316	3712	4107	4501	4895	5289	5682
50	6075	6467	6859	7251	7643	8033	8424	8814	9204	9593
18. 0	9982	0371	0759	1147	1534	1922	2308	2695	3081	3466
10	49.3851	4236	4620	5005	5388	5772	6154	6537	6919	7301
20	7682	8063	8444	8824	9204	9584	9963	0342	0721	1099
30	50.1476	1854	2231	2607	2984	3340	3735	4110	4485	4860
40	5234	5608	5981	6354	6727	7099	7471	7843	8214	8585
50	8256	9326	9696	0065	0434	0803	1172	1540	1907	2275
19. 0	51.2642	3009	3375	3741	4107	4472	4837	5202	5566	5930
10	6294	6657	7020	7382	7745	8107	8468	8829	9190	9551
20	9911	0271	0631	0990	1349	1707	2066	2423	2781	3138
30	52.3495	3852	4208	4564	4920	5275	5630	5984	6339	6693
40	7746	7400	7753	8105	8458	8810	9161	9513	9864	0215
50	53.0505	0915	1265	1614	1963	2312	2661	3009	3357	3704
20. 0	4052	4399	4745	5091	5437	5783	6129	6474	6818	7163
10	7507	7851	8194	8537	8880	9223	9565	9907	0249	0590
20	54.0931	1272	1613	1953	2293	2632	2971	3310	3649	3987
30	4325	4663	5001	5338	5674	6011	6347	6683	7019	7354
40	7689	8024	8358	8693	9026	9360	9693	0026	0359	0691
50	55.1024	1356	1687	2018	2349	2680	3010	3341	3670	4000

D	0	1	2	3	4	5	6	7	8	9
13.0	36.3364	3940	4515	5090	5664	6237	6810	7382	7953	8524
10	9094	9663	0231	0799	1367	1933	2499	3064	3629	4193
20	37.4756	5319	5881	6442	7003	7563	8122	8681	9229	9797
30	38.0354	0910	1465	2020	2575	3128	3682	4234	4786	5337
40	5888	6438	5987	7536	8084	8631	9178	9724	0270	0815
50	39.1359	1903	2447	2989	3531	4073	4614	5154	5694	6233
14.0	6771	7309	7846	8383	8919	9455	9990	0524	1058	1591
10	40.2124	2656	3187	3718	4249	4778	5308	5836	6364	6892
20	7419	7945	8471	8996	9521	0045	0569	1092	1615	2137
30	41.2658	3179	3699	4219	4738	5257	5775	6293	6810	7326
40	7842	8358	8873	9378	9901	0415	0927	1440	1951	2463
50	42.2973	3484	3993	4503	5011	5519	6027	6534	7041	7547
15.0	8052	8557	9062	9566	0070	0573	1075	1577	2079	2580
10	43.3080	3580	4080	4579	5078	5576	6073	6570	7067	7563
20	8059	8554	9048	9543	0036	0529	1022	1514	2006	2497
30	44.2988	3479	3968	4458	4947	5435	5923	6411	6898	7384
40	7870	8356	8841	9326	9810	0294	0777	1260	1743	2225
50	45.2706	3187	3668	4148	4628	5107	5586	6064	6542	7019
16.0	7496	7973	8449	8925	9400	9875	0349	0823	1297	1770
10	46.2242	2714	3186	3658	4128	4599	5069	5539	6008	6476
20	6945	7413	7880	8347	8814	9280	9746	0211	0676	1141
30	47.1605	2668	2532	2995	3457	3919	4381	4842	5303	5763
40	6223	6683	7142	7601	8059	8517	8975	9432	9889	0345
50	48.0801	1257	1712	2167	2621	3075	3529	3982	4435	4887
17.0	5339	5791	6242	6693	7143	7593	8043	8492	8941	9390
10	9838	0286	0733	1180	1627	2073	2519	2965	3410	3854
20	49.4299	4743	5186	5630	6073	6515	6957	7399	7841	8282
30	8722	9162	9603	0042	0481	0920	1359	1797	2235	2672
40	50.3109	3546	3982	4418	4854	5289	5724	6159	6593	7027
50	7460	7893	8326	8759	9191	9622	0054	0485	0916	1346
18.0	51.1776	2206	2635	3064	3493	3921	4349	4777	5204	5631
10	6057	6484	6910	7335	7761	8185	8610	9034	9458	9882
20	52.0305	0728	1151	1573	1995	2417	2838	3259	3679	4100
30	4520	4939	5359	5778	6197	6615	7033	7451	7868	8285
40	8702	9119	9535	9950	0366	0781	1196	1611	2025	2439
50	53.2853	3266	3679	4092	4504	4916	5328	5739	6150	6561
19.0	6772	7382	7792	8202	8611	9020	9429	9837	0245	0653
10	54.1061	1468	1875	2281	2689	3094	3500	3905	4310	4715
20	5119	5524	5927	6331	6735	7138	7540	7943	8345	8747
30	9149	9550	9951	0352	0752	1152	1552	1952	2351	2750
40	55.3149	3548	3946	4344	4741	5139	5536	5933	6329	6724
50	7121	7517	7912	8308	8702	9097	9491	9885	0279	0673
20.0	56.1066	1459	1851	2244	2636	3028	3419	3811	4202	4592
10	4983	5373	5763	6153	6542	6932	7320	7709	8097	8486
20	8873	9261	9648	0035	0422	0809	1195	1581	1967	2351
30	57.2738	3124	3507	3892	4276	4660	5044	5427	5810	6193
40	6576	6958	7341	7723	8104	8486	8867	9248	9629	0009
50	58.0389	0769	1149	1528	1907	2286	2665	3042	3422	3800

\*34 Logar. Sines, Deg. 21, 22, 23, 24, 25, 26, 27, 28. In. 9.

D	0	1	2	3	4	5	6	7	8	9
21.0	4329	4658	4987	5315	5643	5971	6299	6626	6953	7280
10	7606	7932	8258	8583	8909	9234	9558	9883	0207	0531
20	56.0854	1178	1501	1824	2146	2468	2790	3112	3433	3755
30	4075	4396	4716	5036	5356	5676	5995	6314	6632	6951
40	7269	7587	7904	8222	8539	8855	9172	9488	9804	0120
50	57.0435	0751	1066	1380	1695	2009	2323	2636	2949	3263
22.0	3575	3888	4200	4512	4824	5136	5447	5758	6068	6379
10	6689	6999	7309	7618	7927	8236	8545	8853	9162	9469
20	9777	0084	0391	0699	1005	1312	1618	1924	2229	2534
30	58.2840	3144	3449	3753	4058	4361	4665	4968	5272	5574
40	5877	6179	6482	6783	7085	7386	7688	7988	8289	8590
50	8800	9190	9489	9789	0088	0387	0686	0984	1282	1580
23.0	59.1878	2175	2473	2770	3067	3363	3659	3955	4251	4547
10	4842	5137	5432	5727	6021	6315	6609	6903	7196	7490
20	7783	8075	8368	8660	8952	9244	9536	9827	0118	0409
30	60.0700	0990	1280	1570	1860	2150	2439	2728	3017	3308
40	3594	3881	4170	4457	4755	5032	5319	5606	5892	6178
50	6465	6751	7036	7321	7607	7892	8176	8461	8745	9029
24.0	9313	9597	9880	0163	0446	0729	1012	1294	1576	1858
10	61.2140	2421	2702	2983	3264	3545	3825	4105	4385	4665
20	4944	5223	5502	5781	6060	6338	6616	6894	7172	7450
30	7727	8004	8281	8558	8834	9110	9386	9662	9937	0213
40	62.0488	0763	1038	1313	1587	1861	2135	2409	2682	2956
50	3229	3502	3774	4047	4319	4591	4863	5135	5406	5677
25.0	5948	6219	6490	6760	7030	7300	7570	7840	8109	8378
10	8647	8916	9184	9453	9721	9989	0257	0524	0792	1059
20	63.1326	1593	1859	2125	2391	2658	2923	3189	3454	3719
30	3984	4249	4514	4778	5042	5306	5570	5833	6097	6360
40	6623	6886	7148	7411	7673	7935	8197	8458	8720	8981
50	9242	9503	9764	0024	0284	0544	0804	1064	1323	1583
26.0	64.1842	2101	2360	2618	2876	3135	3393	3650	3908	4165
10	4423	4679	4936	5193	5450	5706	5962	6218	6473	6728
20	6984	7239	7494	7749	8004	8258	8512	8766	9020	9274
30	9527	9781	0034	0287	0539	0792	1044	1297	1549	1802
40	65.2052	2303	2555	2806	3057	3307	3558	3808	4058	4309
50	4558	4808	5057	5307	5556	5805	6054	6302	6550	6799
27.0	7047	7295	7542	7790	8037	8284	8531	8778	9025	9271
10	9517	9763	0009	0255	0500	0746	0991	1236	1481	1727
20	66.1970	2214	2459	2703	2946	3190	3433	3677	3920	4163
30	4406	4648	4891	5133	5375	5617	5859	6100	6341	6582
40	6824	7065	7305	7546	7786	8026	8266	8506	8746	8986
50	9225	9464	9703	9942	0181	0419	0658	0896	1134	1372
28.0	67.1609	1847	2084	2321	2558	2795	3032	3268	3505	3742
10	3977	4213	4448	4684	4919	5155	5389	5624	5859	6094
20	6328	6562	6796	7030	7264	7497	7731	7964	8197	8430
30	8663	8895	9278	9360	9592	9824	0056	0287	0519	0751
40	68.0982	1213	1443	1674	1905	2135	2365	2595	2825	3055
50	3281	3514	3743	3972	4201	4430	4658	4887	5115	5344



D	0	1	2	3	4	5	6	7	8	9
21. 0	84177	4555	4932	5309	5686	6062	6439	6815	7190	7566
10	7941	8316	8691	9066	9440	9814	0188	0562	0935	1308
20	9.1581	2054	2426	2798	3170	3542	3914	4285	4656	5027
30	5397	5768	6138	6508	6878	7247	7616	7985	8354	8722
40	9090	9459	9827	0194	0562	0920	1296	1662	2029	2395
50	60.2761	3127	3493	3858	4223	4588	4953	5317	5682	6046
22. 0	6410	5773	7137	7500	7893	8225	8588	8950	9312	9674
10	61.0036	0327	0759	1120	1480	1841	2201	2561	2921	3281
20	3641	4000	4359	4718	5077	5435	5793	6151	6509	6867
30	7224	7581	7938	8295	8652	9008	9364	9720	0076	0432
40	62.0737	1142	1497	1852	2207	2561	2915	3269	3623	3976
50	4330	4682	5036	5388	5701	6093	6445	6797	7149	7501
23. 0	7852	8203	8554	8905	9255	9606	9956	0306	0655	1005
10	63.1344	1704	2053	2401	2750	3098	3447	3795	4143	4490
20	4838	5185	5532	5879	6226	6572	6918	7265	7611	7956
30	8302	8647	8992	9337	9682	0027	0371	0716	1060	1404
40	64.1747	2091	2434	2777	3120	3463	3806	4148	4490	4832
50	5174	5516	5857	6199	6540	6881	7222	7562	7903	8243
24. 0	8583	8923	9263	9602	9942	0281	0620	0959	1297	1636
10	65.1974	2312	2650	2988	3326	3663	4000	4337	4674	5011
20	5348	5684	6020	6356	6692	7028	7363	7699	8034	8369
30	8704	9039	9373	9707	0042	0376	0710	1043	1377	1710
40	66.2043	2376	2709	3042	3374	3707	4039	4371	4703	5035
50	5366	5697	6029	6360	6691	7021	7352	7682	8013	8343
25. 0	8672	9002	9332	9661	9991	0320	0649	0977	1306	1634
10	67.1953	2291	2619	2947	3274	3602	3929	4257	4584	4910
20	5237	5564	5890	6216	6543	6869	7194	7520	7846	8171
30	8496	8821	9146	9471	9795	0120	0444	0768	1092	1416
40	68.1740	2063	2386	2710	3033	3356	3678	4001	4323	4646
50	4968	5290	5612	5934	6255	6577	6898	7219	7540	7861
26. 0	8182	8502	883	9143	9463	9783	0103	0423	0742	1061
10	69.1381	1700	2019	2338	2656	2975	3293	3612	3930	4248
20	4566	4883	5201	5518	5835	6153	6470	6786	7103	7420
30	7736	8053	8369	8685	9001	9316	9632	9947	0263	0578
40	70.0893	1208	1523	1837	2152	2466	2780	3095	3409	3722
50	4036	4349	4663	4975	5290	5603	5916	6228	6541	6853
27. 0	7166	7478	7790	8102	8414	8726	9037	9349	9660	9971
10	71.0282	0593	0904	1215	1525	1836	2146	2456	2766	3076
20	3386	3696	4005	4314	4624	4933	5242	5551	5859	6168
30	6477	6785	7093	7401	7709	8017	8325	8633	8940	9248
40	9555	9852	0169	0476	0782	1089	1395	1702	2008	2315
50	72.2621	2927	3232	3538	3844	4149	4454	4759	5065	5369
28. 0	5674	5979	6284	6588	6892	7197	7501	7805	8109	8412
10	8716	9020	9323	9626	9929	0232	0535	0838	1141	1444
20	73.1746	2048	2351	2653	2955	3257	3558	3861	4162	4463
30	4764	5065	5367	5668	5968	6269	6570	6871	7171	7471
40	7771	8070	8371	8671	8971	9271	9570	9870	0169	0468
50	74.0767	1066	1365	1664	1962	2261	2559	2858	3156	3454

\*30 Logar. Sines, Deg. 29, 30, 31, 32, 33, 34, 35, 36. In. 9.

D	0	1	2	3	4	5	6	7	8	9
29.0	58.5571	5799	6027	6254	6482	6709	6936	7163	7389	7616
10	7842	5069	8295	8521	8747	8972	9198	9423	9648	9873
20	59.0098	0323	0548	0772	0996	1220	1444	1668	1892	2115
30	2339	2562	2785	3008	3231	3453	3676	3898	4120	4342
40	4554	4786	5007	5229	5450	5671	5892	6113	6334	6554
50	6774	6995	7215	7435	7654	7874	8094	8313	8532	8751
30.0	8970	9189	9407	9626	9844	002	0280	0498	0716	0938
10	70.1151	1368	1585	1802	2019	2236	2452	2669	2885	3101
20	3317	3533	3749	3964	4179	4395	4610	4825	5040	5254
30	5457	5683	5897	6112	6325	6539	6753	6967	7180	7393
40	7606	7819	8032	8245	8457	8670	8882	9094	9306	9518
50	9730	9941	0153	0364	0575	0786	0997	1208	1419	1629
31.0	71.1839	2049	2260	2469	2679	2889	3098	3308	3517	3726
10	3935	4141	4352	4561	4769	4978	5186	5394	5601	5809
20	6017	6224	6432	6639	6846	7053	7259	7465	7672	7879
30	8085	8291	8497	8703	8908	9114	9320	9525	9730	9935
40	72.0140	0345	0549	0754	0958	1162	1366	1570	1774	1978
50	2181	2385	2588	2791	2994	3197	3400	3603	3805	4007
32.0	4210	4411	4614	4815	5017	5219	5420	5622	5823	6024
10	6225	6426	6626	6827	7027	7228	7428	7628	7828	8027
20	8227	8427	8626	8825	9024	9223	9422	9621	9819	0018
30	73.0216	0415	0613	0811	1009	1206	1404	1601	1799	1996
40	2193	2390	2587	2784	2980	3177	3373	3569	3765	3961
50	4157	4353	4548	4744	4939	5134	5329	5525	5719	5914
33.0	6109	6303	6498	6692	6886	7080	7274	7467	7661	7854
10	8048	8241	8434	8627	8820	9013	9205	9398	9590	9783
20	9974	0157	0359	0550	0742	0934	1125	1316	1507	1699
30	74.1889	2080	2271	2462	2652	2842	3032	3223	3413	3602
40	3792	3982	4171	4361	4550	4739	4928	5117	5305	5493
50	5682	5871	6059	6248	6436	6624	6811	6999	7187	7374
34.0	7562	7749	7936	8123	8310	8497	8683	8870	9056	9242
10	9429	9615	9801	9987	0172	0358	0543	0728	0914	1099
20	75.1284	1469	1654	1838	2023	2207	2392	2576	2760	2944
30	3128	3312	3495	3679	3862	4046	4229	4412	4595	4778
40	4960	5143	5326	5508	5690	5872	6054	6236	6418	6600
50	6781	6963	7144	7325	7507	7688	7869	8049	8230	8411
35.0	8591	8772	8952	9132	9312	9492	9672	9851	0031	0211
10	76.0390	0569	0748	0927	1106	1285	1464	1642	1821	1999
20	2177	2356	2534	2712	2889	3067	3245	3422	3599	3777
30	3954	4131	4308	4485	4662	4838	5015	5191	5367	5543
40	5720	5896	6071	6247	6423	6598	6774	6949	7124	7299
50	7475	7649	7824	7999	8173	8348	8522	8696	8871	9045
36.0	9219	9392	9566	9740	9913	0087	0260	0433	0606	0779
10	77.0952	1125	1298	1470	1643	1815	1987	2159	2331	2503
20	2675	2847	3018	3190	3361	3532	3704	3875	4046	4216
30	4388	4558	4729	4899	5069	5240	5410	5580	5750	5920
40	6090	6259	6429	6598	6768	6937	7106	7275	7444	7613
50	7781	7950	8119	8287	8455	8623	8791	8959	9127	9295

Logar. Tangents, Deg. 29, 30, 31, 32, 33, 34, 35, 36. In. 9. \*37

D	0	1	2	3	4	5	6	7	8	9
29.0	74.1752	4050	4345	4545	4943	5240	5538	5835	6132	6429
10	6726	7023	7319	7616	7912	8209	8505	8801	9097	9393
20	9689	9985	0281	0576	0872	1167	1462	1757	2052	2347
30	75.2642	2937	3231	3525	3820	4115	4409	4703	4997	5291
40	5585	5878	6172	6465	6759	7052	7345	7638	7931	8224
50	8517	8810	9102	9395	9687	9979	0272	0564	0856	1148
30.0	76.1439	1731	2023	2314	2606	2897	3188	3479	3770	4061
10	4352	4642	4933	5224	5514	5805	6095	6385	6675	6965
20	7255	7545	7834	8124	8413	8703	8992	9281	9570	9859
30	77.0145	0437	0726	1015	1303	1592	1880	2168	2457	2745
40	3033	3321	3608	3895	4184	4471	4759	5046	5333	5621
50	5908	6195	6482	6768	7055	7342	7628	7915	8210	8487
31.0	8774	9060	9346	9632	9918	0203	0489	0775	1060	1346
10	78.1631	1916	2201	2485	2771	3056	3341	3626	3910	4195
20	4479	4764	5048	5332	5616	5900	6184	6468	6752	7036
30	7319	7603	7886	8170	8453	8736	9019	9302	9585	9868
40	79.0151	0433	0716	0999	1281	1563	1846	2128	2410	2692
50	2974	3256	3538	3819	4101	4383	4664	4945	5227	5508
32.0	5789	6070	6351	6632	6913	7194	7474	7755	8036	8316
10	8596	8877	9157	9437	9717	9997	0277	0557	0836	1116
20	80.1359	1675	1955	2234	2513	2792	3072	3351	3630	3908
30	4187	4466	4745	5023	5302	5580	5859	6137	6415	6693
40	6978	7249	7527	7805	8083	8361	8638	8916	9193	9471
50	9748	0025	0302	0580	0857	1134	1410	1687	1964	2241
33.0	81.2517	2794	3070	3347	3623	3899	4175	4452	4728	5004
10	5279	5555	5831	6107	6382	6658	6933	7209	7484	7759
20	8035	8310	8585	8860	9135	9410	9684	9959	0234	0508
30	82.0783	1057	1332	1606	1880	2154	2429	2703	2977	3250
40	3524	3798	4072	4345	4619	4893	5166	5439	5713	5986
50	6259	6532	6805	7078	7351	7624	7897	8170	8442	8715
34.0	8987	9260	9532	9805	0077	0349	0621	0893	1165	1437
10	83.1709	1981	2253	2525	2796	3068	3339	3611	3882	4154
20	4425	4695	4967	5238	5509	5780	6051	6322	6593	6864
30	7134	7405	7675	7946	8216	8487	8757	9027	9297	9568
40	9838	0108	0378	0647	0917	1187	1457	1726	1996	2266
50	84.2535	2805	3074	3343	3612	3882	4151	4420	4689	4958
35.0	5227	5496	5764	6033	6302	6571	6839	7107	7376	7644
10	7913	8181	8449	8717	8985	9254	9522	9790	0057	0325
20	85.0593	0861	1128	1396	1664	1931	2199	2466	2733	3001
30	3268	3535	3802	4069	4336	4603	4870	5137	5404	5671
40	5938	6204	6471	6737	7004	7270	7537	7803	8069	8336
50	8602	8868	9134	9400	9666	9932	0198	0464	0730	0995
36.0	86.1261	1527	1792	2058	2323	2589	2854	3119	3385	3650
10	3915	4180	4445	4710	4975	5240	5505	5770	6035	6300
20	6554	6829	7094	7358	7623	7887	8152	8416	8680	8945
30	9209	9473	9737	0001	0265	0529	0793	1057	1321	1585
40	87.1849	2112	2376	2640	2903	3167	3430	3694	3957	4220
50	4484	4747	5010	5273	5536	5800	6063	6326	6589	6851



38 Logar. Sines, Deg. 37, 38, 39, 40, 41, 42, 43, 44. Index 9.

D	0	1	2	3	4	5	6	7	8	9
37.0	77.9463	9531	9798	9965	0133	0300	0467	0634	0801	0968
10	78.1131	1301	1467	1634	1800	1966	2132	2298	2464	2630
20	2796	2961	3127	3292	3457	3623	3788	3953	4118	4282
30	4447	4612	4776	4941	5107	5269	5433	5597	5761	5925
40	6089	6252	6416	6579	6742	6905	7059	7232	7395	7557
50	7720	7883	8045	8208	8370	8532	8594	8856	9018	9180
38.0	9342	9540	9665	9826	9988	0149	0310	0471	0532	0793
10	79.0954	1115	1275	1436	1596	1757	1917	2077	2237	2397
20	2557	2716	2876	3035	3195	3354	3513	3673	3832	3991
30	4149	4308	4467	4626	4784	4942	5101	5259	5417	5575
40	5733	5891	6048	6206	6364	6521	6679	6836	6993	7150
50	7307	7464	7621	7777	7934	8091	8247	8403	8560	8716
39.0	8872	9028	9184	9339	9495	9651	9806	9962	0117	0272
10	80.0427	0582	0737	0892	1047	1201	1356	1511	1665	1819
20	1973	2128	2282	2435	2589	2743	2897	3050	3204	3357
30	3510	3664	3817	3970	4123	4276	4428	4581	4734	4886
40	5038	5191	5343	5495	5647	5799	5951	6103	6254	6406
50	6557	6709	6860	7011	7163	7314	7465	7615	7766	7917
40.0	8067	8218	8368	8519	8669	8819	8969	9119	9269	9419
10	9569	9718	9867	0017	0167	0316	0465	0614	0763	0912
20	81.1061	1210	1358	1507	1655	1804	1952	2100	2248	2396
30	2544	2692	2840	2987	3135	3283	3430	3578	3725	3872
40	4019	4166	4313	4460	4607	4753	4900	5046	5193	5339
50	5485	5631	5778	5923	6069	6215	6361	6507	6652	6797
41.0	6943	7088	7233	7378	7523	7668	7813	7958	8103	8247
10	8392	8536	8681	8825	8969	9113	9257	9401	9545	9689
20	9832	9976	0120	0263	0406	0550	0693	0836	0979	1122
30	82.1265	1407	1550	1693	1835	1977	2120	2262	2404	2546
40	2688	2830	2972	3114	3255	3397	3538	3680	3821	3963
50	4104	4245	4386	4527	4668	4808	4949	5089	5230	5370
42.0	5511	5651	5791	5931	6071	6211	6351	6491	6631	6770
10	6910	7049	7189	7328	7467	7606	7745	7884	8022	8162
20	8301	8439	8578	8716	8855	8993	9131	9269	9407	9545
30	9683	9821	9959	0097	0234	0372	0509	0646	0784	0921
40	83.1058	1195	1332	1469	1605	1742	1879	2015	2152	2288
50	2425	2561	2697	2833	2969	3105	3241	3377	3512	3648
43.0	3783	3919	4054	4189	4325	4459	4595	4730	4865	4999
10	5134	5269	5403	5538	5672	5806	5941	6075	6209	6343
20	6477	6611	6745	6878	7012	7145	7279	7412	7546	7679
30	7812	7945	8078	8211	8344	8477	8609	8742	8875	9007
40	9139	9272	9404	9536	9668	9800	9932	0064	0196	0327
50	84.0359	0591	0722	0854	0985	1116	1247	1378	1509	1640
44.0	1771	1902	2033	2163	2294	2424	2555	2685	2815	2945
10	3076	3205	3336	3465	3595	3725	3855	3984	4114	4243
20	4372	4502	4631	4760	4889	5018	5147	5276	5404	5533
30	5662	5790	5919	6047	6175	6304	6432	6560	6688	6816
40	6944	7071	7199	7327	7454	7582	7709	7836	7964	8091
50	8218	8345	8472	8599	8726	8852	8979	9106	9232	9359

Logar	D	1
37.0		
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38.0		
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43.0		
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44.0		
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Logar. Tangents, Deg. 37, 38, 39, 40, 41, 42, 43, 44. In. 9. \*39

D	0	1	2	3	4	5	6	7	8	9
37.0	87.7114	7377	7640	7903	8165	8428	8691	8953	9216	9478
10	9741	0003	0265	0528	0790	1052	1314	1576	1839	2101
20	88.2363	2625	2887	3148	3410	3672	3934	4196	4457	4719
30	4980	5242	5503	5765	6026	6288	6549	6810	7072	7333
40	7594	7855	8116	8377	8639	8900	9160	9421	9682	9943
50	89.0204	0465	0725	0986	1247	1507	1768	2028	2289	2549
38.0	2810	3070	3331	3591	3851	4111	4371	4632	4892	5152
10	5412	5672	5932	6192	6452	6712	6971	7231	7491	7751
20	8010	8270	8530	8789	9049	9308	9568	9827	0086	0346
30	90.0605	0864	1124	1383	1642	1901	2160	2419	2679	2938
40	3197	3455	3714	3973	4232	4491	4750	5008	5267	5526
50	5784	6043	6302	6560	6819	7077	7336	7594	7852	8111
39.0	8369	8672	8886	9144	9402	9660	9918	0177	0435	0693
10	91.0951	1209	1467	1724	1982	2240	2498	2756	3014	3271
20	3529	3787	4044	4302	4560	4817	5075	5332	5590	5847
30	6104	6362	6619	6876	7134	7391	7648	7905	8163	8420
40	8677	8934	9191	9446	9705	9962	0219	0476	0733	0990
50	92.1247	1503	1760	2017	2274	2530	2787	3044	3300	3557
40.0	3813	4070	4327	4583	4840	5096	5352	5609	5865	6121
10	6378	6634	6890	7147	7403	7659	7915	8171	8427	8683
20	8940	9196	9452	9708	9964	0219	0475	0731	0987	1243
30	93.1499	1755	2010	2266	2522	2778	3033	3289	3545	3800
40	4056	4311	4567	4822	5078	5333	5589	5844	6100	6355
50	6616	6866	7121	7376	7632	7887	8142	8397	8653	8908
41.0	9163	9418	9673	9928	0183	0438	0694	0949	1204	1458
10	94.1713	1968	2223	2478	2733	2988	3243	3498	3752	4007
20	4262	4517	4771	5026	5281	5535	5790	6045	6299	6554
30	6808	7063	7317	7572	7826	8081	8335	8590	8844	9099
40	9353	9607	9862	0116	0370	0625	0879	1133	1387	1642
50	95.1896	2150	2404	2659	2913	3167	3421	3675	3929	4183
42.0	4437	4691	4945	5199	5453	5707	5961	6215	6469	6723
10	6977	7231	7485	7739	7993	8246	8500	8754	9008	9262
20	9515	9769	0023	0277	0530	0784	1038	1291	1545	1799
30	96.2052	2306	2560	2813	3067	3320	3574	3827	4081	4335
40	4588	4842	5095	5349	5602	5855	6109	6362	6616	6869
50	7122	7376	7629	7883	8136	8389	8643	8896	9149	9403
43.0	9656	9909	0162	0416	0669	0922	1175	1429	1682	1935
10	97.2188	2441	2694	2948	3201	3454	3707	3960	4213	4466
20	4719	4973	5226	5479	5732	5985	6238	6491	6744	6997
30	7250	7503	7756	8009	8262	8515	8768	9021	9274	9527
40	9780	0033	0286	0538	0791	1044	1297	1550	1803	2056
50	98.2309	2562	2814	3067	3320	3573	3826	4079	4331	4584
44.0	4837	5090	5343	5596	5848	6101	6354	6607	6860	7112
10	7365	7618	7871	8123	8376	8629	8882	9134	9387	9640
20	9893	0145	0398	0651	0903	1156	1409	1662	1914	2167
30	99.2420	2672	2925	3178	3430	3683	3936	4189	4441	4694
40	4947	5199	5452	5705	5957	6210	6463	6715	6968	7221
50	7473	7726	7979	8231	8484	8737	8989	9242	9495	9747

D	0	1	2	3	4	5	6	7	8	9
45.0	84.9485	9611	9737	9854	9990	0116	0242	0367	0493	0619
10	85.0745	0870	0996	1121	1246	1372	1497	1622	1747	1872
20	1997	2123	2247	2371	2496	2620	2745	2869	2994	3118
30	3243	3366	3490	3614	3738	3862	3986	4109	4233	4356
40	4480	4603	4727	4850	4973	5096	5219	5342	5465	5588
50	5711	5833	5955	6078	6201	6323	6445	6568	6690	6812
46.0	6934	7056	7178	7300	7421	7543	7665	7786	7908	8029
10	8150	8272	8393	8514	8635	8756	8877	8998	9119	9239
20	9340	9480	9601	9721	9842	9962	0082	0202	0322	0442
30	86.0562	0682	0802	0921	1041	1161	1280	1399	1519	1638
40	1758	1877	1996	2115	2234	2353	2471	2590	2709	2827
50	2946	3064	3183	3301	3419	3537	3656	3774	3892	4010
47.0	4127	4245	4363	4481	4598	4715	4833	4950	5068	5185
10	5302	5419	5536	5653	5770	5887	6004	6120	6237	6353
20	6470	6586	6703	6819	6935	7051	7167	7283	7399	7515
30	7631	7747	7862	7978	8094	8209	8324	8439	8555	8670
40	8785	8900	9015	9130	9245	9360	9474	9589	9704	9818
50	87.9932	0047	0161	0276	0390	0504	0618	0732	0846	0960
48.0	1073	1187	1301	1414	1528	1641	1755	1868	1981	2094
10	2208	2321	2434	2547	2659	2772	2885	2997	3110	3223
20	3335	3448	3560	3672	3784	3896	4008	4120	4232	4344
30	4456	4568	4679	4791	4903	5014	5125	5237	5348	5459
40	5570	5681	5793	5904	6014	6125	6236	6347	6457	6568
50	6678	6789	6900	7010	7120	7230	7340	7450	7560	7670
49.0	7780	7889	7999	8109	8218	8328	8437	8547	8656	8765
10	8875	8984	9093	9202	9311	9420	9529	9637	9746	9855
20	9963	0072	0180	0289	0397	0505	0613	0721	0829	0937
30	88.1045	1153	1261	1369	1477	1584	1692	1799	1907	2014
40	2121	2228	2336	2443	2550	2657	2764	2871	2977	3084
50	3191	3297	3404	3510	3617	3723	3829	3936	4042	4148
50.0	4254	4360	4466	4572	4677	4783	4889	4994	5100	5205
10	5311	5416	5521	5627	5732	5837	5942	6047	6152	6257
20	6362	6466	6571	6675	6780	6885	6989	7093	7198	7302
30	7406	7510	7614	7718	7822	7926	8030	8133	8237	8341
40	8444	8549	8651	8755	8858	8961	9064	8167	9271	9374
50	9476	9579	9682	9785	9888	9990	0093	0195	0297	0400
51.0	890.503	605	707	809	911	013	115	217	319	421
10	891.523	624	726	827	929	030	132	233	334	435
20	892.536	637	738	839	940	041	142	242	343	444
30	893.544	645	745	846	946	046	146	246	346	447
40	894.546	646	746	846	945	045	144	244	343	445
50	895.542	641	741	840	939	038	137	236	335	434
52.0	896.532	631	729	828	926	025	123	222	320	419
10	897.516	614	712	810	908	006	104	201	299	397
20	898.494	592	689	787	884	981	078	176	273	371
30	899.467	564	660	757	854	951	047	144	240	337
40	890.433	529	626	722	818	914	010	106	202	298
50	901.394	489	585	681	776	872	967	063	158	254



D	1	2	3	4	5	6	7	8	9
45.0	00.0000	253	0505	0758	1011	1263	1516	1769	2021
10	2527	2779	3032	3285	3537	3790	4043	4295	4548
20	5053	5305	5559	5811	6064	6317	6569	6822	7075
30	7580	7833	8086	8338	8591	8844	9096	9349	9602
40	01.0107	0360	0613	0866	1118	1371	1624	1877	2129
50	2635	2888	3140	3393	3646	3899	4152	4404	4657
46.0	5163	5416	5668	5921	6174	6427	6680	6932	7185
10	7691	7944	8197	8450	8703	8956	9209	9461	9714
20	02.0220	0473	0726	0979	1232	1485	1738	1991	2244
30	2750	3003	3256	3509	3762	4015	4268	4521	4774
40	5280	5534	5787	6040	6293	6546	6799	7052	7305
50	7812	8065	8318	8571	8825	9078	9331	9584	9837
47.0	03.0344	0597	0851	1104	1357	1611	1864	2117	2371
10	2877	3131	3384	3637	3891	4144	4398	4651	4905
20	5412	5665	5919	6172	6426	6680	6933	7187	7440
30	7947	8201	8455	8708	8962	9216	9469	9723	9977
40	04.0484	0738	0992	1246	1500	1753	2007	2261	2515
50	3023	3277	3531	3785	4038	4292	4546	4800	5054
48.0	5563	5817	6071	6325	6579	6833	7087	7341	7595
10	8104	8358	8612	8867	9121	9375	9629	9884	0138
20	05.0647	0901	1156	1410	1664	1919	2173	2428	2682
30	3191	3445	3701	3955	4210	4465	4719	4974	5229
40	5738	5992	6248	6502	6757	7012	7267	7522	7777
50	8286	8541	8796	9051	9306	9561	9816	0072	0327
49.0	06.0837	1092	1347	1602	1858	2113	2368	2623	2879
10	3389	3645	3900	4156	4411	4666	4922	5177	5433
20	5944	6200	6455	6711	6967	7222	7478	7734	7989
30	8501	8757	9013	9269	9524	9780	0036	0292	0548
40	07.1060	1316	1573	1829	2085	2341	2597	2853	3110
50	3522	3878	4135	4391	4648	4904	5160	5417	5673
50.0	6186	6443	6700	6956	7213	7470	7726	7983	8240
10	8753	9010	9267	9524	9780	0038	0295	0552	0809
20	08.1323	1580	1837	2094	2352	2609	2866	3123	3381
30	3895	4153	4410	4668	4925	5183	5440	5698	5956
40	6471	6728	6986	7244	7502	7760	8018	8275	8533
50	9049	9307	9565	9823	0081	0340	0598	0856	1114
51.0	09.1631	1889	2147	2406	2664	2923	3181	3440	3698
10	4215	4474	4733	4991	5250	5509	5768	6026	6286
20	6803	7062	7321	7580	7839	8099	8358	8617	8876
30	9395	9654	9913	0173	0432	0692	0951	1211	1470
40	10.1990	2249	2500	2769	3028	3288	3548	3808	4068
50	4588	4848	5108	5368	5628	5889	6149	6409	6670
52.0	7190	7451	7711	7971	8232	8493	8753	9013	9275
10	9796	0057	0318	0579	0839	1100	1361	1622	1883
20	11.2406	2667	2928	3189	3451	3712	3974	4235	4496
30	5019	5281	5543	5804	6066	6328	6590	6852	7113
40	7637	7899	8161	8423	8686	8948	9210	9472	9735
50	12.0259	0522	0784	1047	1309	1572	1835	2097	2360

\*42 Log. Sines. Deg. 53, 54, 55, 56, 57, 58, 59, 60. In. 9.

D	0	1	2	3	4	5	6	7	8	9
53.0	902.349	444	539	634	729	824	919	014	108	203
10	903.298	392	487	581	676	770	864	959	053	147
20	904.241	335	429	523	617	711	804	898	991	085
30	905.179	272	366	459	552	645	739	832	925	017
40	906.111	204	296	389	482	574	667	760	852	945
50	907.037	129	222	314	406	498	591	682	774	866
54.0	958	049	141	233	324	416	507	599	690	781
10	908.873	964	055	146	237	328	419	510	601	691
20	909.782	873	963	054	144	235	325	415	506	596
30	910.686	776	866	956	046	136	226	315	405	495
40	911.584	674	763	853	942	031	121	210	299	388
50	912.477	566	655	744	833	921	010	099	187	276
55.0	913.364	453	541	630	718	806	894	982	070	158
10	914.246	334	422	510	598	685	773	860	948	035
20	915.123	210	297	385	472	559	646	733	820	907
30	994	080	167	254	341	427	514	600	687	773
40	916.859	945	032	118	204	290	376	462	548	634
50	917.719	805	891	976	062	147	233	318	404	489
56.0	918.574	659	744	830	915	000	084	169	254	339
10	919.424	508	593	677	762	846	931	015	099	184
20	920.268	352	436	520	604	688	772	855	939	023
30	921.107	190	174	357	441	524	607	691	774	857
40	941	023	106	189	272	355	438	520	603	685
50	922.768	851	933	016	098	180	263	345	427	509
57.0	923.591	673	755	837	919	001	083	164	246	328
10	924.409	491	572	653	735	816	897	978	060	141
20	925.222	303	384	465	545	626	707	787	868	949
30	926.029	110	190	270	351	431	511	591	671	751
40	831	911	991	071	151	231	310	390	469	549
50	927.628	708	787	867	946	025	104	183	262	341
58.0	928.420	499	578	657	736	814	893	972	050	129
10	929.207	286	364	442	521	599	677	755	833	911
20	989	067	145	223	300	378	456	533	611	688
30	930.764	843	920	998	075	152	229	306	383	460
40	931.537	611	691	768	845	921	998	074	151	227
50	932.304	380	457	533	609	685	762	838	914	990
59.0	933.066	141	217	293	369	444	520	596	671	747
10	822	897	973	048	123	198	274	349	424	499
20	934.574	649	723	798	873	948	022	097	171	246
30	935.320	395	469	543	618	692	766	840	914	988
40	936.062	136	210	283	357	431	505	578	652	725
50	799	872	945	019	092	165	238	311	385	458
60.0	937.531	603	674	749	822	895	967	040	113	186
10	938.258	330	402	475	547	619	691	763	835	907
20	979	051	123	195	267	339	410	482	554	625
30	939.697	768	839	911	982	053	125	196	267	338
40	940.409	480	515	622	693	763	834	905	975	046
50	941.117	187	257	328	398	468	539	609	679	749

Log.

D	1
53.0	203
10	147
20	085
30	017
40	945
50	866
54.0	781
10	691
20	596
30	495
40	388
50	276
55.0	158
10	035
20	907
30	773
40	624
50	489
56.0	339
10	184
20	023
30	857
40	685
50	509
57.0	328
10	141
20	949
30	751
40	549
50	341
58.0	129
10	911
20	688
30	460
40	227
50	990
59.0	747
10	499
20	246
30	988
40	725
50	458
60.0	186
10	907
20	625
30	338
40	046
50	749

D	0	1	2	3	4	5	6	7	8	9
53.0	12.2386	3148	3411	3674	3937	4200	4463	4727	4990	5253
10	5516	5780	6043	6306	6570	6833	7097	7360	7624	7888
20	8151	8415	8679	8942	9207	9471	9735	9999	0263	0527
30	13.0791	1055	1320	1584	1848	2113	2377	2642	2906	3171
40	3436	3700	3965	4230	4495	4760	5024	5289	5555	5820
50	6085	6350	6615	6880	7146	7411	7677	7942	8208	8473
54.0	8739	9005	9270	9536	9802	0068	0334	0600	0866	1132
10	14.1398	1664	1931	2197	2463	2730	2996	3263	3529	3796
20	4062	4329	4596	4863	5130	5397	5663	5931	6198	6465
30	6732	6999	7266	7534	7801	8069	8336	8604	8871	9139
40	9407	9675	9942	0210	0478	0746	1014	1283	1551	1819
50	15.2057	2356	2624	2892	3161	3429	3698	3967	4235	4504
55.0	4773	5042	5311	5580	5849	6118	6387	6657	6926	7195
10	7465	7734	8004	8273	8543	8813	9083	9352	9622	9892
20	16.0162	0432	0702	0973	1243	1513	1783	2054	2324	2595
30	2866	3136	3407	3678	3949	4220	4491	4762	5033	5304
40	5575	5846	6118	6389	6661	6932	7204	7475	7747	8019
50	8291	8563	8835	9107	9379	9651	9923	0195	0468	0740
56.0	17.1023	1205	1558	1830	2103	2376	2649	2922	3195	3468
10	3741	4014	4287	4561	4834	5107	5381	5655	5928	6202
20	6476	6749	7023	7297	7571	7845	8120	8394	8668	8943
30	9217	9492	9766	0041	0316	0590	0865	1140	1415	1690
40	18.1965	2240	2516	2791	3066	3342	3618	3893	4169	4445
50	4720	4996	5272	5548	5824	6101	6377	6653	6930	7206
57.0	7483	7759	8036	8313	8589	8866	9143	9420	9697	9975
10	19.0252	0529	0807	1084	1362	1639	1917	2195	2473	2751
20	3029	3307	3585	3863	4141	4420	4698	4977	5255	5534
30	5813	6091	6370	6649	6928	7207	7487	7766	8045	8325
40	8604	8884	9163	9443	9723	0003	0283	0563	0843	1123
50	20.1403	1684	1964	2245	2525	2806	3087	3368	3649	3930
58.0	4211	4492	4773	5054	5336	5617	5899	6180	6462	6744
10	7026	7308	7590	7872	8154	8436	8719	9001	9284	9565
20	9849	0132	0415	0698	0981	1264	1547	1836	2114	2397
30	21.2681	2964	3248	3532	3815	4100	4384	4668	4952	5235
40	5521	5805	6090	6374	6659	6944	7229	7514	7799	8084
50	8369	8654	8940	9225	9511	9797	0082	0368	0654	0940
59.0	22.1226	1512	1799	2085	2371	2658	2945	3231	3518	3805
10	4092	4379	4667	4954	5241	5529	5816	6104	6392	6679
20	6967	7255	7543	7832	8120	8408	8697	8985	9274	9563
30	9851	0140	0429	0719	1008	1297	1586	1876	2166	2455
40	23.2745	3035	3325	3615	3905	4195	4486	4776	5067	5357
50	5648	5939	6230	6520	6812	7103	7394	7685	7977	8268
60.0	8561	8852	9144	9436	9728	0021	0313	0605	0898	1190
10	24.1483	1776	2069	2362	2655	2948	3241	3535	3828	4122
20	4415	4709	5003	5297	5592	5885	6180	6474	6769	7063
30	7358	7653	7948	8243	8538	8833	9128	9424	9719	0015
40	25.0311	0607	0903	1199	1495	1791	2087	2384	2681	2977
50	3274	3571	3868	4165	4462	4760	5057	5355	5652	5950



\*44 Log. Sines Deg. 61, 62, 63, 64, 65, 66, 67, 68. In. g.

D	0	1	2	3	4	5	6	7	8	9
61. 0	941.819	889	959	029	099	169	239	308	378	448
10	942.517	587	656	725	795	864	933	003	072	141
20	943.201	279	348	417	486	555	624	692	761	830
30	898	967	036	104	172	241	309	377	446	514
40	944.582	650	718	786	854	922	990	058	125	193
50	945.261	328	396	464	531	599	666	733	800	868
62. 0	935	002	069	136	203	270	337	404	471	538
10	946.604	671	738	804	870	937	004	070	136	203
20	947.269	335	401	467	533	599	665	731	797	863
30	929	995	060	126	192	257	323	388	453	519
40	948.584	649	715	780	845	910	975	040	105	170
50	949.235	300	364	429	494	558	623	688	752	816
63. 0	881	945	009	074	138	202	266	330	394	458
10	950.522	586	650	714	777	841	905	968	032	096
20	951.159	222	286	349	412	476	539	602	665	728
30	791	854	917	980	043	105	168	231	294	356
40	952.419	481	544	606	668	731	793	855	917	979
50	953.042	104	166	228	290	351	413	475	537	598
64. 0	660	722	783	845	906	967	029	090	152	213
10	954.274	335	396	457	518	579	640	701	762	823
20	883	944	005	065	126	186	247	307	368	428
30	955.488	548	609	669	729	789	849	909	969	029
40	956.089	148	208	268	328	387	447	506	566	625
50	684	744	804	862	921	980	040	099	158	217
65. 0	957.276	335	393	452	511	570	628	687	746	804
10	862	919	979	038	096	154	212	271	329	387
20	958.445	503	561	619	677	734	792	850	908	965
30	959.023	080	138	195	253	310	367	425	482	539
40	590	63	711	768	825	881	938	995	052	109
5	960.165	222	279	335	392	448	505	561	618	674
66. 0	730	786	845	899	955	011	067	123	179	235
10	961.290	346	402	458	513	569	624	680	735	791
20	846	902	957	012	067	122	178	233	288	343
30	962.398	453	508	562	617	672	727	781	836	891
40	945	999	054	108	162	217	271	325	379	434
50	963.488	542	596	650	704	757	811	865	919	972
67. 0	96.026	080	133	187	240	294	347	400	454	507
10	560	613	666	719	773	826	878	931	984	037
20	965.690	143	195	248	301	353	406	458	511	563
30	615	668	720	772	824	879	928	980	032	085
40	966.136	188	240	292	344	395	447	499	550	602
50	653	705	756	807	858	910	961	012	064	115
68. 0	967.166	217	268	319	369	420	471	522	573	623
10	674	725	775	826	876	927	977	027	078	128
20	968.178	228	278	328	379	429	478	528	578	628
30	678	728	777	827	877	926	976	025	075	124
40	969.173	223	272	321	370	420	469	518	567	616
50	665	714	762	811	860	909	957	006	055	103

D	1	0	1	2	3	4	5	6	7	8	9
61.0	25.6248	6546	6844	7142	7441	7739	8038	8336	8635	8934	
10	9233	9532	9831	0130	0430	0730	1029	1329	1629	1928	
20	26.2229	2529	2829	3129	3430	3731	4031	4332	4633	4934	
30	5235	5537	5838	6140	6442	6743	7045	7347	7649	7952	
40	8254	8556	8859	9162	9465	9767	0070	0374	0677	0980	
50	27.1284	1588	1891	2195	2499	2803	3107	3412	3716	4021	
62.0	4326	4630	4935	5240	5546	5851	6156	6462	6768	7073	
10	7379	7685	7991	8298	8604	8911	9217	9524	9831	0138	
20	28.0445	0752	1060	1367	1673	1983	2291	2599	2907	3215	
30	3523	3832	4140	4449	4758	5067	5376	5685	5995	6304	
40	6614	6924	7234	7544	7854	8164	8475	8785	9096	9407	
50	9718	0029	0340	0651	0963	1274	1586	1898	2210	2522	
63.0	29.2834	3146	3459	3772	4084	4397	4710	5023	5337	5650	
10	5964	6277	6591	6905	7219	7534	7848	8163	8477	8792	
20	9167	9422	9737	0053	0368	0684	0999	1315	1631	1947	
30	30.2264	2580	2897	3213	3530	3847	4164	4482	4799	5117	
40	5434	5752	6070	6388	6707	7025	7343	7662	7981	8300	
50	8619	8938	9258	9577	9897	0217	0537	0857	1177	1498	
64.0	31.1818	2139	2460	2781	3102	3423	3745	4066	4388	4710	
10	5032	5354	5676	5999	6321	6644	6967	7290	7613	7937	
20	8260	8584	8908	9232	9556	9880	0205	0529	0854	1179	
30	32.1504	1829	2154	2480	2806	3131	3457	3783	4110	4436	
40	4763	5089	5416	5743	6071	6398	6725	7053	7381	7709	
50	8037	8365	8694	9023	9351	9680	0009	0339	0668	0998	
65.0	33.1327	1657	1987	2318	2648	2979	3309	3640	3971	4302	
10	4634	4965	5297	5629	5961	6293	6625	6958	7291	7623	
20	7957	8290	8623	8957	9290	9624	9958	0292	0627	0961	
30	34.1296	1631	1966	2301	2636	2972	3308	3644	3980	4316	
40	4652	4989	5326	5662	6000	6337	6674	7012	7350	7688	
50	8026	8364	8702	9041	9380	9719	0058	0398	0737	1077	
66.0	35.1417	1757	2097	2438	2778	3119	3460	3801	4142	4484	
10	4826	5168	5510	5852	6194	6537	6880	7223	7566	7909	
20	8253	8596	8940	9284	9629	9973	0318	0662	1008	1353	
30	36.1698	2044	2389	2735	3081	3428	3774	4121	4468	4815	
40	5162	5510	5857	6205	6553	6901	7250	7598	7947	8296	
50	8645	8995	9344	9694	0044	0394	0745	1095	1446	1797	
67.0	37.2148	2499	2851	3203	3555	3907	4259	4612	4965	5317	
10	5671	6024	6377	6731	7085	7439	7793	8148	8503	8858	
20	9213	9568	9924	0279	0635	0992	1348	1705	2061	2418	
30	38.2776	3133	3491	3849	4207	4565	4923	5282	5641	6000	
40	6359	6719	7079	7438	7799	8159	8519	8880	9241	9603	
50	9964	0326	0689	1050	1412	1775	2137	2500	2863	3226	
68.0	39.3590	3954	4318	4683	5047	5412	5777	6142	6507	6873	
10	7239	7605	7971	8337	8704	9071	9438	9806	0173	0541	
20	40.0909	1277	1646	2015	2384	2753	3122	3492	3862	4232	
30	4602	4973	5344	5715	6086	6458	6829	7201	7574	7946	
40	8319	8692	9065	9438	9812	0186	0560	0934	1309	1684	
50	41.2059	2434	2809	3185	3561	3938	4314	4691	5068	5445	

\*46 Logar. Sines. Deg. 69, 70, 71, 72, 73, 74, 75, 76. In. 9.

D	0	1	2	3	4	5	6	7	8	9
69.0	970.152	200	248	297	345	394	442	490	538	586
10	635	683	731	779	826	874	922	970	018	065
20	971.113	161	208	256	303	351	398	446	493	540
30	588	635	682	729	776	823	870	917	964	011
40	972.058	105	151	198	245	291	338	384	431	477
50	524	570	617	662	709	755	802	848	894	940
70.0	986	032	078	124	169	215	261	307	352	398
10	973.443	489	535	580	625	671	716	761	807	852
20	897	942	987	032	077	122	167	212	257	302
30	974.347	391	436	481	525	570	614	659	703	747
40	792	836	881	925	969	013	057	101	145	189
50	975.233	277	321	365	408	452	496	539	583	626
71.0	670	713	757	800	844	887	930	974	017	060
10	976.103	146	189	232	275	318	361	404	446	489
20	532	574	617	660	702	745	787	830	872	914
30	957	999	041	083	125	167	209	251	293	335
40	977.377	419	461	503	544	586	628	669	711	752
50	794	835	877	918	959	001	042	083	124	165
72.0	978.206	247	288	329	370	411	452	493	533	574
10	615	655	696	736	777	817	858	898	939	979
20	979.019	059	100	140	180	220	260	300	340	380
30	420	460	500	539	579	618	658	697	737	776
40	816	855	895	934	973	012	052	091	130	169
50	980.208	247	286	325	364	403	441	480	519	558
73.0	596	635	673	712	750	789	827	866	904	942
10	980	019	057	095	133	171	209	247	285	323
20	981.361	399	436	474	512	549	587	624	662	699
30	737	774	812	849	886	924	961	998	035	072
40	982.109	146	183	220	257	294	331	367	404	440
50	477	514	551	587	623	660	696	733	769	805
74.0	842	878	914	950	986	022	058	094	130	166
10	983.202	238	273	309	345	380	416	452	487	523
20	558	594	629	664	700	735	770	805	840	875
30	910	945	980	015	050	085	120	155	189	224
40	984.259	293	328	363	397	432	466	500	535	569
50	603	637	672	706	740	774	808	843	876	910
75.0	944	978	011	045	079	112	146	180	213	247
10	985.280	314	347	380	414	447	480	513	547	580
20	613	646	679	712	745	778	811	843	876	909
30	945	974	007	039	072	104	137	169	202	234
40	986.266	299	331	363	395	437	459	491	523	555
50	587	619	651	683	714	746	778	809	841	873
76.0	904	936	967	998	030	061	092	124	155	186
10	987.217	248	279	310	341	372	403	434	465	495
20	526	557	587	618	649	679	710	740	771	801
30	831	862	892	922	953	983	013	043	073	803
40	988.133	163	193	222	252	282	312	341	371	401
50	430	460	490	519	548	578	607	636	665	695



Logar. Tang. Deg. 69, 70, 71, 72, 73, 74, 75, 76. In. 10. °47

D	0	1	2	3	4	5	6	7	8	9
69.0	41.5823	6200	6578	6956	7335	7714	8093	8472	8851	9231
10	9611	9991	0371	0752	1133	1514	1896	2277	2659	3041
20	42.3424	3807	4190	4573	4956	5340	5724	6108	6493	6877
30	7262	7648	8033	8419	8805	9191	9578	9964	03 2	0739
40	43.1126	1514	1902	2291	2679	3068	3458	3847	4237	4627
50	5017	5407	5798	6189	6581	6972	7364	7756	8148	8541
70.0	8934	9327	9721	0115	0509	0903	1297	1692	2087	2483
10	44.2879	3274	3671	4067	4464	4861	5258	5656	6054	6452
20	6851	7250	7649	8048	8448	8847	9248	9648	0049	0450
30	45.0851	1253	1655	2057	2459	2862	3265	3669	4072	4476
40	4881	5285	5690	6095	6501	6906	7312	7719	8125	8532
50	8939	9347	9755	0163	0571	0980	1389	1798	2208	2618
71.0	46.3028	3439	3849	4261	4672	5084	5496	5908	6321	6734
10	7147	7561	7975	8389	8804	9219	9634	0049	0465	0881
20	47.1298	1715	2132	2549	2967	3385	3803	4222	4641	5060
30	5480	5900	6320	6741	7162	7583	8005	8427	8849	9272
40	9695	0118	0542	0966	1390	1814	2239	2665	3090	3516
50	48.3942	4369	4796	5223	5651	6079	6507	6936	7365	7794
72.0	8224	8654	9084	9515	9946	0378	0809	1241	1674	2107
10	49.2540	2973	3407	3841	4276	4711	5146	5582	6018	6454
20	6891	7328	7765	8203	8641	9080	9519	9958	0397	0837
30	50.1278	1718	2159	2601	3043	3485	3927	4370	4813	5257
40	5701	6145	6590	7035	7481	7927	8373	8820	9267	9714
50	51.1062	0610	1059	1508	1957	2407	2857	3307	3758	4209
73.0	4661	5113	5565	6018	6471	6925	7379	7833	8288	8743
10	9199	9655	0111	0568	1025	1483	1941	2399	2858	3317
20	52.3777	4237	4697	5158	5619	6081	6543	7005	7468	7931
30	8395	8859	9324	9789	0254	0720	1186	1653	2120	2587
40	53.3055	3523	3992	4461	4931	5401	5871	6342	6814	7285
50	7758	8230	8703	9177	9651	0125	0600	1075	1551	2027
74.0	54.2504	2981	3458	3936	4414	4893	5372	5852	6332	6813
10	7294	7775	8257	8740	9223	9706	0190	0674	1159	1644
20	55.2130	2616	3102	3589	4077	4565	5053	5542	6031	6521
30	7012	7502	7994	8485	8978	9471	9964	0457	0951	1446
40	56.1941	2437	2933	3430	3927	4424	4922	5421	5920	6419
50	6920	7420	7921	8423	8925	9427	9930	0431	0938	1442
75.0	57.1947	2453	2959	3466	3973	4481	4989	5497	6006	6516
10	7026	7537	8048	8560	9072	9585	0099	0613	1127	1642
20	58.2157	2673	3190	3707	4225	4743	5262	5781	6301	6821
30	7342	7863	8385	8908	9431	9955	0479	1003	1529	2055
40	59.2581	3108	3636	4164	4692	5222	5751	6282	6813	7344
50	7876	8409	8942	9476	0010	0545	1081	1617	2154	2691
76.0	60.3229	3767	4306	4846	5386	5927	6469	7011	7553	8097
10	8640	9185	9730	0276	0822	1369	1916	2464	3013	3562
20	61.4112	4663	5214	5766	6318	6871	7425	7979	8534	9090
30	9646	0203	0761	1319	1877	2434	2997	3558	4119	4681
40	62.5244	5807	6371	6935	7501	8067	8633	9201	9768	0337
50	63.0906	1476	2047	2618	3190	3762	4336	4910	5484	6060

48 *Leg. Sines. Deg. 77, 78, 79, 80, 81, 82, 83, 84. In. 9.*

D	0	1	2	3	4	5	6	7	8	9
77.0	724	753	782	811	840	869	898	927	956	985
10	989.014	042	071	100	128	157	185	214	243	271
20	299	328	356	384	413	441	469	497	525	553
30	581	609	637	665	693	721	749	777	804	832
40	859	887	915	942	970	997	025	052	079	107
50	990.134	161	188	215	242	269	296	324	351	377
78.0	404	431	458	485	511	538	565	591	618	644
10	671	697	724	750	777	803	829	855	881	908
20	934	960	986	012	038	064	090	115	141	167
30	991.193	218	244	270	295	321	346	372	397	422
40	448	473	498	524	549	574	599	624	649	674
50	699	724	749	774	799	823	848	873	897	922
79.0	947	971	996	020	044	069	093	117	142	166
10	992.190	214	238	263	287	311	335	358	382	406
20	430	454	478	501	525	549	572	596	619	643
30	666	689	713	736	759	783	806	829	852	875
40	898	921	944	967	990	013	036	058	081	104
50	993.127	149	172	195	217	240	262	284	307	329
80.0	351	374	396	418	440	462	484	506	528	550
10	572	594	616	638	660	681	703	725	746	768
20	789	811	832	853	875	896	918	939	960	981
30	994.003	024	045	066	087	108	129	150	171	191
40	212	233	254	274	295	316	336	357	377	397
50	418	438	459	479	499	519	540	560	580	600
81.0	620	640	660	680	700	719	739	759	779	798
10	818	838	857	877	896	916	935	955	974	993
20	995.013	032	051	070	089	108	127	146	165	184
30	203	222	241	260	278	297	316	334	353	372
40	390	409	427	445	464	482	500	519	537	555
50	573	591	609	628	646	663	681	699	717	735
82.0	753	770	788	806	823	841	859	876	894	911
10	928	946	963	980	998	015	032	049	066	083
20	996.100	117	134	151	168	185	202	218	235	252
30	268	285	302	318	335	351	368	384	400	417
40	434	449	465	482	498	514	530	546	562	578
50	594	610	625	641	657	673	688	704	720	735
83.0	751	766	782	797	812	828	843	858	874	889
10	904	919	934	949	964	979	994	009	024	039
20	997.053	068	083	097	112	127	141	156	170	185
30	199	214	228	242	257	271	285	299	313	327
40	341	355	369	383	397	411	425	439	452	466
50	480	493	507	520	534	547	561	574	588	601
84.0	614	627	641	654	667	680	693	706	719	732
10	745	758	771	784	797	809	822	835	847	860
20	872	885	897	910	922	935	947	959	972	984
30	896	008	020	032	044	056	068	080	092	104
40	116	127	139	151	163	174	186	197	209	220
50	232	243	254	266	277	288	300	311	322	333

Logar. Tang. Deg. 77, 78, 79, 80, 81, 82, 83, 84. Index IO, II. \*49

D	0	1	2	3	4	5	6	7	8	9
77. 0	63 66.6	7213	7790	8368	8947	9526	0106	0687	1269	1551
10	64.2434	3018	3602	4187	4773	5360	5947	6535	7124	7713
20	8303	3894	9486	0078	0671	1265	1859	2455	3051	3647
30	65.4245	4843	5442	6042	6642	7243	7845	8448	9052	9656
40	66.0261	0867	1473	2081	2689	3298	3907	4518	5129	5741
50	6354	6967	7582	8197	8813	9430	0047	0665	1285	1905
78. 0	67.2525	3147	3769	4393	5017	5642	6267	6894	7521	8149
10	8778	9408	0039	0670	1303	1936	2570	3205	3841	4477
20	68.5115	5753	6392	7032	7673	8315	8958	9601	0246	0891
30	69.1537	2184	2832	3481	4131	4782	5433	6086	6739	7393
40	8049	8705	9362	0020	0578	1238	1999	2660	3323	3987
50	70.4651	5316	5983	6650	7318	7987	8658	9329	0001	0674
79. 0	71.1348	2023	2699	3375	4053	4732	5412	6093	6775	7455
10	8141	8826	9512	0199	0887	1576	2266	2957	3649	4342
20	72.5036	5731	6427	7124	7822	8521	9221	9923	0625	1329
30	73.2035	2739	3445	4153	4862	5572	6283	6995	7708	8422
40	9137	9854	0571	1290	2010	2731	3453	4176	4900	5626
50	74.6352	7080	7809	8539	9270	0002	0736	1470	2205	2943
80. 0	75.3681	4421	5151	5903	6646	7390	8135	8881	9629	0378
10	76.1128	1880	2632	3386	4141	4897	5656	6414	7174	7935
20	8698	9461	0226	0993	1761	2529	3300	4071	4844	5618
30	77.6393	7170	7948	8728	9508	0290	1074	1858	2644	3432
40	78.4220	5011	5802	6595	7389	8185	8982	9780	0580	1381
50	79.2183	2987	3793	4600	5408	6217	7029	7841	8655	9471
81. 0	80.0287	1106	1926	2747	3570	4394	5220	6047	6876	7706
10	8538	9371	0206	1042	1880	2720	3561	4403	5247	6093
20	81.6940	7789	8640	9492	0345	1201	2057	2916	3770	4638
30	82.5501	6366	7233	8101	8971	9843	0716	1591	2468	3346
40	83.4226	5108	5992	6877	7764	8653	9543	0435	1329	2225
50	84.3123	4022	4923	5826	6730	7637	8546	9456	0368	1282
82. 0	85.2197	3115	4034	4956	5879	6804	7731	8660	9591	0524
10	86.1458	2295	3333	4274	5216	6161	7107	8056	9006	9959
20	87.0913	1870	2828	3789	4751	5716	6683	7652	8623	9598
30	88.0571	1548	2528	3509	4493	5479	6467	7457	8449	9444
40	89.0441	1440	2441	3444	4450	5458	6468	7481	8495	9513
50	90.0532	1554	2578	3604	4633	5664	6698	7734	8774	9813
83. 0	91.0856	1902	2950	4000	5053	6109	7167	8227	9290	0356
10	92.1424	2495	3567	4644	5722	6803	7887	8973	0061	1153
20	93.2248	3345	4444	5547	6652	7760	8870	9984	1100	2219
30	94.3340	4465	5593	6723	7856	8992	0131	1273	2418	3566
40	95.4716	5870	7027	8187	9349	0515	1684	2856	4031	5209
50	96.6391	7575	8763	9953	1148	2345	3545	4749	5956	7166
84. 0	97.8380	9597	0817	2041	3267	4498	5732	6969	8210	9454
10	99.0702	1953	3208	4466	5728	6993	8262	9535	0812	2192
20	11.00.3376	4663	5955	7250	8549	9851	1158	2468	3783	5101
30	01.6423	7749	9079	0413	1752	3094	4440	5791	7145	8504
40	02.9667	1234	2606	3981	5361	6745	8131	9527	0925	2326
50	04.3733	5144	6560	7979	9403	0832	2266	3705	5148	6596





The Remainder of the Logarithm Tangents to 90.

\*51

Min.	85 Deg.	86 Deg.	87 Deg.	88 Deg.	89 Deg.	Min.
0	11.058048	11.155356	11.280604	11.456916	11.758078	0
1	9506	11. 7175	11. 3028	11. 60553	11. 65379	1
2	11.060968	11. 9092	11. 5465	11. 64221	11. 72805	2
3	11. 2435	11.160837	11. 7917	11. 67920	11. 80359	3
4	11. 3907	11. 2679	11.290381	11. 71651	11. 88047	4
5	11. 5384	11. 4529	11. 2860	11. 75414	11. 95874	5
6	11. 6866	1. 6387	11. 5353	11. 79210	11.803844	6
7	11. 8353	11. 8252	11. 7861	11. 83039	11. 11964	7
8	11. 9845	11.170126	11.300383	11. 86902	11. 20237	8
9	11.071342	11. 2008	11. 2919	11. 90800	11. 28672	9
10	11. 2844	11. 3897	11. 5471	11. 94733	11. 37273	10
11	11. 4351	11. 5795	11. 8037	11. 98702	11. 46048	11
12	11. 5864	11. 7702	11.310618	11.502707	11. 55004	12
13	11. 7381	11. 9616	11. 3216	11. 06750	11. 64149	13
14	11. 8904	11.181539	11. 5828	11. 10830	11. 73490	14
15	11.080432	11. 3471	11. 8456	11. 14949	11. 83037	15
16	11. 1966	11. 5410	11.321100	11. 19108	11. 92797	16
17	11. 3505	11. 7359	11. 3761	11. 23307	11.902783	17
18	11. 5049	11. 9317	11. 6437	11. 27546	11. 13003	18
19	11. 6599	11.191283	11. 9730	11. 31827	11. 23469	19
20	11. 8154	11. 3258	11.331848	11. 36151	11. 34194	20
21	11. 9715	11. 5242	11. 4566	11. 40519	11. 45191	21
22	11.091281	11. 7235	11. 7311	11. 44930	11. 56473	22
23	11. 2853	11. 9236	11.340072	11. 49387	11. 68055	23
24	11. 4430	11.201248	11. 2851	11. 53890	11. 79955	24
25	11. 6013	11. 3269	11. 5648	11. 58440	11. 92191	25
26	11. 7602	11. 5299	11. 8462	11. 63078	12.004781	26
27	11. 9197	11. 7338	11.351296	11. 67685	12. 17746	27
28	11.100797	11. 9387	11. 4147	11. 72382	12. 31111	28
29	11. 2404	11.211446	11. 7017	11. 77131	12. 44900	29
30	11. 4016	11. 3514	11. 9907	11. 81932	12. 59142	30
31	11. 5634	11. 5592	11.362815	11. 86787	12. 73866	31
32	11. 7258	11. 7680	11. 5744	11. 91696	12. 89106	32
33	11. 8888	11. 9778	11. 8692	11. 96662	12.104901	33
34	11.110524	11.221886	11.371660	11.601685	12. 21292	34
35	11. 2167	11. 4005	11. 4648	11. 06766	12. 38326	35
36	11. 3815	11. 6133	11. 7657	11. 11908	12. 56056	36
37	11. 5470	11. 8272	11.380687	11. 17111	12. 74540	37
38	11. 7131	11.230422	11. 3738	11. 22378	12. 93845	38
39	11. 8798	11. 2582	11. 6811	11. 27708	12.214049	39
40	11.120471	11. 47 3	11. 9906	11. 33105	12. 35239	40

\*52 *The Remainder of the Logarithmic Tangents, to 90 Degrees.*

Min.	85 Deg.	86 Deg.	87 Deg.	88 Deg.	89 Deg.	Min.
41	11.122151	11.236935	11.393022	11.638570	12.257516	41
42	11. 3838	11. 5128	11. 6161	11. 44105	12. 80997	42
43	11. 5531	11.241332	11. 9323	11. 49710	12.305821	43
44	11. 7230	11. 3547	11.402508	11. 55379	12. 32151	44
45	11. 8936	11. 5773	11. 5717	11. 61144	12. 60180	45
46	11.130649	11. 8011	11. 8949	11. 66975	12. 90141	46
47	11. 2368	11.250260	11.412205	11. 72886	12.422328	47
48	11. 4094	11. 2521	11. 5486	11. 78878	12. 57091	48
49	11. 5827	11. 4793	11. 8792	11. 84954	12. 94880	49
50	11. 7567	11. 7078	11.422123	11. 91110	12.536273	50
51	11. 9314	11. 9374	11. 5480	11. 97366	12. 82030	51
52	11.141068	11.261683	11. 8863	11.703708	12.633183	52
53	11. 2829	11. 4004	11.432272	11. 10144	12. 91175	53
54	11. 4597	11. 6337	11. 5709	11. 16677	12.758122	54
55	11. 6372	11. 8583	11. 9172	11. 23309	12.837304	55
56	11. 8154	11.271041	11.442654	11. 30044	12.934214	56
57	11. 9943	11. 3412	11. 6183	11. 35885	13.059153	57
58	11.151740	11. 5796	11. 9732	11. 43835	13.235241	58
59	11. 3545	11. 8194	11.453309	11. 50898	13.536274	59
60	11. 5356	11.280604	11. 6916	11. 58078		60



Mr *STREET*'s  
T A B L E

O F

*Logistical Logarithms*

To every  $\left\{ \begin{array}{l} \text{Minute} \\ \text{Second} \end{array} \right\}$  of a  $\left\{ \begin{array}{l} \text{Degree.} \\ \text{Minute.} \end{array} \right\}$

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	0	1	2	3	4	5	6	7	8	9	11
0. 0	0000	35536	32553	3079	9542	48573	27782	2711	26522	2601	0
10	2. 5163	5149	4771	4424	410.	3802	3522	3259	3010	2775	10
20	2553	2341	2139	1946	1761	1584	1413	1249	1091	0939	20
30	0792	0649	0512	037	0148	0122	0000	9881	9765	962	30
40	1. 9542	9435	9331	9228	9128	9031	89	8812	871	8661	40
50	8575	8487	8403	8320	8239	8159	8081	8001	7929	7855	50
1. 0	17. 782	710	639	570	501	434	368	302	238	175	60
10	112	050	990	93	871	812	75	698	642	587	70
20	16. 532	478	425	372	320	269	218	168	118	069	80
30	021	973	625	878	832	786	740	695	651	607	90
40	15. 563	520	477	435	393	351	310	269	229	189	100
50	149	110	071	032	924	956	918	881	844	805	110
2. 0	14. 771	735	699	664	629	594	559	525	491	457	120
10	424	390	357	325	292	260	228	196	165	133	130
20	102	071	040	010	979	949	919	890	860	831	140
30	13. 802	773	745	716	688	660	632	604	576	549	150
40	522	795	468	441	415	388	362	336	310	284	160
50	259	233	208	182	158	133	108	083	059	034	170
3. 0	010	985	962	936	915	891	868	845	821	798	180
10	12. 775	753	730	707	685	663	640	618	596	574	190
20	553	531	510	488	467	445	424	403	382	362	200
30	341	320	300	279	259	239	218	198	178	159	210
40	139	119	099	080	061	041	022	003	984	965	220
50	11. 946	927	908	889	871	852	834	816	797	779	230
4. 0	761	743	725	707	689	671	654	636	619	601	240
10	584	566	549	532	515	498	481	464	447	430	250
20	414	397	380	363	347	331	314	298	282	266	260
30	249	233	217	201	186	170	154	138	123	107	270
40	091	076	061	045	030	015	995	984	969	954	280
50	10. 939	924	909	894	880	865	850	835	821	806	290
5. 0	792	777	763	749	734	720	706	692	678	663	300
10	649	635	621	608	594	580	566	552	539	525	310
20	512	498	484	471	458	444	431	418	404	391	320
30	378	365	352	339	326	313	300	287	274	261	330
40	246	235	223	210	197	185	172	160	147	135	340
50	122	110	098	085	073	061	049	036	024	012	350
6. 0	000	988	976	964	952	940	928	916	905	893	360
10	9. 881	869	858	846	834	823	811	800	784	777	370
20	761	754	742	731	720	708	697	686	675	664	380
30	652	641	630	619	608	597	586	575	564	553	390
40	542	532	521	510	499	488	478	467	456	446	400
50	435	425	414	404	393	383	372	362	351	341	410
7. 0	331	320	310	300	289	275	265	259	249	238	420
10	228	218	208	198	188	178	168	158	148	138	430
20	128	119	109	099	089	075	070	060	050	041	440
30	031	021	012	002	992	983	973	964	954	945	450
40	8. 935	925	917	907	898	888	879	870	861	851	460
50	842	833	824	814	805	796	787	778	769	760	470



\*56 Street's Logist. Logarithms  $\frac{2}{11}$ , 8, 9, 10 11, 12, 13, 14, 15.

	0	1	2	3	4	5	6	7	8	9	
8.0	8. 751	742	733	724	715	706	697	688	679	670	480
10	661	652	643	635	626	617	608	599	591	582	490
20	573	565	556	547	539	530	522	513	504	496	500
30	487	479	470	462	453	445	437	428	420	411	510
40	403	395	386	378	370	361	353	345	337	328	520
50	320	312	304	296	288	279	271	263	255	247	530
9.0	82. 39	31	23	15	07	99	91	83	75	67	540
10	81. 59	52	44	36	28	20	12	04	97	89	550
20	80. 81	73	66	58	50	43	35	27	20	12	560
30	04	97	89	81	74	66	59	51	44	36	570
40	79. 29	21	14	06	99	91	84	77	69	62	580
50	78. 55	47	40	32	25	18	11	03	96	89	590
10.0	77. 82	74	67	60	53	45	38	31	24	17	600
10	10	03	96	88	81	74	67	60	53	46	610
20	76. 39	32	25	18	11	04	97	90	83	77	620
30	75. 70	63	56	49	42	35	28	22	15	08	630
40	01	94	88	81	74	67	61	54	47	41	640
50	74. 34	27	21	14	07	01	94	87	81	74	650
11.0	73. 68	61	54	48	41	35	28	22	15	09	660
10	02	96	89	83	76	70	64	57	51	44	670
20	72. 38	32	25	19	12	06	00	93	87	81	680
30	71. 75	68	62	56	49	43	47	31	24	18	690
40	12	06	00	93	87	81	75	69	63	57	700
50	70. 50	44	38	32	26	20	14	08	02	96	710
12.0	69. 90	84	78	72	66	60	54	48	42	36	720
10	30	24	18	12	06	00	94	88	82	77	730
20	68. 71	65	59	53	47	41	36	30	24	18	740
30	12	07	01	95	89	84	78	72	66	61	750
40	67. 55	49	43	38	32	26	21	15	09	04	760
50	66. 96	92	87	81	76	70	64	59	53	48	770
13.0	42	37	31	25	20	14	09	03	98	92	780
10	65. 87	81	76	70	65	59	54	48	43	38	790
20	32	27	21	16	10	05	00	94	89	84	800
30	64. 78	73	67	62	57	51	46	41	35	30	810
40	25	20	14	09	04	98	93	88	83	77	820
50	63. 72	67	62	57	51	46	41	36	31	25	830
14.0	20	15	10	05	00	94	89	84	79	74	840
10	62. 69	64	59	54	48	43	38	33	28	23	850
20	18	13	08	03	98	93	88	83	78	73	860
30	61. 68	63	58	53	48	43	38	33	28	23	870
40	18	13	08	03	99	94	89	84	79	74	880
50	60. 69	64	59	55	50	45	40	35	30	25	890
15.0	21	16	11	06	01	97	92	87	82	77	900
10	59. 73	68	63	58	54	49	44	39	35	30	910
20	25	20	16	11	06	02	97	92	88	83	920
30	58. 78	74	69	64	60	55	50	46	41	36	930
40	32	27	23	18	13	09	04	00	55	90	940
50	57. 86	81	77	72	68	63	58	54	49	45	950

	11	0	1	2	3	4	5	6	7	8	9	11
16. 0	57.40	36	31	27	22	18	13	09	04	00	960	
10	56.95	91	86	82	77	73	69	64	60	55	970	
20	51	46	42	37	33	29	24	20	15	11	980	
30	07	02	98	94	89	85	80	76	72	67	990	
40	55.63	59	54	50	46	41	37	33	28	24	1000	
50	20	16	11	07	03	98	94	90	86	81	1010	
17. 0	54.77	73	69	64	60	56	52	47	43	39	1020	
10	35	30	26	22	18	14	09	05	01	97	1030	
20	53.93	89	84	80	76	72	68	64	59	55	1040	
30	51	47	43	39	35	31	26	22	18	14	1050	
40	10	06	02	98	94	90	85	81	77	73	1060	
50	52.69	65	61	57	53	49	45	41	37	33	1070	
18. 0	29	25	21	17	13	09	05	01	97	93	1080	
10	51.89	85	81	77	73	69	65	61	57	53	1090	
20	49	45	41	37	33	29	25	22	18	14	1100	
30	10	06	02	98	94	90	86	82	79	75	1110	
40	50.71	67	63	59	55	51	48	44	40	36	1120	
50	32	28	25	21	17	13	09	05	02	98	1130	
19. 0	49.94	90	86	83	79	75	71	67	64	60	1140	
10	56	52	49	45	41	37	33	30	26	22	1150	
20	18	15	11	07	03	00	96	92	89	85	1160	
30	48.81	77	74	70	66	63	59	55	52	48	1170	
40	44	41	37	33	30	26	22	19	15	11	1180	
50	08	04	00	97	93	89	86	82	78	75	1190	
20. 0	47.71	68	64	60	57	53	50	46	42	39	1200	
10	35	32	28	24	21	17	14	10	07	03	1210	
20	46.99	96	93	89	85	82	78	75	71	68	1220	
30	64	60	57	53	50	46	43	39	36	32	1230	
40	29	25	22	18	15	11	08	04	01	97	1240	
50	45.94	90	87	84	80	77	73	70	66	63	1250	
21. 0	59	56	52	49	46	42	39	35	32	28	1260	
10	25	22	18	15	11	08	05	01	98	94	1270	
20	44.91	88	84	81	77	74	71	67	64	60	1280	
30	57	54	50	47	44	40	37	34	30	27	1290	
40	24	20	17	14	10	07	04	00	97	94	1300	
50	43.90	87	84	80	77	74	70	67	64	60	1310	
22. 0	57	54	51	47	44	41	38	34	31	28	1320	
10	25	21	18	15	11	08	05	02	98	95	1330	
20	42.92	89	85	82	79	76	73	69	66	63	1340	
30	60	56	53	50	47	44	40	37	34	31	1350	
40	28	24	21	18	15	12	09	05	02	99	1360	
50	41.96	93	89	86	83	80	77	74	71	67	1370	
23. 0	64	61	58	55	52	49	45	42	39	36	1380	
10	33	30	27	24	20	17	14	11	08	05	1390	
20	02	99	96	92	89	86	83	80	77	74	1400	
30	40.71	68	65	62	59	55	52	49	46	43	1410	
40	40	37	34	31	28	25	22	19	16	13	1420	
50	10	07	04	01	98	95	91	88	85	82	1430	

\*58 Street's Logist. Logar.  $\frac{0}{11}$ , 24, 25, 26, 27, 28, 29, 30, 31.

"	0	1	2	3	4	5	6	7	8	9	"
24.0	39.79	76	73	70	67	64	61	58	55	52	1440
10	49	46	43	40	37	34	31	28	25	22	1450
20	19	17	14	11	08	05	02	99	96	93	1460
30	38.90	87	84	81	78	75	72	69	66	63	1470
40	60	57	54	51	49	46	43	40	37	34	1480
50	31	28	25	22	20	17	14	11	08	05	1490
25.0	03	99	96	93	91	88	85	82	79	76	1500
10	37.73	70	68	65	62	59	56	53	50	47	1510
20	45	42	39	36	33	30	27	25	22	19	1520
30	16	13	10	08	05	02	99	96	93	91	1530
40	36.88	85	82	79	77	74	71	68	65	63	1540
50	60	57	54	51	49	46	43	40	37	35	1550
26.0	32	29	26	23	21	18	15	12	10	07	1560
10	04	01	98	96	93	90	87	85	82	79	1570
20	35.76	74	71	68	65	63	60	57	55	52	1580
30	49	46	44	41	38	35	33	30	27	25	1590
40	22	19	16	14	11	08	06	03	00	97	1600
50	34.95	92	89	87	84	81	79	76	73	71	1610
27.0	68	65	63	60	57	54	52	49	46	44	1620
10	41	38	36	33	31	28	25	23	20	17	1630
20	15	12	09	07	04	01	99	96	93	91	1640
30	33.88	86	83	80	77	73	72	70	67	65	1650
40	62	59	57	53	51	49	46	44	41	38	1660
50	36	33	31	28	25	23	20	18	15	13	1670
28.0	10	07	05	02	00	97	94	92	89	87	1680
10	32.84	82	79	76	74	71	69	66	64	61	1690
20	59	56	53	51	48	46	43	41	38	36	1700
30	33	31	28	25	23	20	18	15	13	10	1710
40	08	05	03	00	98	95	93	90	88	85	1720
50	31.83	80	78	75	73	70	68	65	63	60	1730
29.0	58	55	53	50	48	45	43	40	38	35	1740
10	33	30	28	25	23	20	18	15	13	10	1750
20	08	05	03	01	98	96	93	91	88	86	1760
30	83	81	78	76	73	71	69	66	64	61	1770
40	59	56	54	52	49	47	44	42	39	37	1780
50	34	32	30	27	25	22	20	18	15	13	1790
30.0	10	08	05	03	01	98	96	93	91	89	1800
10	29.86	84	81	79	77	74	72	69	67	65	1810
20	62	60	58	55	53	50	48	46	43	41	1820
30	39	36	34	31	29	27	24	22	20	17	1830
40	15	12	10	08	05	03	01	98	96	94	1840
50	28.91	89	87	84	82	80	77	75	73	70	1850
31.0	68	65	63	61	59	56	54	52	49	47	1860
10	45	42	40	38	35	33	31	28	26	24	1870
20	21	19	17	15	12	10	08	05	03	01	1880
30	98	96	94	92	89	87	85	82	80	78	1890
40	75	73	71	69	66	64	62	60	57	55	1900
50	53	50	48	46	44	41	39	37	35	32	1910



Street's Logist. Logar.  $\frac{0}{11}$ , 32, 33, 34, 35, 36, 37, 38, 39. \*59

I	II	0	1	2	3	4	5	6	7	8	9	II
32.0	27.30	28	25	23	21	19	16	14	12	10		1920
10	07	05	03	01	98	96	94	92	89	87		1930
20	26.85	83	81	78	76	74	72	69	67	65		1940
30	63	60	58	56	54	52	49	47	45	43		1950
40	40	38	36	34	32	29	27	25	23	21		1960
50	18	16	14	12	10	07	05	03	01	99		1970
33.0	25.96	94	92	90	88	85	83	81	79	77		1980
10	74	72	70	68	66	44	41	39	37	35		1990
20	53	51	48	46	44	42	40	38	35	33		2000
30	31	29	27	25	22	20	18	16	14	12		2010
40	10	07	05	03	01	99	97	94	92	90		2020
50	24.88	86	84	82	80	77	75	73	71	69		2030
34.0	67	65	62	60	58	56	54	52	50	48		2040
10	45	43	41	39	37	35	33	31	29	26		2050
20	24	22	20	18	16	14	12	10	08	05		2060
30	03	01	99	97	95	93	91	89	87	84		2070
40	23.82	80	78	76	74	72	70	68	66	64		2080
50	61	59	57	55	53	51	49	47	45	43		2090
35.0	41	39	37	35	33	31	28	26	24	22		2100
10	20	18	16	14	12	10	08	06	04	02		2110
20	00	98	96	94	91	89	87	85	83	81		2120
30	22.79	77	75	73	70	69	67	65	63	61		2130
40	59	57	55	53	51	49	47	45	43	41		2140
50	39	37	35	33	31	29	27	25	23	20		2150
36.0	18	16	14	12	10	08	05	04	02	00		2160
10	21.98	96	94	92	90	88	86	84	82	80		2170
20	78	76	74	72	70	69	67	65	63	61		2180
30	59	57	55	53	51	49	47	45	43	41		2190
40	39	37	35	33	31	29	27	25	23	21		2200
50	19	17	15	13	11	09	07	05	03	01		2210
37.0	20.99	98	96	94	92	90	88	86	84	82		2220
10	80	78	76	74	72	70	68	66	64	62		2230
20	60	59	57	55	53	51	49	47	45	43		2240
30	41	39	37	35	33	32	30	29	26	24		2250
40	22	20	18	16	14	12	10	09	07	05		2260
50	03	01	99	97	95	93	91	89	87	86		2270
38.0	19.84	82	80	78	76	74	72	70	68	67		2280
10	65	63	61	59	57	55	53	51	50	48		2290
20	46	44	42	40	38	36	34	33	31	29		2300
30	27	25	23	21	19	18	16	14	12	10		2310
40	08	06	04	03	01	99	97	95	93	91		2320
50	89	88	86	84	82	80	78	76	75	73		2330
39.0	71	69	67	65	63	62	60	58	56	54		2340
10	52	50	49	47	45	43	41	39	38	36		2350
20	34	32	30	28	27	25	23	21	19	17		2360
30	16	14	12	10	08	06	05	03	01	99		2370
40	97	95	94	92	90	88	86	85	83	81		2380
50	79	77	75	74	72	70	68	66	65	63		2390

\*60 Street's Logist. Logar.  $\frac{0}{11}$ , 40, 41, 42, 43, 44, 45, 46, 47.

'	11	0	1	2	3	4	5	6	7	8	9	11
40.0		17.61	59	57	55	54	52	50	48	46	45	2400
10		43	41	39	37	36	34	32	30	28	27	2410
20		25	23	21	19	18	16	14	12	11	09	2420
30		07	05	03	02	00	98	96	94	92	91	2430
40		16.89	87	86	84	82	80	78	77	75	73	2440
50		71	70	68	66	64	62	61	59	57	55	2450
41.0		54	52	50	48	47	45	43	41	40	38	2460
10		36	34	33	31	29	27	26	24	22	20	2470
20		19	17	15	13	12	10	08	06	05	03	2480
30		01	99	98	96	94	92	91	88	87	85	2490
40		15.84	82	80	78	77	75	73	71	70	68	2500
50		66	65	63	61	59	58	56	54	52	51	2510
42.0		49	47	46	44	42	40	39	37	35	34	2520
10		32	30	28	27	25	23	22	20	18	16	2530
20		15	13	11	10	08	06	04	03	01	99	2540
30		14.98	96	94	93	91	89	87	86	84	82	2550
40		81	79	77	76	74	72	70	69	67	65	2560
50		64	62	60	59	57	55	54	52	50	49	2570
43.0		47	45	43	42	40	38	37	35	33	32	2580
10		30	28	27	25	23	22	20	18	17	15	2590
20		13	12	10	08	07	05	03	02	00	98	2600
30		13.97	95	93	92	90	88	87	85	83	82	2610
40		80	78	77	75	73	72	70	68	67	65	2620
50		63	62	60	59	57	55	54	52	50	49	2630
44.0		47	45	44	42	40	39	37	35	34	32	2640
10		31	29	27	26	24	22	21	19	17	16	2650
20		14	13	11	09	08	06	04	03	01	00	2660
30		12.98	96	95	94	91	90	88	87	85	83	2670
40		82	80	78	77	75	74	72	70	69	68	2680
50		66	64	62	61	59	57	56	54	53	51	2690
45.0		49	48	46	45	43	41	40	38	37	35	2700
10		33	32	30	29	27	25	24	22	21	19	2710
20		17	16	14	13	11	09	08	06	05	03	2720
30		01	00	98	97	95	94	92	90	89	87	2730
40		11.86	84	82	81	79	78	76	74	73	71	2740
50		70	68	67	65	63	62	60	59	57	56	2750
46.0		54	52	51	49	48	46	45	43	41	40	2760
10		38	37	35	34	32	30	29	27	26	24	2770
20		23	21	19	18	16	15	13	12	10	09	2780
30		07	05	04	02	01	99	98	96	95	93	2790
40		10.91	90	88	87	85	84	82	81	79	78	2800
50		76	74	73	71	70	68	67	65	64	62	2810
47.0		61	59	57	56	54	53	51	50	48	47	2820
10		45	44	42	41	39	37	36	34	33	31	2830
20		30	28	27	25	24	22	21	19	18	16	2840
30		15	13	12	10	08	07	05	04	02	01	2850
40		9.99	98	96	95	93	92	90	89	87	86	2860
50		84	83	81	80	78	77	75	74	72	71	2870

Street's Logist. Logar.  $\frac{0}{11}$ , 48, 49, 50, 51, 52, 53, 54, 55. \*61

	I	II	O	I	2	3	4	5	6	7	8	9	II
400	48.0		9.69	68	66	65	63	62	60	59	57	56	2880
410		10	54	53	51	50	48	47	45	44	42	41	2890
420		20	39	38	36	35	33	32	30	29	27	26	2900
430		30	24	23	21	20	18	17	15	14	12	11	2910
440		40	09	08	06	05	03	02	00	99	97	96	2920
450		50	8.94	93	91	90	88	87	85	84	83	82	2930
460	49.0		80	78	77	75	74	72	71	69	68	66	2940
470		10	65	63	62	60	59	57	56	55	54	53	2950
480		20	50	49	47	46	44	43	41	40	38	37	2960
490		30	35	34	33	31	30	28	27	25	24	22	2970
500		40	21	19	18	16	15	14	12	11	09	08	2980
510		50	06	05	03	02	01	99	98	96	95	93	2990
520	50.0		7.92	90	89	87	86	85	83	82	80	79	3000
530		10	77	76	74	73	72	70	69	67	66	64	3010
540		20	63	62	60	59	57	56	54	53	51	50	3020
550		30	49	47	46	44	43	40	41	39	37	36	3030
560		40	34	33	32	30	29	27	26	24	23	21	3040
570		50	20	19	17	16	14	13	11	10	09	07	3050
580	51.0		06	04	03	02	00	99	97	96	94	93	3060
590		10	6.92	90	89	87	86	85	83	82	80	79	3070
600		20	78	76	73	73	72	70	69	68	66	65	3080
610		30	63	62	61	59	58	56	55	54	52	51	3090
620		40	49	48	47	45	44	42	41	40	38	37	3100
630		50	35	34	33	31	30	28	27	26	24	23	3110
640	52.0		21	20	19	17	16	15	13	12	11	09	3120
650		10	08	06	05	03	02	01	99	98	96	95	3130
660		20	5.94	92	91	90	88	87	85	84	83	81	3140
670		30	80	79	77	76	74	73	72	70	69	68	3150
680		40	66	65	63	62	61	59	58	57	55	54	3160
690		50	52	51	50	48	47	46	44	43	41	40	3170
700	53.0		39	37	36	35	33	32	31	29	28	26	3180
710		10	25	24	22	21	20	18	17	16	14	13	3190
720		20	12	10	09	07	06	05	03	02	01	99	3200
730		30	4.98	96	95	94	93	91	90	89	87	86	3210
740		40	84	83	82	80	79	78	76	75	74	72	3220
750		50	71	70	68	67	66	64	63	62	60	59	3230
760	54.0		58	56	55	54	52	51	50	48	47	46	3240
770		10	44	43	42	40	39	38	36	35	34	32	3250
780		20	31	30	28	26	24	23	22	21	20	19	3260
790		30	18	16	15	14	12	11	10	08	07	06	3270
800		40	04	03	02	00	99	98	96	95	94	92	3280
810		50	3.91	90	88	87	86	84	83	82	81	79	3290
820	55.0		78	77	75	74	73	71	70	69	67	66	3300
830		10	65	63	62	61	59	58	57	56	54	53	3310
840		20	52	50	49	48	46	45	44	42	41	40	3320
850		30	39	37	36	35	33	32	31	29	28	27	3330
860		40	26	24	23	22	20	19	18	16	15	14	3340
870		50	13	11	10	09	07	06	05	04	03	01	3350



\*62 Street's Logist. Logar.  $\frac{1}{11}$ , 56, 57, 58, 59.

	0	1	2	3	4	5	6	7	8	9	
56.0	3.00	98	07	05	94	93	92	91	89	88	
10	2.87	85	84	83	82	80	79	78	76	75	3360
20	74	73	71	70	69	67	66	65	64	62	3370
30	61	60	58	57	56	55	53	52	51	50	3380
40	48	47	46	44	43	42	41	39	38	37	3390
50	35	34	33	32	30	29	28	27	25	24	3400
57.0	23	21	20	19	18	16	15	14	13	11	3410
10	10	09	08	06	05	04	02	01	00	99	3420
20	1.97	96	95	94	92	91	90	89	87	86	3430
30	85	84	82	81	80	79	77	76	75	74	3440
40	72	71	70	69	67	66	65	63	62	61	3450
50	60	58	57	56	55	53	52	51	50	48	3460
58.0	47	46	45	43	42	41	40	39	37	36	3470
10	35	34	32	31	30	29	27	26	25	24	3480
20	22	21	20	19	17	16	15	14	12	11	3490
30	10	09	07	06	05	04	03	01	00	99	3500
40	98	96	95	94	93	91	90	89	88	87	3510
50	85	84	83	82	80	79	78	77	75	74	3520
59.0	73	72	71	69	68	67	66	64	63	62	3530
10	61	60	58	57	56	55	53	52	51	50	3540
20	49	47	46	45	44	42	41	40	39	38	3550
30	36	35	34	33	31	30	29	28	27	25	3560
40	24	23	22	21	19	18	17	16	15	13	3570
50	12	11	10	8	7	6	5	4	2	1	3580
60.0	0	—	—	—	—	—	—	—	—	—	3590
											3600.





Fig: I.

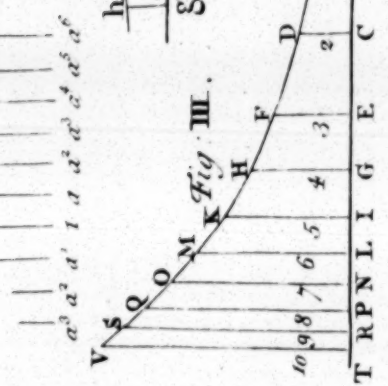


Fig: II.

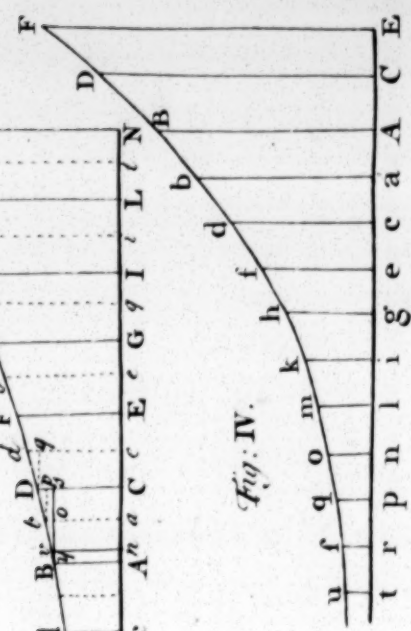


Fig: III.



Fig: IV.

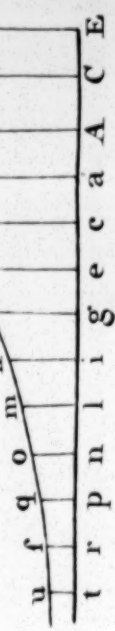


Fig: V.

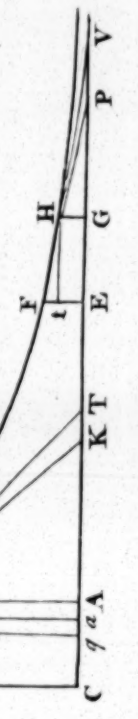


Fig: VI.



Fig: VII.



Fig: IX.



Fig: XI.

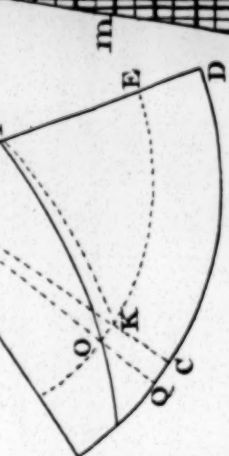
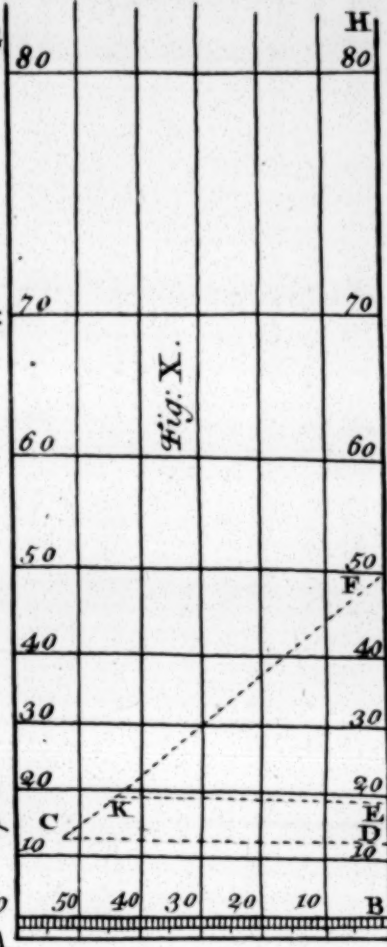


Fig: X.



XVII

XVIII

XIX

XX

XXI

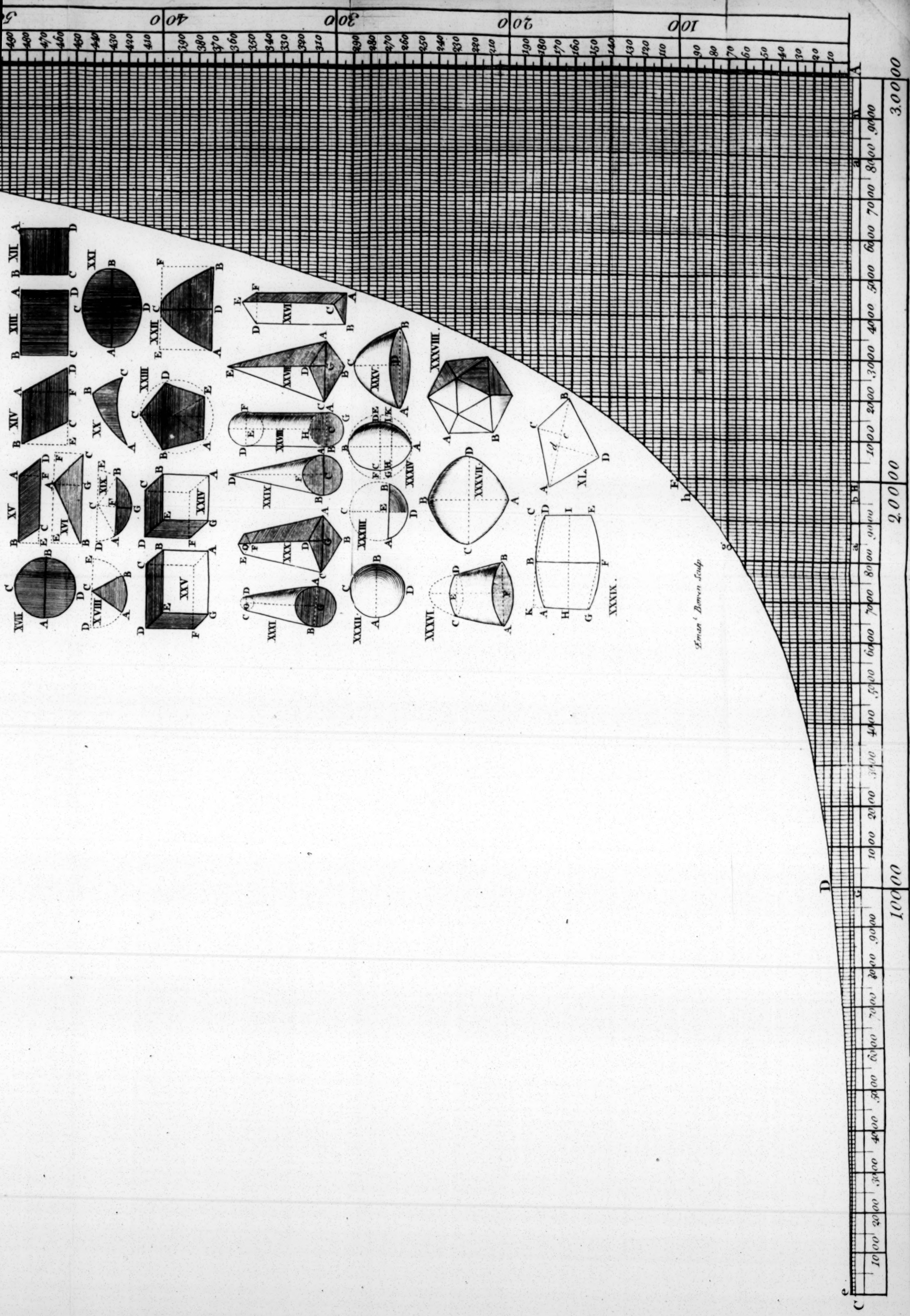
XXII

XXIII

XXIV

XXV





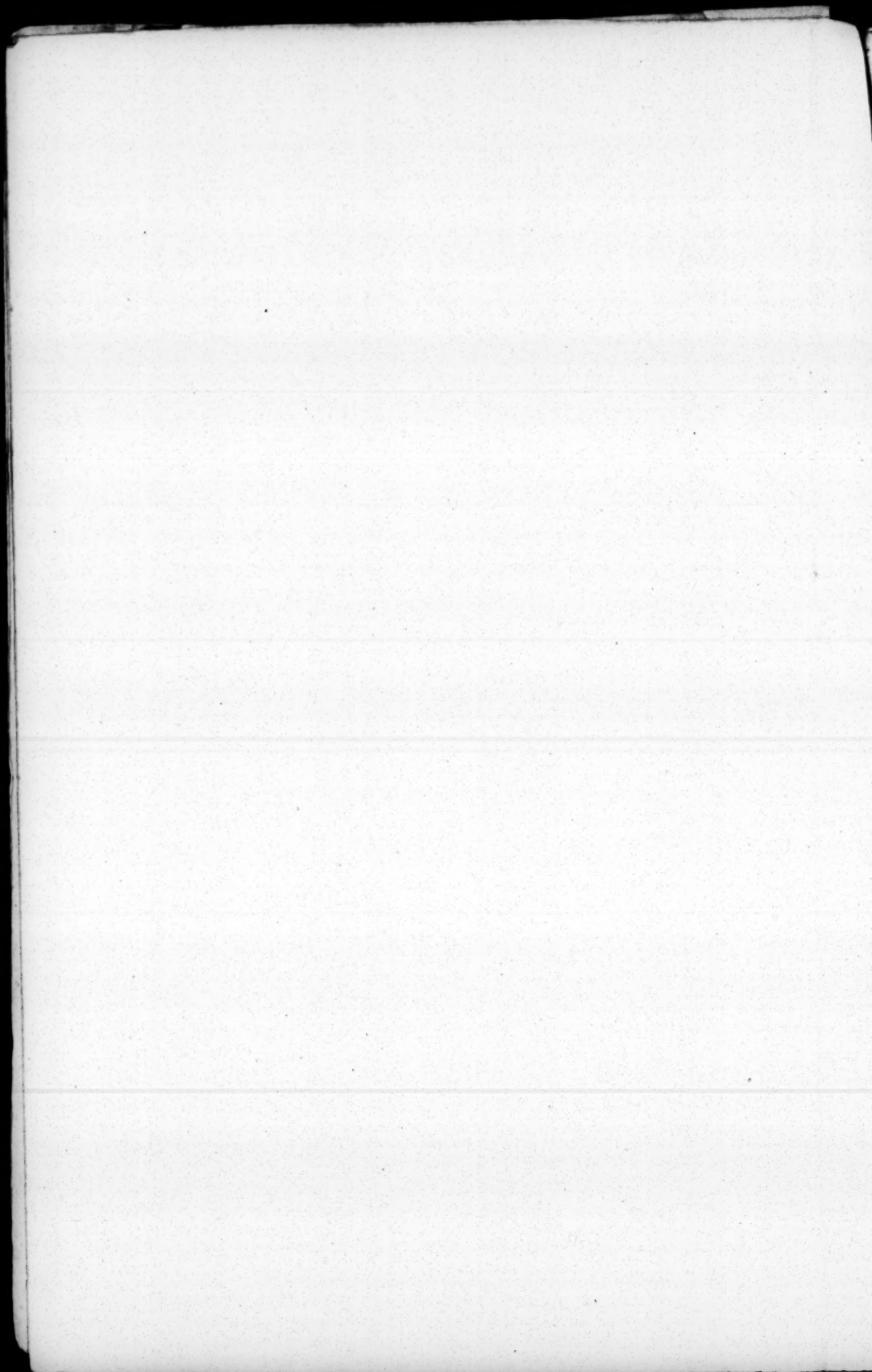
D

10000

20000

30000

Eman. & Bowen Sculp.





Hours	Deg. 1	M.	Deg. 1	M.	Deg. 1
I	2 30	1	0 3	31	1 18
II	5 00	2	0 5	32	1 20
III	7 30	3	0 8	33	1 23
IV	10 00	4	0 10	34	1 25
V	12 30	5	0 13	35	1 28
VI	15 00	6	0 15	36	1 30
VII	17 30	7	0 18	37	1 33
VIII	20 00	8	0 20	38	1 35
IX	22 30	9	0 23	39	1 38
X	25 00	10	0 25	40	1 40
XI	27 30	11	0 28	41	1 43
XII	30 00	12	0 30	42	1 45
XIII	32 30	13	0 33	43	1 48
XIV	35 00	14	0 35	44	1 50
XV	37 30	15	0 38	45	1 53
XVI	40 00	16	0 40	46	1 55
XVII	42 30	17	0 43	47	1 58
XVIII	45 00	18	0 45	48	2 00
XIX	47 30	19	0 48	49	2 03
XX	50 00	20	0 50	50	2 05
XXI	52 30	21	0 53	51	2 08
XXII	55 00	22	0 55	52	2 10
XXIII	57 30	23	0 58	53	2 13
XXIV	60 00	24	1 00	54	2 15
		25	1 03	55	2 18
		26	1 05	56	2 20
		27	1 08	57	2 23
		28	1 10	58	2 25
		29	1 13	59	2 28
		30	1 15	60	2 30



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